MAP 4170 Name:______________________

Test 1 Date: September 18, 2014

Show sufficient work and clearly mark your answers. Each problem is worth 10 points.

1. An account pays 5% simple interest for the first two years, 5% compounded semiannually for the next two years, 5% compounded biannually for the next two years, and 5% compounded continuously for the next two years. Determine the equivalent annual effective interest rate over the 8-year period.

   (A) 4.98%

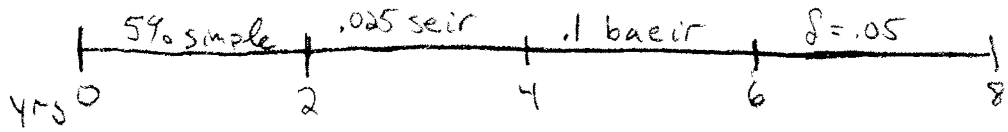
   (B) 4.99%

   (C) 5.00%

   (D) 5.01%

   (E) 5.02%

$i = aeir$



$$\therefore (1+i)^8 = (1+.05(2))(1.025)^4(1.1)^1 e^{.05(2)}$$

$$\Longrightarrow i \doteq 4.99\%$$

2. Jane is to receive payments of X in 2 years and $2X$ in 5 years. Using an interest rate of 8%, compounded monthly, the present value of the payments is 2960. Determine X.

   (A) 1250

   (B) 1300

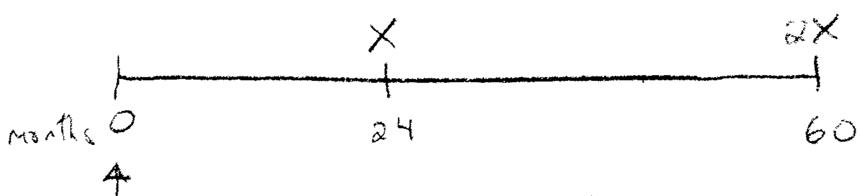
   (C) 1350

   (D) 1400

   (E) 1450

$$i^{(12)} = .08 \Longrightarrow \frac{.08}{12} = meir$$

$$\Longrightarrow v = \left(1+\frac{.08}{12}\right)^{-1} = mdf$$



$$2960 = Xv^{24} + 2Xv^{60} = X(v^{24} + 2v^{60})$$

$$\Longrightarrow X \doteq 1348.50$$

3. An account credits interest using a simple interest rate of i for the first half of year 1, and a nominal interest rate of i compounded semiannually thereafter. A deposit at the beginning of year 1 doubles after 5 years. Determine i.

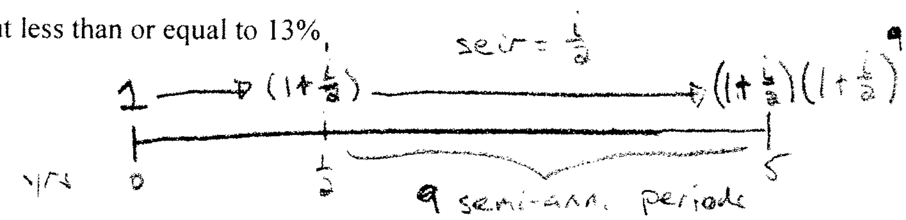
   (A) less than or equal to 7%

   (B) greater than 7% but less than or equal to 9%

   (C) greater than 9% but less than or equal to 11%

   (D) greater than 11% but less than or equal to 13%

   (E) greater than 13%



$$\therefore \left(1+\frac{i}{2}\right)^{10} = 2 \implies i \doteq 14.35\%$$

4. David can receive one of the following two payment streams:

   (i) 100 at time 0, 500 at time n, and 800 at time $2n$
   (ii) 915 at time 10

   At an annual effective discount rate of d, the present values of the two streams are equal.

   Given $v^n = 0.392375$, where v is the annual discount factor, determine d.

   (A) 7.51%

   (B) 7.66%

   (C) 7.81%

   (D) 7.96%

   (E) 8.11%

$$100 + 500v^n + 800v^{2n} = 915v^{10} = 915(1-d)^{10}$$

$$v^{2n} = (.392375)^2$$

$$\implies d \doteq .0751$$

5. Given a simple discount rate of 6%, determine the ratio $\frac{d_2}{i_3}$ where d_2 is the annual effective discount rate for the second year, and i_3 is the annual effective interest rate for the third year.

(A) $\frac{82}{94}$

(B) $\frac{82}{88}$

(C) $\frac{94}{82}$

(D) $\frac{94}{88}$

(E) None of the above

$$a(t) = (1 - .06t)^{-1}$$

$$d_2 = \frac{a(2) - a(1)}{a(2)}, \quad i_3 = \frac{a(3) - a(2)}{a(2)} \quad \therefore \frac{d_2}{i_3} = \frac{a(2) - a(1)}{a(3) - a(2)}$$

$$a(1) = .94^{-1} \quad a(2) = .88^{-1} \quad a(3) = .82^{-1}$$

$$\therefore \frac{d_2}{i_3} = \frac{\frac{1}{.88} - \frac{1}{.94}}{\frac{1}{.82} - \frac{1}{.88}} = \frac{\frac{.06}{(.88)(.94)}}{\frac{.06}{(.82)(.88)}} = \frac{82}{94}$$

6. Karen deposits X into an account that pays interest using an interest rate of i, annual effective, at the same time that Catherine deposits $2X$ into an account that pays interest using a simple interest rate of i. At the end of 1 year, Catherine has 55 more than Karen. At the end of 3 years, Karen has 66.55. Determine the amount in Catherine's account at the end of 3 years.

