MAP 4170 Name:______________________________

Test 1 Date: September 19, 2013

Show sufficient work and clearly mark your answers. Each problem is worth 10 points.

1. Determine the force of interest at time $t=3$ for an account with accumulation function $a(t) = 0.08t^2 + 0.32t + 1$.

   (A) 0.15

   (B) 0.20

   (C) 0.25

   (D) 0.30

   (E) 0.35

$$a'(t) = 0.16t + 0.32$$

$$\therefore \delta_t = \frac{0.16t + 0.32}{.08t^2 + 0.32t + 1} \Rightarrow \delta_3 = \frac{0.8}{2.68}$$

$$\Rightarrow \delta_3 \doteq 0.29851$$

2. An account earns interest according to $\delta_t = 0.15\sqrt{t}$, where t is the number of years after July 1, 2012. On July 1, 2012, 100 is deposited into the account and on July 1, 2013, another 100 is deposited into the account. If there are no other deposits into the account, determine the amount in the account on July 1, 2016.

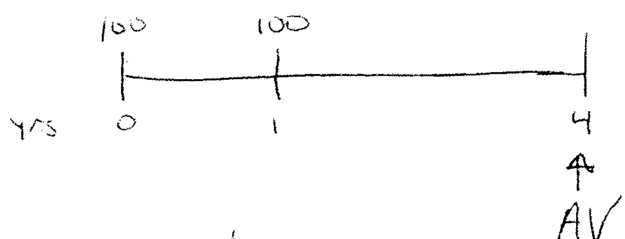
   (A) 325

   (B) 350

   (C) 375

   (D) 400

   (E) 425



$$a(t) = e^{\int_0^t 0.15 r^{1/2} dr} = e^{0.15 \cdot \frac{2}{3} r^{3/2} \Big|_0^t}$$

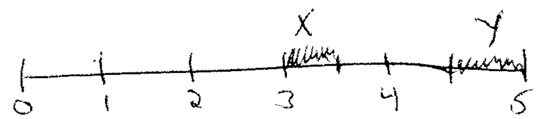
$$\therefore a(t) = e^{.1t^{3/2}}$$

$$a(1) = e^{.1} \qquad a(4) = e^{.8}$$

$$\therefore AV = 100 \cdot a(4) + 100\frac{a(4)}{a(1)} = 100e^{.8} + 100e^{.7}$$

$$\therefore AV \doteq 423.93$$

3. Account A credits interest using a 5% simple interest rate and Account B credits interest using a 5% simple discount rate. Define X to be the semiannual effective discount rate for Account A for the first half of the 4^{th} year and define Y to be the semiannual effective interest rate for Account B for the second half of the 5^{th} years. Determine the ratio $\frac{X}{Y}$.



(A) less than or equal to 0.65

(B) greater than 0.65 but less than or equal to 0.80

(C) greater than 0.80 but less than or equal to 0.95

(D) greater than 0.95 but less than or equal to 1.10

(E) greater than 1.10

A: $a(t) = 1 + .05t$

$a(3) = 1.15 \quad a(3.5) = 1.175$

$\therefore X = \frac{1.175 - 1.15}{1.175} = \frac{.025}{1.175}$

B: $a(t) = (1 - .05t)^{-1} \Rightarrow a(5) = (.75)^{-1}$ and $a(4.5) = (.775)^{-1}$

$\therefore Y = \frac{(.75)^{-1} - (.775)^{-1}}{(.775)^{-1}} = \frac{.025}{.75}$

$\therefore \frac{X}{Y} = \frac{.75}{1.175} \doteq 0.63830$

4. John is to receive payments of 10000 in 5 years and 20000 in 10 years. Using a discount rate of 6%, compounded semiannually, determine the present value of the payments.

(A) 18110

(B) 18250

(C) 18405

(D) 18515

(E) 18640

$d^{(2)} = .06 \Rightarrow d = \frac{.06}{2} = .03 = \text{sedr}$

$\therefore v = 0.97 = \text{sdf}$

$PV = 10000 v^{10} + 20000 v^{20}$

$\Rightarrow PV \doteq 18250.13$

5. Determine which of the following equations represents the correct relationship between a nominal interest rate compounded semiannually and its equivalent nominal discount rate compounded quarterly.

(A) $i^{(2)} = 2\left[\frac{1}{\sqrt{\left(1-\frac{d^{(4)}}{4}\right)}} - 1\right]$

(B) $i^{(2)} = 2\left[\frac{1}{\sqrt[4]{\left(1-\frac{d^{(4)}}{4}\right)}} - 1\right]$

(C) $i^{(2)} = 2\left[\frac{1}{\left(1-\frac{d^{(4)}}{4}\right)^2} - 1\right]$

(D) $i^{(2)} = 2\left[\frac{1}{\left(1-\frac{d^{(4)}}{4}\right)^4} - 1\right]$

(E) None of the above

$$\left(1+\frac{i^{(2)}}{2}\right)^2 = \left(1-\frac{d^{(4)}}{4}\right)^{-4}$$

$$\Rightarrow i^{(2)} = 2\left[\left(1-\frac{d^{(4)}}{4}\right)^{-2} - 1\right]$$

$$= 2\left[\frac{1}{\left(1-\frac{d^{(4)}}{4}\right)^2} - 1\right]$$

6. For the first 5 years, an account pays interest using an interest rate of i, compounded bi-annually. Thereafter interest is credited using 10% compounded continuously. If a deposit doubles over a 7 year period, determine i.

