Show sufficient work and clearly mark your answers. Each problem is worth 10 points.

1. A deposit of 500 accumulates to 625 after 2.5 years using a simple interest rate $i$. Determine the accumulated value after 2.5 years if 500 is deposited into an account the earns an annual effective interest rate of $i$.

   \[ 625 = 500 \left(1 + 2.5i\right) \implies i = 0.10 \]

   (A) 615  
   (B) 620  
   (C) 625  
   (D) 630  
   (E) 635

2. A deposit of $X$ accumulates to 1000 after 6 years. During the first two years, interest is credited using a simple discount rate of 6%. During the second two-year period, interest is credited using a nominal interest rate of 6% compounded bi-annually. During the third two-year period, interest is credited using a force of interest $\delta = 6\%$. Determine $X$.

   \[ X \xrightarrow{d=.06} i = .06 \xrightarrow{1000} s = .06 \]

   (A) 697  
   (B) 700  
   (C) 702  
   (D) 705  
   (E) 707

   \[ X \cdot (1 - 2(.06)) \cdot (1 + \frac{.06}{.5}) \cdot e^{.06(2)} = 1000 \]

   \[ \implies X = 696.866\ldots \]
3. Given a simple interest rate of 5%, determine the equivalent nominal discount rate, compounded semi-annually, for the second half of the first year.

(A) 1.8%
(B) 2.4%
(C) 3.6%
(D) 4.8%
(E) 7.2%

\[ a(t) = 1 + 0.05t \]
\[ a(0.5) = 1.025 \]
\[ a(1) = 1.05 \]
\[ \Rightarrow d^{(a)} = \frac{0.0476}{2} \]

4. Using an interest rate of \( i \) compounded monthly, a payment of 5000 at the end of two years together with a payment of 10,000 at the end of four years have a total present value of 9375. Using the same interest rate, a deposit of 27,000 accumulates to \( Y \) after six years. Determine \( Y \).

(A) 36,000
(B) 48,000
(C) 64,000
(D) 72,000
(E) 81,000

\[ PV = 9375 \]
\[ \begin{align*}
\text{let } & \quad v = 2\text{-year discount factor} \\
9375 &= 5000v + 10000v^2 \\
& \text{ (quadratic in } v) \\
\Rightarrow & \quad v = \frac{-5000 \pm \sqrt{5000^2 - 4(10000)(-9375)}}{2(10000)} = 0.75
\end{align*} \]
\[ Y: \quad 27000 \] 6 years \[ Y = 27000 \cdot 0.75^3 = 27000(0.75)^3 \]
\[ \Rightarrow Y = 64000 \]
5. An account credits interest using \( \delta_t = k \cdot \frac{t}{t^2 + 2} \) where \( t \) is the number of years after January 1, 2017. A deposit of \( X \) made on January 1, 2017, accumulates to \( 3X \) on January 1, 2021. Determine the accumulated value of this deposit on July 1, 2019.

(A) 2.01X
\[
S_t = k \cdot \frac{2t}{t^2 + 2} \Rightarrow a(t) = \left( \frac{t^2 + 2}{2} \right)^{\frac{1}{2}}
\]

(B) 2.03X
\[
\begin{align*}
X & \quad \quad \quad \quad \quad \quad \quad \quad \quad 3X \\
\text{Jan} & \quad 1, 2017 \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \text{Jan} & \quad 1, 2021
\end{align*}
\]
\[3 = X \cdot a(4) \Rightarrow k = 1\]

(C) 2.05X
\[
\text{AV} = X \cdot a(2.5) = X \cdot \left( \frac{2.5^2 + 2}{2} \right)^{\frac{1}{2}} = X \cdot 2.031\ldots
\]

(D) 2.07X

(E) 2.09X

6. An account credits interest using a simple interest rate of 5%. Determine \( i_5 \), the annual effective interest rate for year 5.

(A) 4.2%
\[
a(t) = 1 + 0.05 \cdot t
\]

(B) 4.3%

(C) 4.4%

(D) 4.5%

(E) 4.6%

\[
a(5) = a(4) \cdot (1 + i_5)
\]
\[
a(5) = 1.25 \iff 1.25 = 1.2 \cdot (1 + i_5)
\]
\[\Rightarrow i_5 = 0.0416\ldots\]
7. A single deposit of \( X \) is made into an account that credits interest using a simple discount rate of \( d \) over a 10-year period. At the end of 3 years, the amount in the account is 1000, whereas at the end of 5 years, the amount in the account is 1100. Calculate \( d \).

(A) 2%

(B) 3%

(C) 4%

(D) 5%

(E) 6%

\[
1100 = 1000 \cdot \frac{a(5)}{a(3)} = 1000 \frac{(1-5d)^{-1}}{(1-3d)^{-1}} = 1000 \frac{1-3d}{1-5d}
\]

\[
\therefore 1.1 (1-5d) = 1-3d \implies d = .04
\]

8. A deposit of 1000 is made into account A, which credits interest using a simple interest rate of 12%. At the same time, a deposit of 1000 is made into account B, which credits interest using a quarterly effective discount rate of 2%. Let \( T \) denote the time at which the forces of interest in the two accounts are equal. If \( \alpha \) and \( \beta \) denote the amounts in accounts A and B, respectively, at time \( T \), determine \( \alpha - \beta \).

(A) -100

(B) -50

(C) 0

(D) 50

(E) 100

\[
\alpha = 1000 \left(1 + .12T\right)
\]

\[
\beta = 1000 \left(.98\right)^{-4T}
\]

\[
A: s_t = \frac{.12}{1+.12t} \quad B: s_t = s \implies e^s = (.98)^{-4}
\]

\[
\implies s = -4 \ln (.98)
\]

\[
\therefore \frac{.12}{1+.12T} = -4 \ln (.98)
\]

\[
\implies T = 4.041\ldots
\]

\[
\therefore \alpha = 1484.949\ldots \quad \beta = 1386.214\ldots \quad \implies \alpha - \beta = 98.735\ldots
\]
9. Determine $\frac{d}{dd} (v^2)$.

(A) $2v$

(B) $-2v$

(C) $2v^3$

(D) $-2v^3$

(E) none of the above

\[ v^2 = (1 - d)^2 \]

\[ \Rightarrow \frac{d}{dd} (v^2) = 2(1 - d) \cdot (-1) \]

\[ = -2(1 - d) = -2v \]

10. Given a nominal interest rate of $i$; converted semiannually, let $d$ denote the equivalent nominal discount rate, converted semiannually. Determine $d$ in terms of $i$.

(A) $d = \frac{i}{1+i}$

(B) $d = \frac{2i}{1+i}$

(C) $d = \frac{2i}{1+2i}$

(D) $d = \frac{2i}{2+i}$

(E) none of the above

\[ \frac{i}{2} = s e i r \quad \frac{d}{2} = s e d r \]

\[ \therefore \quad \frac{d}{2} = \frac{\frac{i}{2}}{1 + \frac{i}{2}} \]

\[ \Rightarrow \quad d = \frac{i}{1 + \frac{i}{2}} = \frac{2i}{2 + i} \]