MAP 4170
Test 1

Name: KEY

Date: May 29, 2013

Show sufficient work and clearly mark your answers. Each problem is worth 10 points.

1. Payments of 1500 in four years and 2500 in six years are to be exchanged for a single payment of X in eight years. Using a discount rate of 8% compounded quarterly, determine X.

(A) 4970

(B) 4980

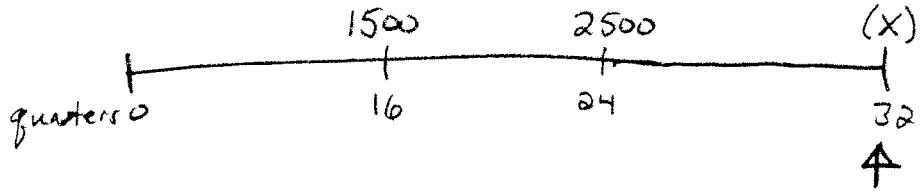
(C) 4990

(D) 5000

(E) 5010

$d^{(4)} = .08 \Rightarrow d = .02 = qedr$

$\Rightarrow v = .98 = qdf \; ; \; v^{-1} = (.98)^{-1} = qaf$



$X = 1500(.98)^{-16} + 2500(.98)^{-8}$

$\doteq 5010$ **E**

2. A deposit of 1000 is made into an account in which interest is credited using a nominal discount rate of d %, compounded semiannually. Six months later, a withdrawal of 500 is taken from the account. One year after the original deposit of 1000, the account has a balance of 531.03. Determine d.

(A) 1

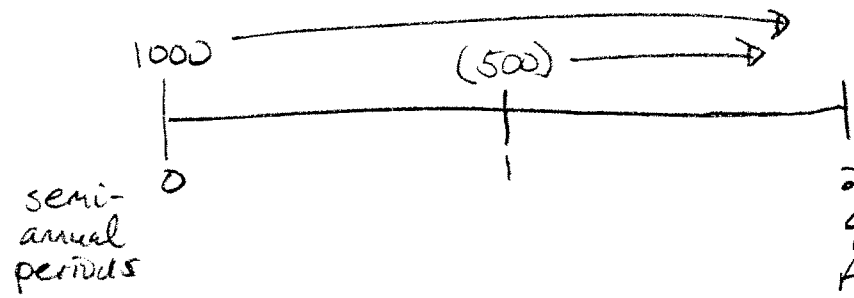
(B) 2

(C) 3

(D) 4

(E) 5

$d = d^{(2)} \Rightarrow \frac{d}{2} = sedr$



$1000\left(1-\frac{d}{2}\right)^{-2} - 500\left(1-\frac{d}{2}\right)^{-1} = 531.03$ (quadratic in $(1-\frac{d}{2})^{-1}$)

$a = 1000$
$b = -500$
$c = -531.03$

$\Rightarrow \left(1-\frac{d}{2}\right)^{-1} = \frac{500 \pm \sqrt{(-500)^2 - 4(1000)(-531.03)}}{2(1000)}$

$\Rightarrow d \doteq .04$ **D**

3. Alice and Beth are to compute the value on July 1, 2013 of a deposit of 100 made into an account on January 1, 2013. The account credits interest using a force of interest that was given to them as "t divided by 1 plus the square of t", where t is the number of years after January 1, 2012. Alice interprets this statement as $\delta_t = \frac{t}{1} + t^2$ and she determines the answer to be A, whereas Beth interprets it as $\delta_t = \frac{t}{1+t^2}$ and determines the answer to be B. Determine the ratio $\frac{A}{B}$.

