MAP 4170 Name:____________________
Test 1 Date: February 4, 2014

Show sufficient work and clearly mark your answers. Each problem is worth 10 points.

1. On January 1, 2012, Kevin deposits 15,000 into an account that credits interest using an interest rate of i, compounded monthly ($i > 0$). On January 1, 2013, Kevin deposits another 25,000 into the account. On January 1, 2014, the two deposits have accumulated to a total of 45,670. Determine i.

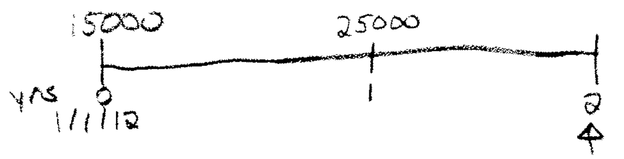
(A) 8.14%

(B) 8.75%

(C) 9.13%

(D) 9.42%

(E) 9.60%



$i = i^{(12)}$

$AV = 45670$

$$45670 = 15000\left(1+\frac{i}{12}\right)^{24} + 25000\left(1+\frac{i}{12}\right)^{12}$$

(quadratic in $\left(1+\frac{i}{12}\right)^{12}$) $a = 15000$, $b = 25000$, $c = -45670$

$$\Rightarrow \left(1+\frac{i}{12}\right)^{12} \doteq 1.1003\ldots \Rightarrow i = i^{(12)} \doteq .096$$

2. John deposits 1000 into an account. After $4n$ years the value of the deposit is 16,000. David deposits X into another account. After $3n$ years the value of David's deposit is 24,000. Both accounts earn the same nominal discount rate, compounded monthly. Determine X.

(A) 2950

(B) 3000

(C) 3050

(D) 3100

(E) 3150

Since compounding, we can use v-notation

Let $v = adf$. Then $v^{-1} = aaf$.

$$\therefore \Big[\overbrace{16000 = 1000\, v^{-4n}}^{\text{John's account}}\Big]^{1/4} \Rightarrow v^{-n} = 2$$

David's account

$$24000 = X v^{-3n} = X \cdot (v^{-n})^3 = 8X$$

$$\therefore \quad X = 3000$$

3. Determine which of the following equations represents the correct relationship between a nominal discount rate compounded semiannually and its equivalent nominal interest rate compounded quarterly.

(A) $d^{(2)} = -2\left[\dfrac{1}{\sqrt{\left(1+\frac{i^{(4)}}{4}\right)}} - 1\right]$

(B) $d^{(2)} = -2\left[\dfrac{1}{\sqrt[4]{\left(1+\frac{i^{(4)}}{4}\right)}} - 1\right]$

(C) $d^{(2)} = -2\left[\dfrac{1}{\left(1+\frac{i^{(4)}}{4}\right)^2} - 1\right]$

(D) $d^{(2)} = -2\left[\dfrac{1}{\left(1+\frac{i^{(4)}}{4}\right)^4} - 1\right]$

(E) None of the above

$$\left[\left(1-\frac{d^{(2)}}{2}\right)^{-2} = \left(1+\frac{i^{(4)}}{4}\right)^4\right]^{-1/2}$$

$$1-\frac{d^{(2)}}{2} = \left(1+\frac{i^{(4)}}{4}\right)^{-2}$$

$$\Rightarrow d^{(2)} = 2\left[1-\left(1+\frac{i^{(4)}}{4}\right)^{-2}\right]$$

$$= -2\left[\frac{1}{\left(1+\frac{i^{(4)}}{4}\right)^2} - 1\right]$$

4. Determine $\frac{d}{dv}(d)$, where d is the periodic effective discount rate that corresponds to the periodic discount factor, v.

(A) v

(B) $-v$

(C) 1

(D) -1

(E) None of the above

$$v = 1-d$$

$$\Rightarrow d = 1-v$$

$$\Rightarrow d' = -1$$

5. An initial deposit of X is made into an account that credits interest using a simple interest rate of 5% for the first three years and a constant force of interest equal to 4% thereafter. The accumulated value of the initial deposit after seven years is 11,735. Determine the accumulated value of the initial deposit after two years.

(A) 9565

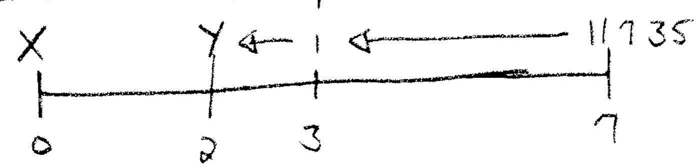
(B) 9575

(C) 9585

(D) 9595

(E) 9605

$i = .05$ simple | $\delta = .04$



$a(t) = 1 + .05t$

$$Y = 11735\, e^{-.04(4)} \frac{a(2)}{a(3)}$$

$$= 11735\, e^{-.16} \frac{1.1}{1.15} \doteq 9565$$

6. Account A credits interest using a simple discount rate, d. Account B credits interest using a nominal discount rate of 8%, compounded quarterly. The forces of interest in the two accounts are equal after 1 year. If 1000 is deposited into Account A at time 0, determine the accumulated value of the deposit after 4 years.

