MAP 4170
Test 1

Name:____________________________

Date: May 26, 2016

Show sufficient work and clearly mark your answers. Each problem is worth 10 points.

1. An account credits interest using $\delta_t = \frac{kt}{t^2+2}$ where t is the number of years after January 1, 2016. A deposit of 100 made on July 1, 2016, accumulates to 230 on January 1, 2021. Determine the accumulated value of this deposit on January 1, 2019.

(A) 155

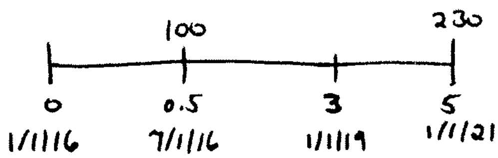
(B) 160

(C) 165

(D) 170

(E) 175

$$\delta_t = \frac{k}{2} \cdot \frac{2t}{t^2+2} \xRightarrow[\text{case}]{\text{special}} a(t) = \left(\frac{t^2+2}{2}\right)^{k/2}$$



$$\therefore\ 230 = 100 \cdot \frac{a(5)}{a(0.5)}$$

$$a(5) = (13.5)^{k/2} \quad a(0.5) = (1.125)^{k/2} \implies \frac{a(5)}{a(0.5)} = \left(\frac{13.5}{1.125}\right)^{k/2} = (12)^{k/2}$$

$$\therefore\ 2.30 = 12^{k/2} \implies k/2 = \frac{\ln(2.3)}{\ln(12)}$$

$$AV_{1/1/19} = 100 \cdot \frac{a(3)}{a(0.5)} = 100 \cdot \left(\frac{5.5}{1.125}\right)^{k/2} = 170.22$$

2. Using a nominal interest rate of i, compounded semiannually, payments of 100 at the end of 1 year and 300 at the end of 2 years have a total present value of X. Using the same interest rate, payments of 125 at the end of 4 years and 375 at the end of 5 years have a total present value of $0.64X$. Determine i.

(A) 10.9%

(B) 11.8%

(C) 16.6%

(D) 21.8%

(E) 23.6%

(compounding) Let v = adf

$$X = 100v + 300v^2$$

$$.64X = 125v^4 + 375v^5 = 1.25v^3\underbrace{(100v + 300v^2)}_{=X}$$

$$\therefore\ .64X = 1.25v^3 \cdot X \qquad v = \frac{1}{1+j} \quad j = \text{aeir}$$

$$\implies (1+j)^3 = \frac{1.25}{.64} \implies j = \text{aeir} = .25$$

$$\left(1 + \frac{i}{2}\right)^2 = 1.25 \implies i = i^{(2)} = .236\cdots$$

3. Determine the sum, $\frac{d}{dd}(i) + \frac{d}{di}(d)$, where d is the periodic effective discount rate that is equivalent to the periodic effective interest rate, i.

(A) $v^2 + v^{-2}$

(B) $v^2 - v^{-2}$

(C) $2v^2$

(D) $2v^{-2}$

(E) none of the above

$$i = \frac{d}{1-d} \Rightarrow \frac{d}{dd}(i) \overset{QR}{=} \frac{(1-d)-d(-1)}{(1-d)^2} = \frac{1}{(1-d)^2}$$

$$1-d = v \Rightarrow \frac{d}{dd}(i) = \frac{1}{v^2} = v^{-2}$$

$$d = \frac{i}{1+i} \Rightarrow \frac{d}{di}(d) \overset{QR}{=} \frac{(1+i)(1) - i(1)}{(1+i)^2} = \frac{1}{(1+i)^2}$$

$$\Rightarrow \frac{d}{di}(d) = v^2$$

$$\therefore \frac{d}{dd}(i) + \frac{d}{di}(d) = v^{-2} + v^2$$

4. Given a simple interest rate of 5%, determine the equivalent nominal discount rate, compounded quarterly, for the second half of the second year.

