Show sufficient work and clearly mark your answers. Each problem is worth 10 points.

1. Eli owes Archie payments of 2000 in 1 year and 1000 in 2 years. Eli offers to make a single payment of 2610, immediately, claiming that the total present value of the 2 future payments is 2610. Determine the nominal interest rate compounded monthly that would make Eli’s claim true.

   \[ v = \text{monthly discount factor} \]

   \[ PV = 2610 = 2000 v^2 + 1000 v^{24} \]

   (A) 0.88%
   (B) 0.93%
   (C) 5.55%
   (D) 10.58%
   (E) 11.1%

2. At time 0, Peyton deposits an amount into an account that credits interest using a simple discount rate \( d \). There were no other deposits made into the account. At the end of year 3 there is 1000 in the account and at the end of year 12 there is 1500 in the account. Determine \( d \).

   \[ a(t) = (1 - dt)^{-1} \]

   (A) 3.33%
   (B) 3.57%
   (C) 3.71%
   (D) 3.92%
   (E) 4.07%
3. Betty deposits an amount at time 0 into a fund which credits interest using a simple interest rate \( i \). The force of interest in the account at time 10 is equal to 0.05.

Charlie deposits 1000 into a separate account in which interest is credited using a nominal interest rate of \( i \), compounded quarterly. Determine the amount in Charlie’s account at the end of 10 years.

\[
\text{(A) } 1645 \\
\text{(B) } 2685 \\
\text{(C) } 4065 \\
\text{(D) } 5830 \\
\text{(E) } 7040
\]

\[
R : a(t) = 1 + it \implies s_t = \frac{t}{1 + it} \\
10s = 0.10 \implies 0.10 = \frac{t}{1 + 10i} \implies i = 0.10
\]

\[
C : a(t) = \left(1 + \frac{i}{4}\right)^t \quad t = \# \text{ of quarters} \\
i = 0.10 \\
A \text{V} = 1000 \left(1 + \frac{0.10}{4}\right)^{40} = 2685
\]

4. Determine which of the following equations represents the correct relationship between a nominal interest rate compounded monthly and a nominal interest rate compounded quarterly.

\[
\text{(A) } i^{(4)} = 4 \left[ \left(1 + \frac{0.12}{12}\right)^4 - 1 \right] \\
\text{(B) } i^{(4)} = 4 \left[ \left(1 + \frac{0.12}{12}\right)^{12} + 1 \right] \\
\text{(C) } i^{(4)} = 4 \left[ \left(1 + \frac{0.12}{12}\right)^4 + 1 \right] \\
\text{(D) } i^{(4)} = 4 \left[ \left(1 + \frac{0.12}{12}\right)^3 - 1 \right] \\
\text{(E) } i^{(4)} = 4 \left[ \left(1 + \frac{0.12}{12}\right)^{12} - 1 \right]
\]
5. A fund credits interest using an interest rate of 10% compounded every other year for
the first four years, and a nominal discount rate of 12% compounded monthly
thereafter. Determine the accumulated value after 8 years of a deposit of 1000.

(A) 2320
(B) 2330
(C) 2340
(D) 2350
(E) 2360

\[ AV = 1000 \left( 1 + \frac{0.10}{2} \right)^2 \left( 1 - \frac{0.12}{12} \right)^{-48} \]

\[ = 1000 \left( 1 + 0.05 \right)^2 \left( 0.99^{-48} \right) \approx 2332.78 \]

6. At time 0, Jason deposits 500 into an account in which the force of interest is
\[ \delta = \frac{0.5t}{2+t^2}, \text{ for } t > 0 \]. At the end of year 4, Jason makes an additional deposit of \( X \)
to the account. The amount of interest earned between years 3 and 5 is 200.
Determine \( X \).

(A) 30
(B) 50
(C) 70
(D) 90
(E) 110

Since an additional deposit of \( X \) is
made at time 4, \[ I_{[3,5]} = AV_5 - AV_3 - X \]

\[ AV_5 = 500 a(5) + X a(5) = 500 \left( \frac{27}{3} \right)^{14} + X \left( \frac{27/2}{18/2} \right)^{14} \]

\[ AV_3 = 500 a(3) = 500 \left( \frac{15}{6} \right)^{14} \]

\[ I_{[3,5]} = 200 = 500 \left( \frac{13.5}{8} \right)^{14} + X \left( \frac{13/2}{15/2} \right)^{14} - 500 \left( \frac{5.5}{3} \right)^{14} - X \]

\[ \Rightarrow X = 68.32 \]
7. Determine \( \frac{d}{dd}(v) \), the derivative of \( v \) with respect to \( d \), where \( d \) denotes a periodic effective discount rate and \( v \) is the corresponding periodic discount factor.

(A) \(-v^{-2}\)
(B) \(-v^{-1}\)
(C) \(-1\)
(D) \(-v\)
(E) \(-v^2\)

\[ v = 1 - d \]

\[ \frac{d}{dd}(v) = -1 \]

8. The force of interest at time \( t \) for a certain account is \( \delta_t = 0.02t \), \( t > 0 \). Determine the corresponding annual effective discount rate for year 2 for this account.

(A) 2.96%
(B) 2.99%
(C) 3.02%
(D) 3.05%
(E) 3.08%

\[ d_2 = \frac{a(2) - a(1)}{a(2)} \]

\[ a(1) = e^{\int_0^{0.01} 0.02t \, dt} = e^{0.01^2_0} = e^{0.01} \]

\[ a(2) = e^{\int_0^{0.04} 0.02t \, dt} = e^{0.04^2_0} = e^{0.04} \]

\[ d_2 = \frac{e^{0.04} - e^{0.01}}{e^{0.04}} = 2.96\% \]
9. Willie deposits 500 into an account that credits interest using a simple discount rate \( d \) for the first year and then a semiannual effective discount rate of \( d \) thereafter. At the end of 2 years, the account balance is 1000. Determine \( d \).

\[
\begin{align*}
\text{(A) } & 0.10 \\
\text{(B) } & 0.15 \\
\text{(C) } & 0.20 \\
\text{(D) } & 0.25 \\
\text{(E) } & 0.30 \\
\end{align*}
\]

\[
\begin{align*}
1000 &= 500 (1-d)^{-1} \cdot (1-d^{-2}) = 500 (1-d)^{-3} \\
\Rightarrow d &= 0.2
\end{align*}
\]

10. Determine the constant force of interest that is equivalent to an interest rate of 10% compounded quarterly.

\[
\begin{align*}
\text{(A) } & 2.38\% \\
\text{(B) } & 2.47\% \\
\text{(C) } & 9.35\% \\
\text{(D) } & 9.53\% \\
\text{(E) } & 9.88\% \\
\end{align*}
\]

\[
\begin{align*}
\ln(1+i) &= i = ae^{\epsilon r} \\
\left(1 + \frac{i}{4}\right)^4 &= 1 + \epsilon \\
\Rightarrow s &= \ln(1+i) = \ln(1.025)^4 \\
&= 4 \ln(1.025) = 9.88\%
\end{align*}
\]