MAP 4170
Test 1

Name: Key
Date: May 28, 2015

Show sufficient work and clearly mark your answers. Each problem is worth 10 points.

1. Payments of 200, 300, and 500, to be paid at the end of 2 years, 3 years, and 5 years, respectively, are exchanged for a single payment of X to be paid at the end of 1 year. The implied nominal discount rate, compounded monthly, for this exchange is 12%. Determine X.

$d^{(12)} = .12 \Rightarrow d = .01 = \text{medr} \Rightarrow v = .99 = \text{mdf}$

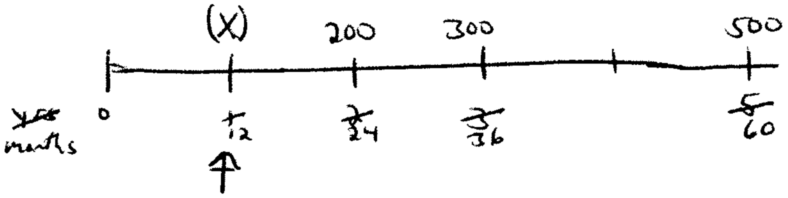
(A) 715

[✓ (B) 720]

(C) 725

(D) 730

(E) 735



$$X = 200v^{12} + 300v^{24} + 500v^{48}$$

$$v = .99$$

$$\Rightarrow X = 721.63$$

2. Determine the derivative, $\frac{d}{di}(d)$, where d is the periodic effective discount rate that is equivalent to the periodic effective interest rate, i.

(A) v^{-2}

(B) v^{-1}

(C) v

[✓ (D) v^2]

(E) none of the above

$$d = \frac{i}{1+i}$$

$$\Rightarrow d' = \frac{(1+i)(1) - i(1)}{(1+i)^2} = \frac{1}{(1+i)^2}$$

$$v = \frac{1}{1+i} \Rightarrow d' = v^2$$

3. An account credits interest using $\delta_t = \frac{t^{-0.5}}{1+2\sqrt{t}}$ where t is the number of years after January 1, 2015. For a deposit on January 1, 2015, the accumulated value on January 1, 2016, is one-third the accumulated value on January 1 of year Y. Determine Y.

(A) 2029

(B) 2030

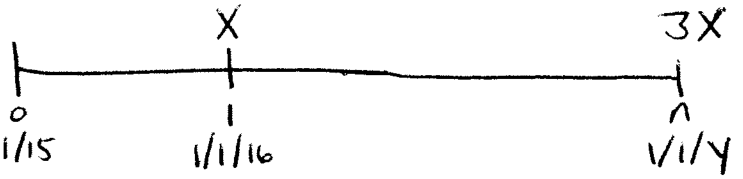
[✓ (C) 2031]

(D) 2032

(E) 2033

$$f(t) = 1+2\sqrt{t} \Longrightarrow f'(t) = t^{-1/2}$$

$$\Longrightarrow \delta_t = \frac{f'(t)}{f(t)} \Longrightarrow a(t) = 1+2\sqrt{t}$$



$$\therefore \quad 3X = X \cdot \frac{a(n)}{a(1)} \Longrightarrow 3 = \frac{1+2\sqrt{n}}{1+2\sqrt{1}} \Longrightarrow n = 16$$

$$n = 16 \Longrightarrow Y = 2031$$

4. Given a simple interest rate of 5%, determine the equivalent quarterly effective interest rate for the first half of the second year.