(A) 1.1%

(B) 2.3%

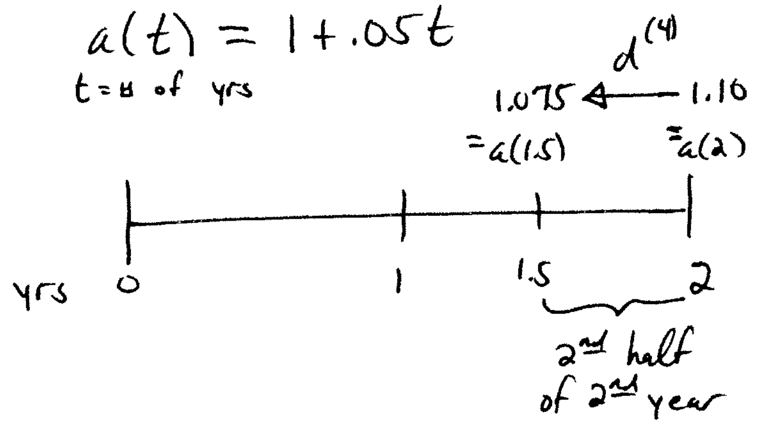
(C) 4.6%

(D) 8.8%

(E) 9.1%

$a(t) = 1 + .05t$

t = # of yrs



$$\frac{d^{(4)}}{4} = d = \text{qedr}$$

$$\Rightarrow 1.075 = 1.1(1-d)^2 \Rightarrow d = 1 - \sqrt{\frac{1.075}{1.1}}$$

$$\therefore d^{(4)} = 4d \doteq .0457\ldots$$

5. Amanda makes a deposit into an account that pays interest using a simple discount rate, d. Five years after the deposit, Amanda has 10,000 in the account. Ten years after the deposit, Amanda has 17,500 in the account. Determine d.

(A) 0.05

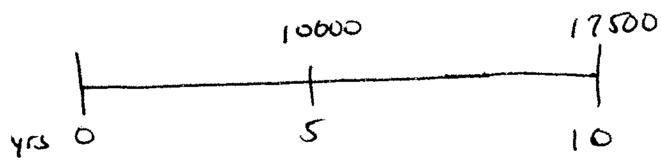
(B) 0.06

(C) 0.07

(D) 0.08

(E) 0.09

$a(t) = (1 - dt)^{-1}$ $t = \#$ of years



$$17500 = 10000 \cdot \frac{a(10)}{a(5)} = 10000 \, \frac{(1-10d)^{-1}}{(1-5d)^{-1}} = 10000 \, \frac{1-5d}{1-10d}$$

$$\therefore \; 17500(1-10d) = 10000(1-5d)$$

$$\Rightarrow d = .06$$

6. Dale deposits 1000 into an account that credits interest using an interest rate of i, compounded quarterly. Two years later, Beth deposits 1000 into an account the credits interest using the same interest rate. Four years after Dale's deposit, Dale has 220 more in his account than Beth has in hers. Determine the amount Beth has in her account four years after her deposit.