(A) 9%

(B) 10%

(C) 11%

(D) 12%

(E) 13%

$$i = i^{(1/2)} \Longrightarrow \frac{i}{(1/2)} = 2i = \text{baeir}$$

$$1 \longrightarrow (1+2i)^{2.5} \xrightarrow{\delta=0.1} (1+2i)^{2.5}\cdot e^{0.2}$$

yrs 0 — 5 (2.5 bi-annual periods) — 7, AV=2

$$\therefore (1+2i)^{2.5}\cdot e^{0.2} = 2$$

$$\Longrightarrow i \doteq 0.10903$$

7. Mary's account is credited interest using an interest rate of $2i$, compounded semiannually. Ellen's account is credited interest using an interest rate of i, compounded annually. Mary and Ellen each deposit X into their respective accounts. At the end of n years, Mary has twice as much in her account than does Ellen have in her account, and the sum of the amounts in their accounts is 6000. Determine X.

(A) 1000

(B) 1025

(C) 1050

(D) 1075

(E) 1100

M: $i^{(2)} = 2i \implies \frac{i^{(2)}}{2} = i = $ seir

$X \longrightarrow X(1+i)^{2n}$

yrs 0 … n

E: $i = $ aeir

$X \longrightarrow X(1+i)^{n}$

yrs 0 … n

$\cancel{X}(1+i)^{2n} = 2\cdot \cancel{X}(1+i)^n$

$\implies (1+i)^n = 2$

$\therefore (1+i)^{2n} = 4$

$$6000 = X(1+i)^{2n} + X(1+i)^n = 4X + 2X$$

$$\implies X = 1000$$

8. On January 1, 2002, Mike opened an account with an initial deposit that is credited with interest using a 6% simple interest rate. On July 1, 2010, the account balance was 943.75. Determine the account balance on January 1, 2008.

(A) 815

(B) 827

(C) 836

(D) 844

(E) 850

$i = .06$ simple $\qquad a(t) = 1 + .06t$

$X \longleftarrow 943.75$

0 (1/1/02) … 6 (1/1/08) … 8.5 (7/1/10)

$$X = 943.75\,\frac{a(6)}{a(8.5)}$$

$a(6) = 1.36$

$a(8.5) = 1.51$

$$\therefore X = 943.75\,\frac{1.36}{1.51} \doteq 850$$

9. An account credits interest using $\delta_t = \frac{t}{2+t^2}$ for $t > 0$. An initial deposit grows to 20400 after 24 years. Determine the amount in the account 4 years after the initial deposit.

(A) 3400

(B) 3600

(C) 3800

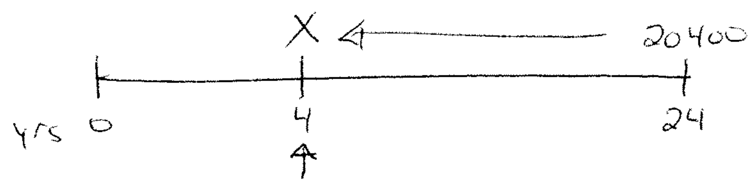
(D) 4000

(E) 4200

$$\delta_t = \frac{t}{2+t^2} = \frac{1}{2} \cdot \frac{2t}{2+t^2} \qquad \begin{cases} f(t) = 2+t^2 \\ c = 1/2 \end{cases}$$

(Special Case)

$$\therefore a(t) = \left(\frac{2+t^2}{2}\right)^{1/2} = \sqrt{1 + t^2/2}$$



$$X = 20400 \frac{a(4)}{a(24)} \qquad \begin{matrix} a(4) = 3 \\ a(24) = 17 \end{matrix}$$

$$\therefore X = 20400 \frac{3}{17} = 3600$$

10. Payments of 20000 in two years and 10000 in four years have a total present value of 22761 when determined using a discount rate of d compounded semiannually. Determine d.

(A) 5.1%

(B) 5.4%

(C) 5.6%

(D) 10.3%

(E) 10.8%

$$d = d^{(2)} \Rightarrow \frac{d}{2} = \text{sedr}$$

$$v = 1 - \frac{d}{2} = \text{sdf}$$

$$22761 = 20000 v^4 + 10000 v^8$$

quadratic in v^4 $\qquad \begin{matrix} a = 10000 \\ b = 20000 \\ c = -22761 \end{matrix}$

$$\therefore v^4 = \frac{\overset{(-b)}{-20000} \pm \sqrt{b^2 - 4ac}}{2a} = 0.81$$

$$\therefore \left(1 - \frac{d}{2}\right)^4 = 0.81 \Rightarrow d = 0.10263$$