[✓ (A) 1.18%]

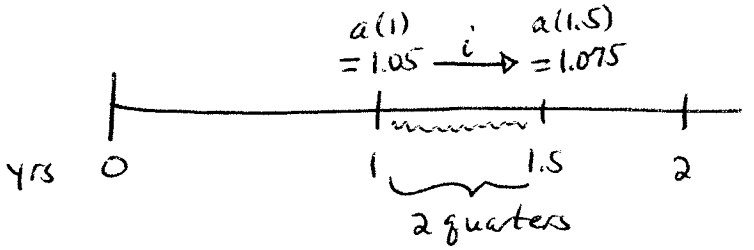
(B) 2.38%

(C) 3.56%

(D) 4.82%

(E) 4.94%

$$a(t) = 1+.05t \qquad i = \text{qeir} \Longrightarrow 1+i = \text{qaf}$$



$$\therefore \quad 1.05(1+i)^2 = 1.075$$

$$\Longrightarrow i \doteq 1.18\%$$

5. An account earns interest according to $\delta_t = C \cdot t$, where C is a constant. A deposit of X at time 1.2 accumulates to $1.14X$ at time 3.8. Determine C.

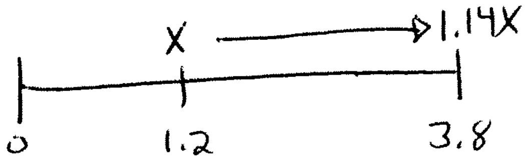
(A) 0.005

(B) 0.010

(C) 0.015

[✓ (D) 0.020]

(E) 0.025



$$1.14X = X e^{\int_{1.2}^{3.8} C \cdot t \, dt}$$

$$\Rightarrow 1.14 = e^{\frac{c}{2} \cdot t^2 \big|_{1.2}^{3.8}} = e^{6.5c}$$

$$\Rightarrow c = \frac{\ln(1.14)}{6.5} \approx .020$$

6. Jack and Dianne each invest 100 in separate accounts. Jack's account credits interest using a nominal discount rate of 6% compounded semiannually. Dianne's account credits interest using a 6% simple discount rate for the first two years, and a nominal interest rate of i, compounded semiannually, thereafter. At the end of 10 years, Jack has 10 less than Dianne. Determine i.

(A) less than or equal to 3.82%

(B) greater than 3.82% but less than or equal to 4.82%

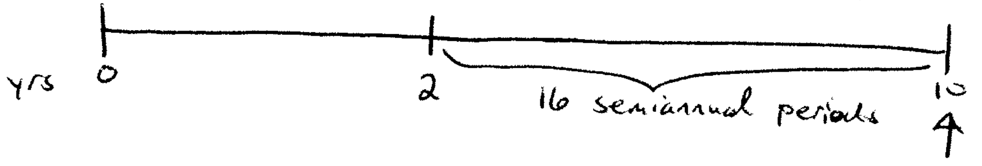
(C) greater than 4.82% but less than or equal to 5.82%

[✓ (D) greater than 5.82% but less than or equal to 6.82%]

(E) greater than 6.82%

$$J: \; AV^{J}_{10 \text{ years}} = 100\left(1 - \frac{.06}{2}\right)^{-20} = 100(.97)^{-20}$$

$$D: \; 100 \xrightarrow{d=.06 \text{ simple}} 100(\overset{.88}{1-2(.06)})^{-1} \xrightarrow{\frac{i}{2} = \text{semi}} AV^{J}_{10} + 10$$



$$\therefore \; 100(.97)^{-20} + 10 = 100(.88)^{-1}\left(1 + \frac{i}{2}\right)^{16}$$

$$\Rightarrow i = 6.79\%$$

7. Philip and Susan each deposit 1000 into separate accounts at time 0. Philip's account credits interest using a semiannual effective interest rate of 4%. Susan's account credits interest using a simple interest rate of 10%. If *T* is the time value at which the forces of interest in the two accounts are equal, determine the amount by which the accumulated value at time *T* in Susan's account exceeds the accumulated value at time *T* in Philip's account.

