MAP 4170
Test 1

Name:____________________________

Date: May 28, 2014

Show sufficient work and clearly mark your answers. Each problem is worth 10 points.

1. Maria deposits 10,000 into an account that pays a 4% interest rate, compounded quarterly. At the end of 5 years she withdraws 4000 and at the end of 8 years she withdraws 3000. Determine the account balance immediately after the withdrawal of 3000.

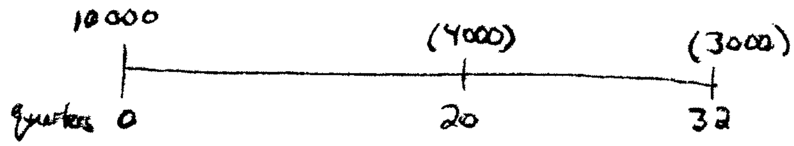
(A) 6200

(B) 6220

(C) 6240

(D) 6260

(E) 6280



$\uparrow$ AV

$i^{(4)} = .04 \Rightarrow i = .01 = \text{qeir}$

$\therefore AV = 10000(1.01)^{32} - 4000(1.01)^{12} - 3000$

$\doteq 6242.11$

2. Scott is to receive 2000 in two years. He determines the present value of this payment to be 1850 when using a simple discount rate, d. Determine d.

(A) 3.75%

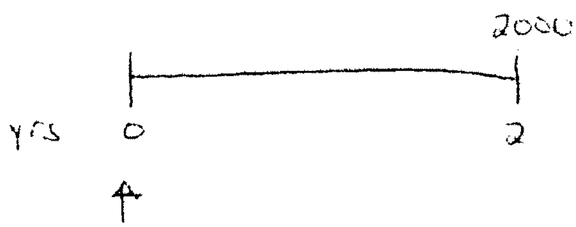
(B) 3.82%

(C) 3.93%

(D) 7.50%

(E) 7.65%

$a(t) = (1-dt)^{-1}$



$PV = 1850 = \dfrac{2000}{a(2)} = \dfrac{2000}{(1-2d)^{-1}} = 2000(1-2d)$

$\Rightarrow d = .0375$

3. Tim deposits 100 into an account at the same time that Todd deposits 50. Tim's account earns 6% simple interest whereas Todd's account earns a nominal interest rate of i, compounded semiannually. The forces of interest in the two accounts are equal at the end of 14.6 years. Determine the excess that Tim has in his account over the amount that Todd has in his account exactly 5 years after their deposits.

(A) 61.2

(B) 63.7

(C) 66.3

(D) 68.8

(E) 71.3

Tim: $i = .06$ simple $\Rightarrow \delta_t^{Tim} = \dfrac{.06}{1+.06t}$

Todd: Let j = aeir $\Rightarrow \delta_t^{Todd} = \ln(1+j) = \delta$

$t = 14.6 \Rightarrow \dfrac{.06}{1+.06(14.6)} = \ln(1+j)$

$\Rightarrow \ln(1+j) = \dfrac{.06}{1.876} \Rightarrow 1+j = e^{(.06/1.876)} = e^{\delta}$

E = Excess after 5 years $= 100(1+.06(5)) - 50(1+j)^5 \doteq 71.33$

4. Given a simple discount rate of 6%, determine the ratio $\frac{i_3}{d_2}$, where i_3 is the annual effective interest rate for the third year and d_2 is the annual effective discount rate for the second year.

(A) 1.03

(B) 1.07

(C) 1.11

(D) 1.15

(E) 1.17

$a(t) = (1-.06t)^{-1}$

$i_3 = \dfrac{a(3)-a(2)}{a(2)} \qquad d_2 = \dfrac{a(2)-a(1)}{a(2)}$

$\therefore \dfrac{i_3}{d_2} = \dfrac{a(3)-a(2)}{a(2)-a(1)}$

$$\left.\begin{aligned} a(1) &= (.94)^{-1} \\ a(2) &= (.88)^{-1} \\ a(3) &= (.82)^{-1} \end{aligned}\right\} \therefore \frac{i_3}{d_2} \doteq 1.146$$

5. Given $\delta_t = \frac{t}{2+t^2}$ for $t > 0$, determine the equivalent semiannual effective discount rate for the time period extending from the end of the second year to the end of the fourth year.

(A) 12.83%

(B) 14.72%

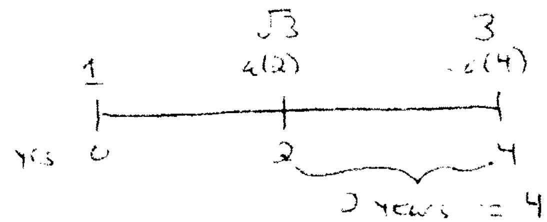
(C) 18.76%

(D) 24.02%

(E) 31.61%

$$\delta_t = \frac{1}{2} \cdot \frac{2t}{2+t^2} \xrightarrow[c=\frac{1}{2}]{f(t)=2+t^2} a(t) = \left(\frac{2+t^2}{2}\right)^{1/2}$$

$$a(t) = \sqrt{1 + \frac{t^2}{2}}$$



$$d = \text{seadr} \Rightarrow \sqrt{3}\,(1-d)^{-4} = 3$$

$$\Rightarrow d = 12.831\%$$

6. Using a nominal discount rate of i, compounded monthly, an amount X accumulates to Y after $3n$ years. The accumulated value after $5n$ years is $3Y$. Determine the ratio $\frac{X}{Y}$.