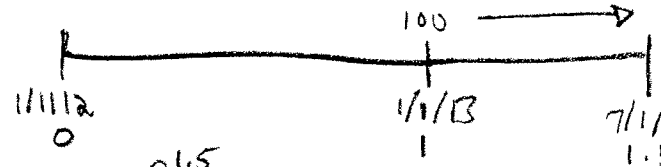
(A) 2.5

(B) 3.2

(C) 4.5

(D) 5.9

(E) 6.3



$A = 100\, e^{\int_1^{1.5} (t+t^2)\,dt} = 100\, e^{(t^2/2 + t^3/3)\Big|_1^{1.5}} \doteq 412.34$

Beth: $\delta_t = \frac{t}{1+t^2}$ special case $f(t) = 1+t^2$, $c = 1/2$ $\quad a(t) = \sqrt{1+t^2}$

$B = 100\,\frac{a(1.5)}{a(1)} = 100\,\frac{\sqrt{3.25}}{\sqrt{2}} \doteq 127.48$

$\therefore \quad \frac{A}{B} \doteq 3.2$ $\quad$ B

4. An account credits interest using a simple interest rate $i = 0.05$, for $0 < t < 10$. Let r denote the equivalent constant force of interest for the 3rd year, and let s denote the equivalent constant force of interest for the 5th year. Determine the ratio $\frac{r}{s}$.

$a(t) = 1 + .05t$

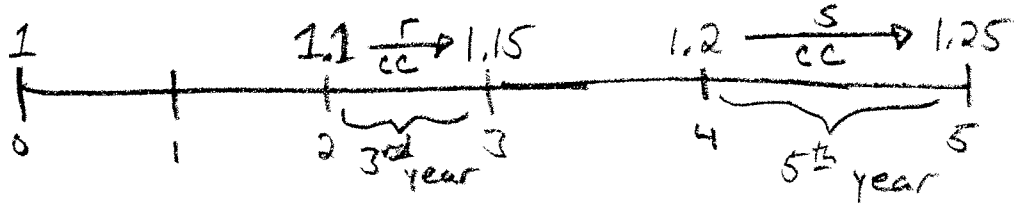
(A) 0.95

(B) 1.00

(C) 1.04

(D) 1.09

(E) 1.15



$1.1\, e^r = 1.15$ $\qquad$ $1.2\, e^s = 1.25$

$\Rightarrow r = \ln\left(\frac{1.15}{1.1}\right)$ $\qquad$ $s = \ln\left(\frac{1.25}{1.2}\right)$

$\therefore \quad \frac{r}{s} \doteq 1.09$

D

5. Determine the semiannual effective discount rate that is equivalent to $\delta = 0.05$.

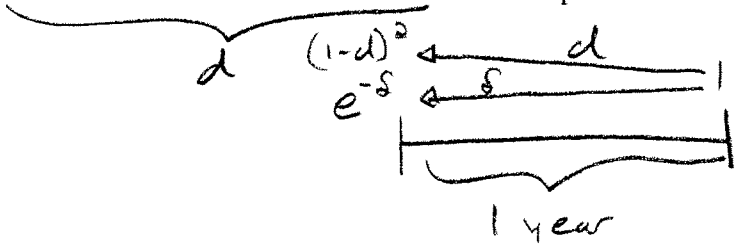
(A) 2.47%

(B) 2.53%

(C) 3.91%

(D) 4.94%

(E) 5.06%



$$\therefore \quad (1-d)^2 = e^{-.05}$$

$$\Rightarrow d = .02469$$

A

6. The provisions from the settlement of a lawsuit state that Judy is to receive a payment of 25000 exactly 15 years from today. Judy would like to receive a payment today in exchange for this future payment. Using an interest rate of 10%, compounded biannually, determine the present value of the future payment.

(A) 5,145

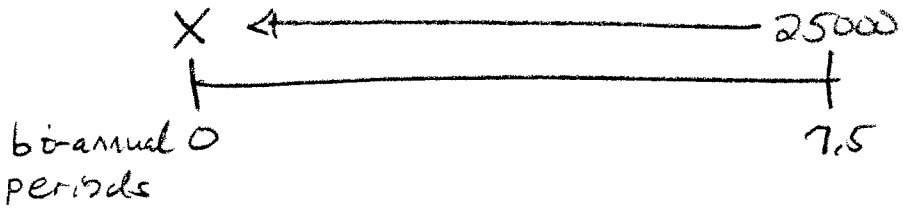
(B) 5,785

(C) 6,370

(D) 16,780

(E) 17,340

$i^{(1/2)} = 10\% \Rightarrow i = \frac{.1}{1/2} = .2 = \text{baeir}$



$$X = 25000\, v_{.2}^{7.5} = 25000(1.2)^{-7.5} \doteq 6369.14$$

C

7. A deposit of 10,000 is made into an account in which interest is credited as follows:

   a simple discount rate, d, for the first three years, then
   a discount rate of $3d$, compounded annually for the next three years, then
   a force of interest equal to 3% thereafter.