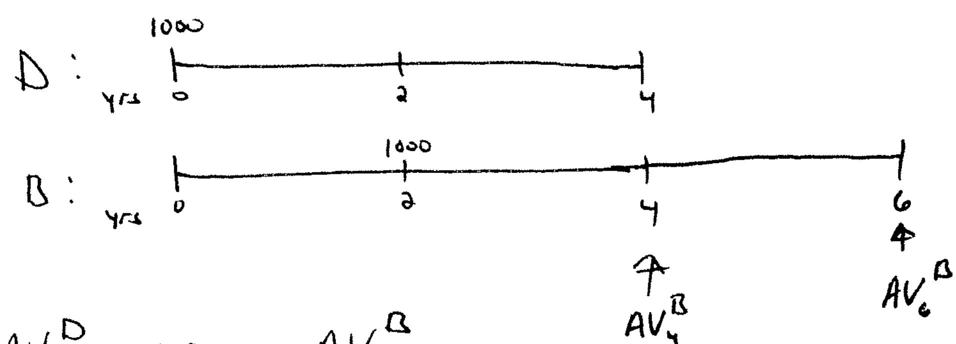
(A) 1185

(B) 1240

(C) 1295

(D) 1350

(E) 1405



$$AV_4^D = 220 + AV_4^B$$

compounding $\Rightarrow$ can use any time period as units

Letting v = 2-year df $\Rightarrow v^{-1}$ = 2-year af

$AV_4^D = 1000\,v^{-2}$ $AV_4^B = 1000\,v^{-1}$

$\therefore \; 1000\,v^{-2} = 220 + 1000\,v^{-1}$ (quadratic in v^{-1}: $a = 1000$, $b = -1000$, $c = -220$)

$$\Rightarrow v^{-1} = \frac{1000 \pm \sqrt{(-1000)^2 - 4(1000)(-220)}}{2(1000)} = 1.185565\ldots$$

$$\therefore \; AV_6^B = 1000\,v^{-2} = 1405.57$$

7. You are given that $\delta_t = 0.01 + 0.02t$. If 1000 is deposited at time $t = 1.5$, determine the accumulated value of the deposit at time $t = 3$.

(A) 1085

(B) 1095

(C) 1105

(D) 1115

(E) 1125

$$AV = 1000 \cdot \frac{a(3)}{a(1.5)} = 1000\, e^{\int_{1.5}^{3} (.01 + .02t)\,dt}$$

$$= 1000\, e^{(.01t + .01t^2)\big|_{1.5}^{3}}$$

$$= 1000\, e^{.12 - .0375} = 1000\, e^{.0825} = 1086.00$$

8. Using the equivalent constant force of interest to a monthly effective discount rate of 0.5%, determine the present value of a payment of 5000 made two years from now.

(A) 4430

(B) 4450

(C) 4470

(D) 4490

(E) 4510

(compounding)

$$PV = 5000\, v^2 \qquad v = adf = (1 - .005)^{12}$$

$$\therefore PV = 5000(.995)^{24} = 4433.27$$

9. Account A credits interest using a quarterly effective interest rate of 2%. Account B credits interest using a simple interest rate of i. The forces of interest in the two accounts are equal at the end of 3 years. If 100 is invested into each of Accounts A and B at time 0, determine which of the following is true after 3 years.

(A) Account A has 7.41 more than Account B

(B) Account A has 4.35 more than Account B

(C) Account B has 7.41 more than Account A

(D) Account B has 4.35 more than Account A

(E) There's not enough information to make such a statement

Acct. A: $a(t) = (1.02)^t$ t - quarters

$\Rightarrow a(t) = (1.02)^{4t}$ t - years

$\Rightarrow \delta_t^A = \ln(1.02)^4 = 4 \cdot \ln(1.02)$

Acct. B: $a(t) = 1 + it$ t - years

$\Rightarrow \delta_t^B = \frac{i}{1+it}$

$$\delta_3^A = \delta_3^B \Rightarrow 4 \cdot \ln(1.02) = \frac{i}{1+3i} \Rightarrow i = \frac{4\ln(1.02)}{1 - 12\ln(1.02)}$$

$\therefore i = .1039\ldots$

A: $AV_3 = 100(1.02)^{12} = 126.82$

B: $AV_3 = 100(1+3i) = 131.17$

$\therefore$ Account B has 4.35 more than Account A

10. Determine the nominal interest rate compounded quarterly that's equivalent to a nominal discount rate of 6% compounded semiannually.

(A) 1.53%

(B) 3.07%

(C) 6.14%

(D) 6.59%

(E) 12.28%

$$\left(1 + \frac{i^{(4)}}{4}\right)^2 = \left(1 - \frac{.06}{2}\right)^{-1}$$

$$\Rightarrow i^{(4)} = .06138\ldots$$