[✓ (A) 35]

(B) 50

(C) 65

(D) 80

(E) 95

P: seir = .04   Let j = aeir. $1+j = (1.04)^2 = 1.0816$

$$\therefore \ \delta_t = \delta = \ln(1+j) = \ln(1.0816)$$

$$S: \ a(t) = 1+.1t \implies \delta_t = \frac{.1}{1+.1t}$$

$$\therefore \ \ln(1.0816) = \frac{.1}{1+.1T} \implies T \doteq 2.748\ldots \text{ years} \ \boxed{1}$$

$$E = 1000\left(1+.1(\boxed{1})\right) - 1000(1.04)^{2\cdot\boxed{1}}$$

$$\doteq 34.25$$

8. Luke makes a deposit into an account that pays 5% simple discount. Ten years after the deposit, Luke has 12000 in the account. Determine the amount that Luke has in the account five years after the deposit.

[✓ (A) 8000]

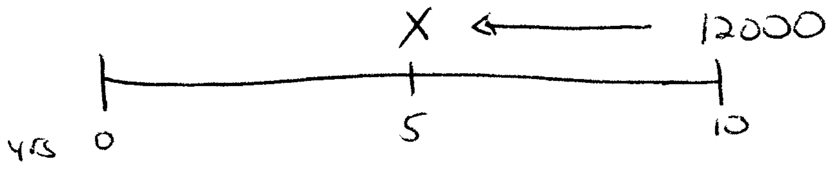
(B) 8500

(C) 9000

(D) 9500

(E) 10000

$$d = .05 \text{ simple} \implies a(t) = (1-.05t)^{-1}$$



$$X = 12000 \cdot \frac{a(5)}{a(10)} = 12000\,\frac{(.75)^{-1}}{(.5)^{-1}} = 12000\,\frac{.5}{.75}$$

$$\implies X = 8000$$

9. A payment of 3000 at the end of k years, together with a payment of 6000 at the end of $2k$ years has a total present value of 3960 when calculated using a constant force of interest, $\delta > 0$. Using the monthly effective interest rate that's equivalent to δ, determine the accumulated value after $3k$ years of a deposit of 432.

(A) 1800

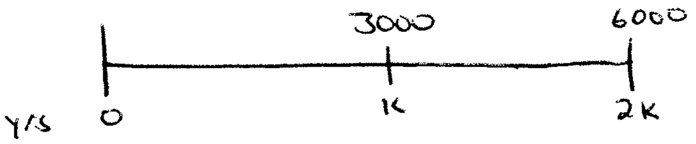
(B) 1850

(C) 1900

(D) 1950

[✓ (E) 2000]

(Compounding) Let $v = \text{adf}$



$PV = 3960 = 3000\,v^{K} + 6000\,v^{2K}$ (quadratic in v^{K})

$\Rightarrow v^{K} = .6$

Answer: $AV = 432 \cdot v^{-3K} = 432(.6)^{-3} = 2000$

10. Determine the nominal interest rate compounded semiannually that's equivalent to a nominal discount rate of 6% compounded quarterly.

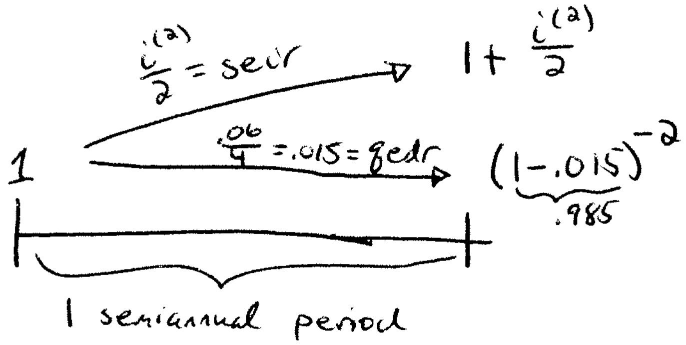
(A) 3.07%

[✓ (B) 6.14%]

(C) 6.59%

(D) 12.28%

(E) 13.17%



$$\therefore \quad 1 + \frac{i^{(2)}}{2} = (.985)^{-2}$$

$$\Rightarrow i^{(2)} \doteq 6.14\%$$