   After 10 year, the account has a balance of 17,185. Determine d.

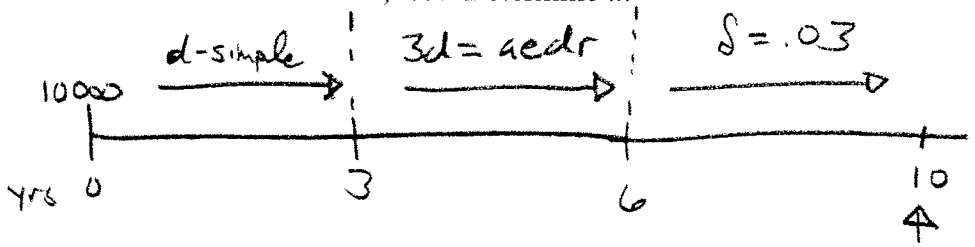
   (A) 3.00%

   (B) 3.33%

   (C) 3.67%

   (D) 4.00%

   (E) 4.33%



$$17185 = 10000(1-3d)^{-1}(1-3d)^{-3}\cdot e^{.03(4)}$$

$$\Rightarrow d \doteq .03333$$

B

8. A payment of 2000 at the end of 2 years and another payment of 1000 at the end of 4 years have a total present value of 2671.83 when using a nominal interest rate of i compounded quarterly. Determine i.

   (A) .036

   (B) .040

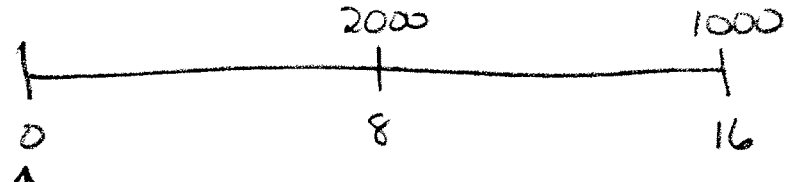
   (C) .044

   (D) .048

   (E) .052

$$i = i^{(4)} \Rightarrow \frac{i}{4} = \text{qeir}$$

$$\Rightarrow v = \frac{1}{1+\frac{i}{4}} = \text{qdf}$$



$$PV = 2671.83 = 2000v^8 + 1000v^{16} \quad \text{(quadratic in } v^8\text{)}$$

$a = 1000$
$b = 2000$
$c = -2671.83$

$$\therefore v^8 = \left(1+\frac{i}{4}\right)^{-8} = \frac{-2000 \pm \sqrt{2000^2 - 4(1000)(-2671.83)}}{2(1000)}$$

$$\Rightarrow i \doteq .044$$

C

9. Determine $\frac{d}{dv}(i)$, where v is the periodic discount factor corresponding to the periodic effective interest rate, i.

$v = (1+i)^{-1}$

(A) $-v^{-2}$

$\therefore \quad i = v^{-1} - 1$

(B) $-v^{-1}$

$\Rightarrow \frac{d}{dv}(i) = -v^{-2}$

(C) -1

(D) $-v$

A

(E) $-v^2$

10. A deposit of 100 grows to 110 after 1 year. Given that the amount function for this account is given by $A(t) = \sqrt{B + Ct}$ for $0 < t < 5$, determine the force of interest for this account at time $t = 3$.

(A) .060

$A(0) = \sqrt{B} = 100 \Rightarrow B = 10000$

(B) .064

$A(1) = \sqrt{B+C} = \sqrt{10000+C} = 110 \Rightarrow C = 2100$

(C) .068

(D) .072

$A(t) = (B+Ct)^{1/2}$

(E) .076

$\Rightarrow A'(t) = \frac{1}{2}(B+Ct)^{-1/2} \cdot C$

$\therefore \delta_t = \frac{C}{2(B+Ct)} \Rightarrow \delta_3 = \frac{2100}{2(10000+2100(3))}$

$\doteq .06442$

B