(A) 1400

(B) 1425

(C) 1450

(D) 1475

(E) 1500

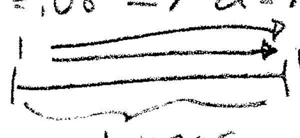
A: $a(t) = (1-dt)^{-1} \Rightarrow a'(t) = -(1-dt)^{-2}(-d)$

$$\Rightarrow \delta_t^A = \frac{(1-dt)^{-2} \cdot d}{(1-dt)^{-1}} = \frac{d}{1-dt}$$

$$\delta_1^A = \frac{d}{1-d}$$

B: Compounding $\Rightarrow \delta_t = \delta$ (constant)

$d^{(4)} = .08 \Rightarrow d = .02 =$ sedr

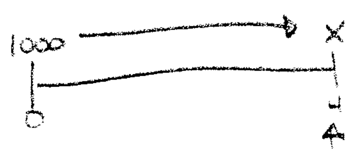


$(1-.02)^{-4} = (.98)^{-4}$ $\Rightarrow e^{\delta} = (.98)^{-4}$

$\Rightarrow \delta = -4\ln(.98)$

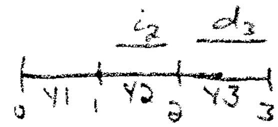
$$\therefore \frac{d}{1-d} = -4\ln(.98) \Rightarrow d = \frac{-4\ln(.98)}{1-4\ln(.98)} \doteq .0747\ldots$$

A:



$$X = 1000(1-4d)^{-1} \doteq 1426.69$$

7. An account credits interest using $\delta_t = 0.15\sqrt{t}$ for $t > 0$. Determine $\frac{i_2}{d_3}$



(A) 0.85

(B) 0.90

(C) 0.95

(D) 1.00

(E) 1.05

$$i_2 = \frac{a(2) - a(1)}{a(1)} \qquad d_3 = \frac{a(3) - a(2)}{a(3)}$$

$$a(n) = e^{\int_0^n .15 t^{1/2} dt} = e^{.1 t^{3/2} \big|_0^n} = e^{.1 n^{3/2}}$$

$$a(1) = e^{.1} \doteq 1.105\ldots \boxed{1}$$

$$a(2) = e^{.1(2)^{3/2}} \doteq 1.326\ldots \boxed{2}$$

$$a(3) = e^{.1(3)^{3/2}} \doteq 1.681\ldots \boxed{3}$$

$$i_2 = \frac{\boxed{2} - \boxed{1}}{\boxed{1}} \doteq 0.2006\ldots \boxed{4} \qquad d_3 = \frac{\boxed{3} - \boxed{2}}{\boxed{3}} \doteq 0.2108\ldots \boxed{5}$$

$$\therefore \quad \frac{i_2}{d_3} \doteq 0.95$$

8. An account credits interest using $\delta_t = \frac{t}{4+2t^2}$ where t is the number of years after January 1, 2013. Determine the value on July 1, 2016, of a deposit of 1000 made on January 1, 2014.

(A) 1375

(B) 1400

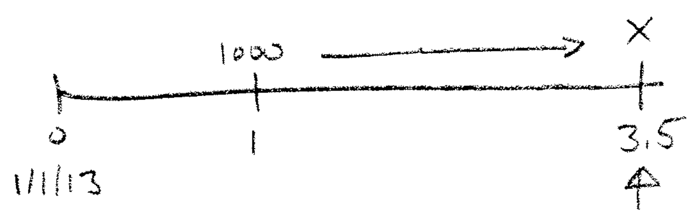
(C) 1425

(D) 1450

(E) 1475

$$\delta_t = \frac{1}{4} \cdot \frac{4t}{4+2t^2} \implies a(t) = \left[\frac{4+2t^2}{4}\right]^{1/4}$$

$$\therefore \quad a(t) = (1 + .5t^2)^{.25}$$



$$X = 1000 \, \frac{a(3.5)}{a(1)} = 1000 \left(\frac{7.125}{1.5}\right)^{1/4}$$

$$\doteq 1476.30$$

9. Chris owes Chuck two payments: a payment of 2000 on January 1, 2016 and another payment of 3000 on October 1, 2017. Chris agrees to make a single payment on January 1, 2015, in exchange for making the two later payments. Determine the amount of the single payment assuming a quarterly effective discount rate of 2%.

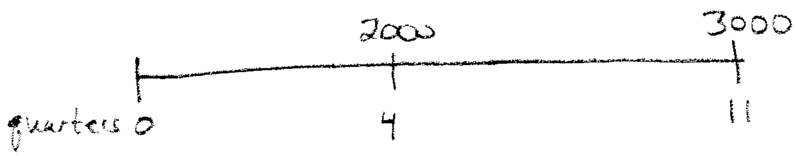
(A) 4250

(B) 4260

(C) 4270

(D) 4280

(E) 4290



1/1/15

$d = .02$ qedr $\Rightarrow v = .98 =$ qdf

$$PV = 2000\,v^4 + 3000\,v^{11}$$

$$= 2000(.98)^4 + 3000(.98)^{11}$$

$$\doteq 4246.93$$

10. An account credits interest using a simple interest rate of *i*. An initial deposit grows to 11,500 after 3 years. The value of the initial deposit after 8 years is 14,000. Determine *i*.

i - simple $\quad a(t) = 1 + it$

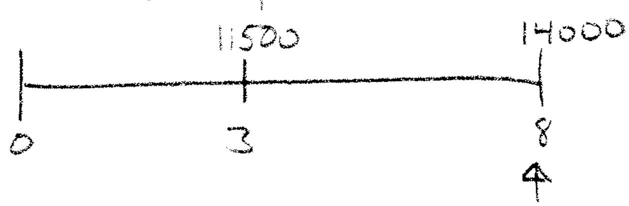
(A) 3.5%

(B) 4.0%

(C) 4.5%

(D) 5.0%

(E) 5.5%



$$14000 = 11500\,\frac{a(8)}{a(3)} = 11500\,\frac{1+8i}{1+3i}$$

$$\Rightarrow i \doteq .05$$