Show sufficient work and clearly mark your answers. Each problem is worth 10 points.

1. Determine the present value of a 20-year annuity due with annual payments of 250 using an annual effective interest rate of 3% for the first 8 years and 6% thereafter.

(A) 3540  (B) 3550  (C) 3560  (D) 3570  (E) 3580

\[
P_V = 250 \cdot \ddot{a}_{8\%} + 250 \cdot \ddot{a}_{12\%} \cdot u_{3\%}^8 \approx 3561.42
\]

\[c\]

2. The accumulated value of a level annuity after 20 payments is \(c\) times the accumulated value of the annuity after 10 payments, when using a periodic discount factor \(v\) such that \(v^{10} = 0.4\). Determine \(c\).

(A) 1.5  (B) 2.0  (C) 2.5  (D) 3.0  (E) 3.5

\[
A_{V_{20}} = c \cdot A_{V_{10}}
\]

\[
v^{10} = 0.4 \implies (1+c)^{10} = 2.5
\]

\[
(1+c)^{20} = (1+c)^{10} \cdot (1+c)^{10} = 6.25
\]

\[
(1+c)^{20} - 1 = c \left[ (1+c)^{10} - 1 \right]
\]

\[
6.25 - 1 = c \left[ 2.5 - 1 \right] \implies c = 3.5
\]

\[E\]
3. An annual payment annuity has an initial payment of 1. Subsequent payments increase by 1 until reaching a payment of \(n\). Payments then decrease by 1 until reaching a final payment of 1. Using an annual effective interest rate of 5%, the present value of the annuity two years before the first payment is 198.64. Determine \(n\).

\[
PV(\text{rainbow annuity due - peak } n) = \left(\frac{a^{-1}}{d^{-1}}\right)^2
\]

\[
198.64 \times (1.05)^2 = \left(\frac{a^{-1}}{d^{-1}}\right)^2 \implies n = 25
\]

4. A perpetuity due with annual payments has an initial payment of 4 and each subsequent payment is 9 more than its preceding payment. The present value of the perpetuity, when calculated using an annual effective discount rate of \(d\), is 850. Determine \(d\).

\[
PV = \frac{4}{d} + \frac{9}{d^2} = 850 \quad \Rightarrow \frac{4}{d} + \frac{9}{d^2} = \frac{850}{1+d} \implies d^2(1+d) = 850d^2
\]

\[
4 \cdot d(1+d) + 9(1+d) = 850d^2
\]

\[
846 \cdot d^2 + 13 \cdot d + 9 = 0 \implies d = \frac{-13 \pm \sqrt{306.25}}{2(846)} = 0.1
\]

\[
\Rightarrow d = \frac{d}{1+d} = 0.1
\]
5. A 25-year annuity with semi-annual payments has first payment equal to 2 and each subsequent payment is 20% more than its preceding one. Determine the accumulated value of the annuity one year after the last payment, using an annual effective interest rate of 10.25%.

\[
A = a_{\frac{1}{2}}^{49} (1.05)^2 + a_{\frac{1}{2}}^{48} (1.05)^3 + \cdots
\]

\[
= 2(1.2)^{49} (1.05)^2 \left[ 1 + \frac{1.05}{1.2} + \cdots \right]
\]

\[
r = \frac{1.05}{1.2} < 1
\]

\[
= 2(1.2)^{49} (1.05)^2 \cdot \frac{1}{50} \left( \frac{1.2}{1.05} - 1 \right)
\]

\[
\approx 133,607.87
\]

6. A perpetuity due with annual payments has the following payment schedule: 100, 200, 300, 400, 500, 400, 300, 200, 200, 200, ... . Determine the present value of the perpetuity using an annual effective interest rate of 2%.

\[
PV = 100 \left( \frac{1}{51.02} \right)^2 + 100 \left( \frac{2}{.02} \right)^8 + \frac{200}{.02} \cdot \left( \frac{2}{.02} \right)^8
\]

\[
\approx 10,931.68
\]
7. Sue invests 100 at the end of each year for 15 years into an account that pays interest annually at an annual effective interest rate of $i$. The interest payments are reinvested at an annual effective interest rate of 5%. At the end of the 15 year period, Sue has a total accumulated value of 1921. Determine $i$.

(A) 0.032  (B) 0.034  (C) 0.036  (D) 0.038  (E) 0.040

\[ \text{Principal} \quad (\text{aeir} = i) \]

\[ 100 \quad 100 \quad 100 \quad 100 \quad \ldots \quad 100 \quad 100 \]

\[ \text{Interest} \quad (\text{aeir} = 0.05) \]

\[ 0 \quad 1 \quad 2 \quad 3 \quad 4 \quad \ldots \quad 14 \quad 15 \]

\[ 1000 \quad 2000 \quad 3000 \quad \ldots \quad 13000 \quad 14000 \]

\[ \therefore \text{AV} = 1921 \]

\[ \therefore 1921 = 100 (1 + s) + 100 \int_{14}^{15.05} \]

\[ \Rightarrow \overline{i} = 0.032 \]

8. At an annual effective interest rate $i$, both of the following annuities have a present value of $X$.

(i) a 10-year annuity due with annual payments of 15
(ii) a 15-year annuity due with annual payments of 10 for the first 5 years, 20 for the second 5 years, and 30 for the last five years

Determine $X$.

(A) 54.25  (B) 67.60  (C) 72.30  (D) 74.80  (E) 88.15

\[ PV_i = 15 \ddot{a}_{10\ i} = 15 \ddot{a}_{10} + 15 \ddot{a}_{5\ i} u^5 \]

\[ PV_{ii} = 10 \ddot{a}_{5\ i} + 20 \ddot{a}_{5\ i} u^5 + 30 \ddot{a}_{5\ i} u^{10} \]

\[ PV = PV_{ii} \Rightarrow x_{15} (15 + 15 u^5) = x_{15} (10 + 20 u^5 + 30 u^{10}) \]

\[ \Rightarrow 30 u^{10} + 5 u^5 - 5 = 0 \]

\[ \Rightarrow u^5 = \frac{-5 \pm \sqrt{625}}{2(30)} = \frac{1}{3} \Rightarrow i = 3^{\frac{1}{5}} - 1 \]

\[ X = 15 \ddot{a}_{10\ i} = 67.6 \]
9. An annuity due with semiannual payments has an initial payment of 60 and each subsequent payment decreases by 7 until reaching a final payment of 4. Determine the present value of the annuity using an annual effective interest rate of 12.36%.

\[ \text{secr} = \sqrt{1.1236} - 1 = 0.06 \]

\[ PV = 4 \cdot a_{\overline{91.06}} + 7(10 \cdot a_{\overline{81.06}}) = 250.23 \]

D

10. A perpetuity due with annual payments has the following payment pattern:
1, 2, 3, 1, 2, 3, ...
Determine the present value of the perpetuity at an annual effective interest rate of 5%.

\[ j = t \cdot e^{cr} = 1.05^3 - 1 \]

\[ PV = \frac{(T \cdot a_{\overline{31.05}})}{j} (1 + j) = 41.3 \]

E