MAP 4170 Name: KEY

Test 1 Date: June 1, 2017

Show sufficient work and clearly mark your answers. Each problem is worth 10 points.

1. A deposit of 500 accumulates to 625 after 2.5 years using a simple interest rate i. Determine the accumulated value after 2.5 years if 500 is deposited into an account the earns an annual effective interest rate of i.

   (A) 615

   (B) 620

   (C) 625

   (D) 630

   (E) 635

$625 = 500(1+2.5i) \Rightarrow i = .10$

If $i = .10 = \text{aeir}$, then

$AV = 500(1.1)^{2.5} = 634.529\ldots$

2. A deposit of X accumulates to 1000 after 6 years. During the first two years, interest is credited using a simple discount rate of 6%. During the second two-year period, interest is credited using a nominal interest rate of 6% compounded bi-annually. During the third two-year period, interest is credited using a force of interest $\delta = 6\%$. Determine X.

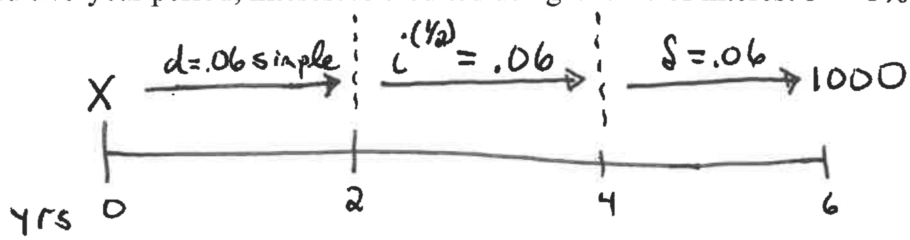
   (A) 697

   (B) 700

   (C) 702

   (D) 705

   (E) 707



$\therefore\ X\cdot(1-2(.06))^{-1}\cdot\left(1+\frac{.06}{1/2}\right)^{1}\cdot e^{.06(2)} = 1000$

$\Rightarrow X = 696.866\ldots$

3. Given a simple interest rate of 5%, determine the equivalent nominal discount rate, compounded semi-annually, for the second half of the first year.

(A) 1.8%

(B) 2.4%

(C) 3.6%

(D) 4.8%

(E) 7.2%

$a(.5) \xrightarrow{d^{(2)}} a(1)$

yrs 0   .5   1

$\underbrace{.5 \quad 1}_{\text{2nd half of 1st year}}$

$\frac{d^{(2)}}{2} = \text{sedr} \Rightarrow v = 1 - \frac{d^{(2)}}{2} = \text{sdf}$

$a(t) = 1 + .05t$

$\therefore a(.5) = 1.025$

$a(1) = 1.05$

$a(.5) = a(1) \cdot v$

$\therefore 1.025 = 1.05\left(1 - \frac{d^{(2)}}{2}\right)$

$\Rightarrow d^{(2)} = .0476\ldots$

4. Using an interest rate of i compounded monthly, a payment of 5000 at the end of two years together with a payment of 10,000 at the end of four years have a total present value of 9375. Using the same interest rate, a deposit of 27,000 accumulates to Y after six years. Determine Y.

(A) 36,000

(B) 48,000

(C) 64,000

(D) 72,000

(E) 81,000

5000 at 2; 10000 at 4

yrs 0   2   4

$PV = 9375$

Let v = 2-year discount factor

$\therefore 9375 = 5000v + 10000v^2$ (quadratic in v)

$\left.\begin{array}{l} a = 10000 \\ b = 5000 \\ c = -9375 \end{array}\right\} \Rightarrow v = \frac{-5000 \pm \sqrt{5000^2 - 4(10000)(-9375)}}{2(10000)} = .75$

Y: $27000 \longrightarrow Y$, $\underbrace{\qquad}_{\text{6 years}}$

$Y = 27000 \cdot v^{-3} = 27000(.75)^{-3}$

$\Rightarrow Y = 64000$

5. An account credits interest using $\delta_t = k \cdot \frac{t}{t^2+2}$ where t is the number of years after January 1, 2017. A deposit of X made on January 1, 2017, accumulates to $3X$ on January 1, 2021. Determine the accumulated value of this deposit on July 1, 2019.