(A) 0.16

(B) 0.19

(C) 0.25

(D) 0.33

(E) 0.48

Let $v = \text{adf}$

$$X = Y\, v^{3n} \Rightarrow \frac{X}{Y} = v^{3n}$$

$$X = 3Y\, v^{5n}$$

$$\therefore\ Y v^{3n} = 3Y v^{5n} \Rightarrow v^{2n} = \frac{1}{3}$$

$$\therefore\ v^{3n} = \left(v^{2n}\right)^{3/2} = \left(\frac{1}{3}\right)^{3/2}$$

$$\frac{X}{Y} = v^{3n} = \left(\frac{1}{3}\right)^{1.5} \doteq .192$$

7. A deposit of 1000 is made into an account in which interest is credited as follows:

   a simple interest rate, i, for the first two years, then
   a nominal interest rate of i, compounded biannually thereafter $\Rightarrow \frac{i}{1/2} = 2i = \text{baeir}$

   After t years, the account has a balance of 1437.66. After $t + 2$ years, the account has a balance of 1574.24. Determine t.

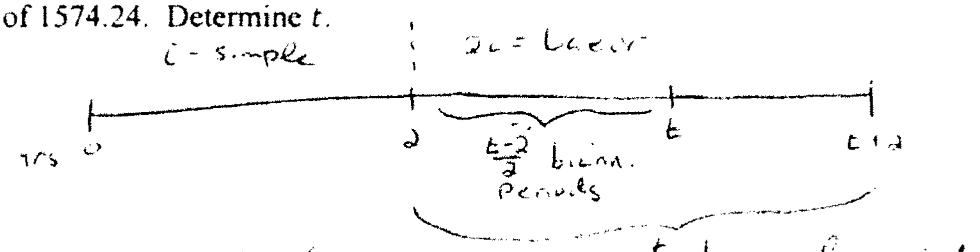
   (A) 4

   (B) 5

   (C) 6

   (D) 7

   (E) 8



$$1437.66 = \underbrace{1000(1+2i)}_{\text{value after 2 years}} \cdot (1+2i)^{\frac{t-2}{2}} = 1000(1+2i)(1+2i)^{t/2}(1+2i)^{-1}$$

$$\therefore (1+2i)^{t/2} = 1.43766$$

$$1574.24 = 1000(1+2i)(1+2i)^{t/2} = 1000(1+2i)(1.43766) \Rightarrow 1+2i = \frac{1574.24}{1437.66}$$

$$\therefore \left(\frac{1574.24}{1437.66}\right)^{t/2} = 1.43766 \Longrightarrow t = 8$$

8. Using an annual effective interest rate of 5%, the present value of payments of 10000 at the end of n years and another 10000 at the end of $2n$ years is 8669. Determine n.

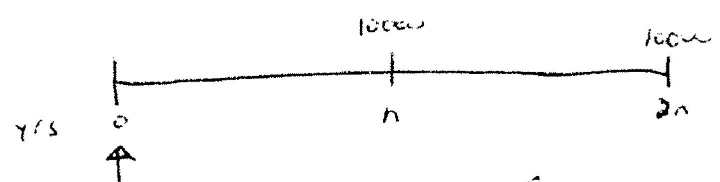
   (A) 11

   (B) 12

   (C) 13

   (D) 14

   (E) 15



$v = \text{adf} = \frac{1}{1.05}$

$$PV = \underbrace{8669 = 10000\, v^n + 10000\, v^{2n}}_{\text{quadratic in } v^n}$$

$a = b = 10000$
$c = -8669$

$$\therefore v^n = \frac{-10000 \pm \sqrt{10000^2 - 4(10000)(-8669)}}{2(10000)}$$

$$= .55683\ldots \boxed{1}$$

$$\therefore (1.05)^{-n} = \boxed{1}$$

$$\Rightarrow n = -\frac{\ln(\boxed{1})}{\ln(1.05)} = 12$$

9. Determine $\frac{d}{di}(v)$, where v is the periodic discount factor corresponding to the periodic effective interest rate, i.

(A) $-v^{-2}$

(B) $-v^{-1}$

(C) -1

(D) $-v$

(E) $-v^{2}$

$$v = (1+i)^{-1}$$

$$\Rightarrow \frac{d}{di}(v) = -(1+i)^{-2} = -v^{2}$$

10. An account credits interest using a simple interest rate of 5%. An initial deposit has an accumulated value of 9000 at the end of 10 years. Determine the amount in the account 5 years after the initial deposit.

(A) 7200

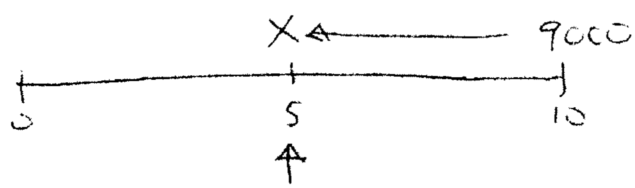
(B) 7300

(C) 7400

(D) 7500

(E) 7600

$$a(t) = 1 + .05t$$



$$X = 9000 \frac{a(5)}{a(10)}$$

$$= 9000 \frac{1+.05(5)}{1+.05(10)} = 7500$$