(A) 115

(B) 120

(C) 125

(D) 130

(E) 135

After 1 year: Karen has $X(1+i)$

Catherine has $2X(1+i)$

$$\therefore \underbrace{2X(1+i) - X(1+i)}_{= X(1+i)} = 55 \quad \therefore X(1+i) = 55$$

After 3 years Karen has $X(1+i)^3 = 66.55$

$$= \underbrace{X(1+i)}_{=55}(1+i)^2$$

$$\therefore 55(1+i)^2 = 66.55$$

$$\Rightarrow i = .1 \Rightarrow X = 50$$

After 3 years Catherine has $2X(1+3i)$

$$= 100(1+3(.1)) = 130$$

7. An account earns interest according to $\delta_t = \frac{Ct}{t^2+9}$, where t is the number of years after January 1, 2010. An initial amount of 3000 is deposited into the account on January 1, 2010. On January 1, 2014, the initial deposit had accumulated to 5000. Determine C.

(A) 1/2

(B) 1

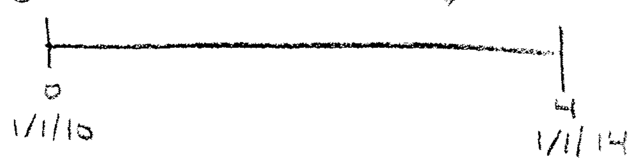
(C) 2

(D) 3

(E) 4

$$\delta_t = \frac{c}{2} \cdot \frac{2t}{t^2+9} \xrightarrow[f(t)=t^2+9]{\text{special case}} a(t) = \left(\frac{t^2+9}{9}\right)^{c/2}$$

$$3000 \longrightarrow 3000\,a(4) = 3000\left(\frac{25}{9}\right)^{c/2}$$
$$= 5000$$



$$3000\left(\frac{25}{9}\right)^{c/2} = 5000 \implies c = 1$$

8. An account earns interest according to $\delta_t = 0.01t^{1.5}$, where t is the number of years after July 1, 2014. A deposit of X on January 1, 2016 accumulates to 10000 on July 1, 2018. Determine X.

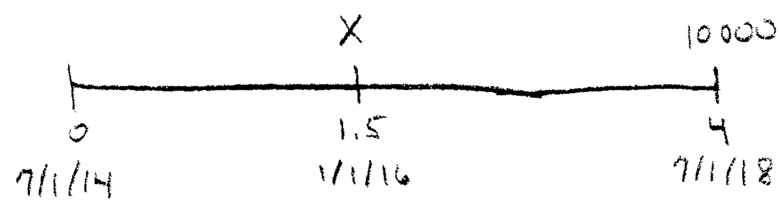
(A) 8800

(B) 8825

(C) 8850

(D) 8875

(E) 8900



$$X = 10000\, e^{\int_4^{1.5} .01 t^{1.5}\, dt}$$

$$= 10000\, e^{\frac{.01}{2.5} t^{2.5} \Big|_4^{1.5}} = 10000\, e^{.004\left((1.5)^{2.5} - 4^{2.5}\right)}$$

$$\doteq 8896.05$$

9. Using a nominal discount rate, $d^{(2)}$, the present value of payments of 400 at the end of two years and 600 at the end of four years is 908.20. Determine $d^{(2)}$.

(A) 1.50%

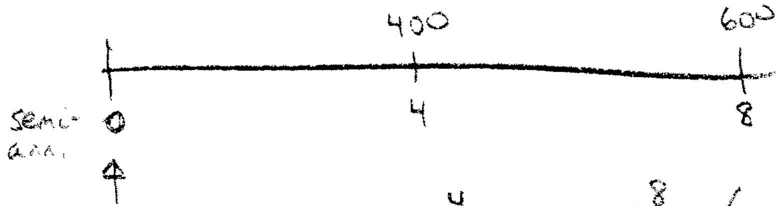
(B) 1.52%

(C) 3.00%

(D) 3.05%

(E) 6.00%

$d = \frac{d^{(2)}}{2} = \text{sedr}$ $\qquad v = 1-d = \text{sdf}$



$908.20 = 400v^4 + 600v^8$ (quadratic in v^4)

$a=600$
$b=400$
$c=-908.20$

$\Rightarrow v^4 \doteq .9413 \; (= (1-d)^4)$

$\Rightarrow d \doteq .015 \Rightarrow d^{(2)} \doteq .03$

10. Greg makes a deposit into an account that pays 5% simple interest. Five years after the deposit, Greg has 1750 in the account. Determine the amount that Greg has in the account 10 years after the deposit.

(A) 2025

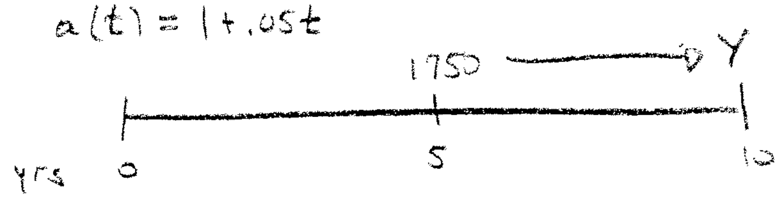
(B) 2050

(C) 2075

(D) 2100

(E) 2125

$a(t) = 1 + .05t$



$$Y = 1750 \cdot \frac{a(10)}{a(5)} = 1750 \, \frac{1.5}{1.25}$$

$$\Rightarrow Y = 2100$$