(A) $2.01X$

(B) $2.03X$

(C) $2.05X$

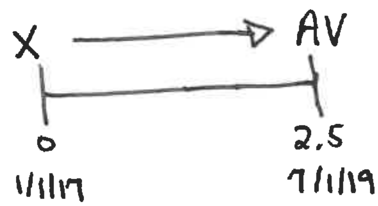
(D) $2.07X$

(E) $2.09X$

$$\delta_t = \frac{k}{2} \cdot \frac{2t}{t^2+2} \Rightarrow a(t) = \left(\frac{t^2+2}{2}\right)^{k/2}$$

$$\therefore 3X = X \cdot a(4)$$

$$\Rightarrow 3 = \left(\frac{4^2+2}{2}\right)^{k/2} \Rightarrow k = 1$$



$$AV = X \cdot a(2.5)$$

$$= X \cdot \left(\frac{2.5^2+2}{2}\right)^{1/2} = X \cdot 2.031\ldots$$

6. An account credits interest using a simple interest rate of 5%. Determine i_5, the annual effective interest rate for year 5.

(A) 4.2%

(B) 4.3%

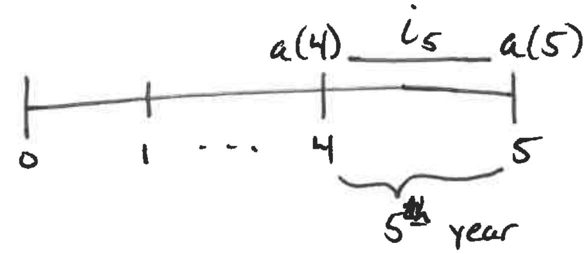
(C) 4.4%

(D) 4.5%

(E) 4.6%

$$a(t) = 1 + .05t$$

i_5:



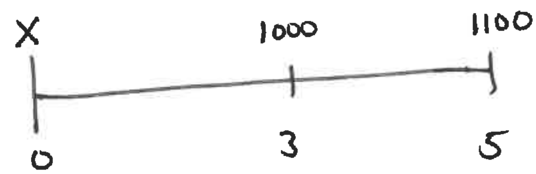
$$a(5) = a(4) \cdot (1 + i_5)$$

$$\left.\begin{array}{l} a(5) = 1.25 \\ a(4) = 1.2 \end{array}\right\} \Rightarrow 1.25 = 1.2(1 + i_5)$$

$$\Rightarrow i_5 = .0416\ldots$$

7. A single deposit of X is made into an account that credits interest using a simple discount rate of d over a 10-year period. At the end of 3 years, the amount in the account is 1000, whereas at the end of 5 years, the amount in the account is 1100. Calculate d.

$$a(t) = (1-dt)^{-1}$$



(A) 2%

(B) 3%

(C) 4%

(D) 5%

(E) 6%

$$1100 = 1000 \cdot \frac{a(5)}{a(3)} = 1000 \frac{(1-5d)^{-1}}{(1-3d)^{-1}} = 1000 \frac{1-3d}{1-5d}$$

$$\therefore \ 1.1(1-5d) = 1-3d \implies d = .04$$

8. A deposit of 1000 is made into account A, which credits interest using a simple interest rate of 12%. At the same time, a deposit of 1000 is made into account B, which credits interest using a quarterly effective discount rate of 2%. Let T denote the time at which the forces of interest in the two accounts are equal. If α and β denote the amounts in accounts A and B, respectively, at time T, determine $\alpha - \beta$.

(A) -100

(B) -50

(C) 0

(D) 50

(E) 100

$$\alpha = 1000(1+.12T)$$

$$\beta = 1000(.98)^{-4T}$$

$$A: \ \delta_t = \frac{.12}{1+.12t} \qquad B: \ \delta_t = \delta \implies e^{\delta} = (.98)^{-4}$$

$$\implies \delta = -4\ln(.98)$$

$$\therefore \ \frac{.12}{1+.12T} = -4\ln(.98)$$

$$\implies T = 4.041\ldots$$

$$\therefore \ \left.\begin{aligned} \alpha &= 1484.949\ldots \\ \beta &= 1386.214\ldots \end{aligned}\right\} \implies \alpha - \beta = 98.735\ldots$$

9. Determine $\frac{d}{dd}(v^2)$.

(A) $2v$

(B) $-2v$

(C) $2v^3$

(D) $-2v^3$

(E) none of the above

$$v^2 = (1-d)^2$$

$$\Rightarrow \frac{d}{dd}(v^2) = 2(1-d)^1 \cdot (-1)$$

$$= -2(1-d) = -2v$$

10. Given a nominal interest rate of i, converted semiannually, let d denote the equivalent nominal discount rate, converted semiannually. Determine d in terms of i.

(A) $d = \frac{i}{1+i}$

(B) $d = \frac{2i}{1+i}$

(C) $d = \frac{2i}{1+2i}$

(D) $d = \frac{2i}{2+i}$

(E) none of the above

$\frac{i}{2}$ = seir $\qquad$ $\frac{d}{2}$ = sedr

$$\therefore \frac{d}{2} = \frac{i/2}{1+i/2}$$

$$\Rightarrow d = \frac{i}{1+i/2} = \frac{2i}{2+i}$$