MAP 4170
Test 2

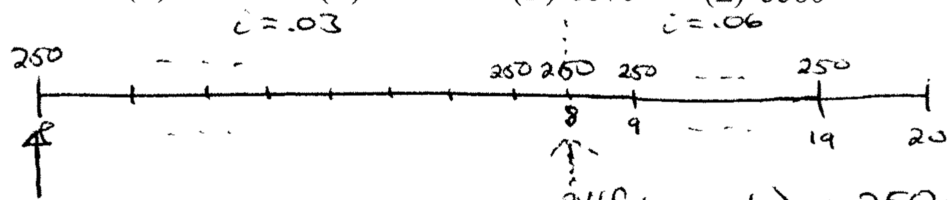
Name:____________________________

Date: October 27, 2011

Show sufficient work and clearly mark your answers. Each problem is worth 10 points.

1. Determine the present value of a 20-year annuity due with annual payments of 250 using an annual effective interest rate of 3% for the first 8 years and 6% thereafter.

(A) 3540 (B) 3550 (C) 3560 (D) 3570 (E) 3580



PV(future pmts) $= 250 \cdot \ddot{a}_{\overline{12}|.06}$

$$PV = 250 \cdot \ddot{a}_{\overline{8}|.03} + 250 \cdot \ddot{a}_{\overline{12}|.06} \cdot v^8_{.03} \doteq 3561.42$$

C

2. The accumulated value of a level annuity after 20 payments is c times the accumulated value of the annuity after 10 payments, when using a periodic discount factor v such that $v^{10} = 0.4$. Determine c.

(A) 1.5 (B) 2.0 (C) 2.5 (D) 3.0 (E) 3.5

$$AV_{20} = c \cdot AV_{10}$$

$v^{10} = .4 \Rightarrow (1+i)^{10} = 2.5$

$\therefore (1+i)^{20} = 6.25$

(Can use immediate or due, Won't matter!)

$$\cancel{R} \cdot s_{\overline{20}|} = c \cancel{R} s_{\overline{10}|}$$

$$(1+i)^{20} - 1 = c[(1+i)^{10} - 1]$$

$$\therefore 6.25 - 1 = c[2.5 - 1] \Rightarrow c = 3.5$$

E

3. An annual payment annuity has an initial payment of 1. Subsequent payments increase by 1 until reaching a payment of n. Payments then decrease by 1 until reaching a final payment of 1. Using an annual effective interest rate of 5%, the present value of the annuity two years before the first payment is 198.64. Determine n.

(A) 23 (B) 24 (C) 25 (D) 26 (E) 27

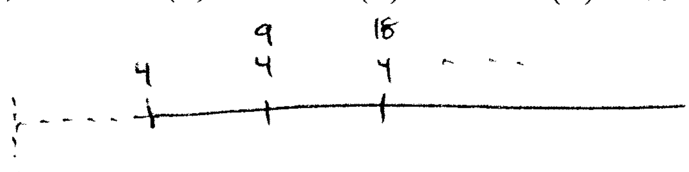
$PV(\text{rainbow annuity due - peak } n) = (\ddot{a}_{\overline{n}|})^2$

$\therefore \quad 198.64(1.05)^2 = (\ddot{a}_{\overline{n}|})^2 \Rightarrow n = 25$

C

4. A perpetuity due with annual payments has an initial payment of 4 and each subsequent payment is 9 more than its preceding payment. The present value of the perpetuity, when calculated using an annual effective discount rate of d, is 850. Determine d.

(A) 8% (B) 9% (C) 10% (D) 11% (E) 12%



$PV = \frac{4}{i} + \frac{9}{i^2} = 850v = \frac{850}{1+i}$ $\left[\frac{4}{i} + \frac{9}{i^2} = \frac{850}{1+i}\right] i^2(1+i)$

$\therefore \; 4i(1+i) + 9(1+i) = 850i^2$

$\Rightarrow 846i^2 \mp 13i \mp 9 = 0 \Rightarrow i = \frac{+13 \pm \sqrt{30625}}{2(846)} = 0.\overline{1}$

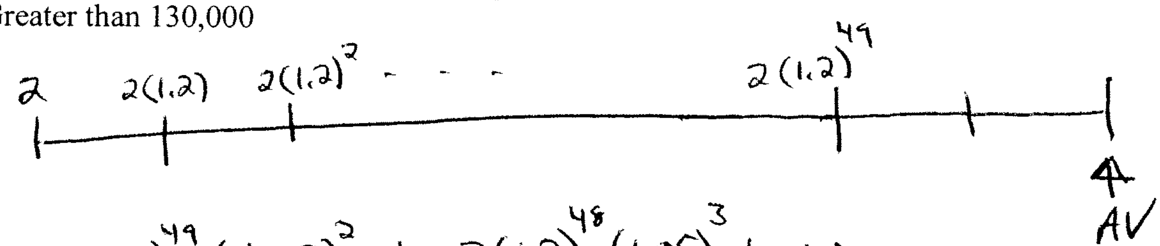
$\Rightarrow d = \frac{i}{1+i} = 0.1$

C

5. A 25-year annuity with semi-annual payments has first payment equal to 2 and each subsequent payment is 20% more than its preceding one. Determine the accumulated value of the annuity one year after the last payment, using an annual effective interest rate of 10.25%.

(A) Less than or equal to 100,000
(B) Greater than 100,000 but less than or equal to 110,000
(C) Greater than 110,000 but less than or equal to 120,000
(D) Greater than 120,000 but less than or equal to 130,000
(E) Greater than 130,000

50 pmts
$i = \text{seir} = .05$



$$AV = 2(1.2)^{49}(1.05)^2 + 2(1.2)^{48}(1.05)^3 + \cdots$$

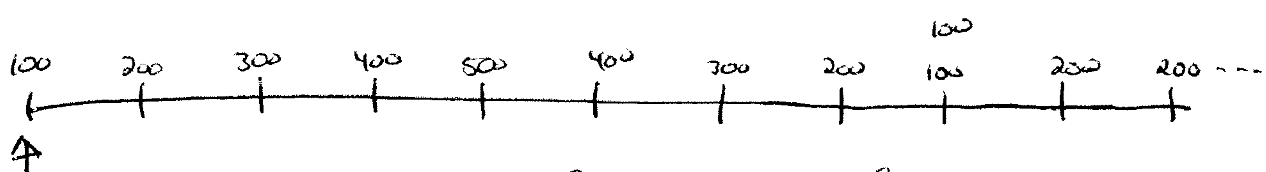
$$= 2(1.2)^{49}(1.05)^2 \left[1 + \frac{1.05}{1.2} + \cdots \right. \qquad r = \frac{1.05}{1.2} < 1$$

$$= 2(1.2)^{49}(1.05)^2 \cdot \ddot{a}_{\overline{50}|\left(\frac{1.2}{1.05}-1\right)} \doteq 133607.87$$

E

6. A perpetuity due with annual payments has the following payment schedule: 100, 200, 300, 400, 500, 400, 300, 200, 200, 200, …. Determine the present value of the perpetuity using an annual effective interest rate of 2%.

(A) 10,675 (B) 10,760 (C) 10,845 (D) 10,930 (E) 11,015



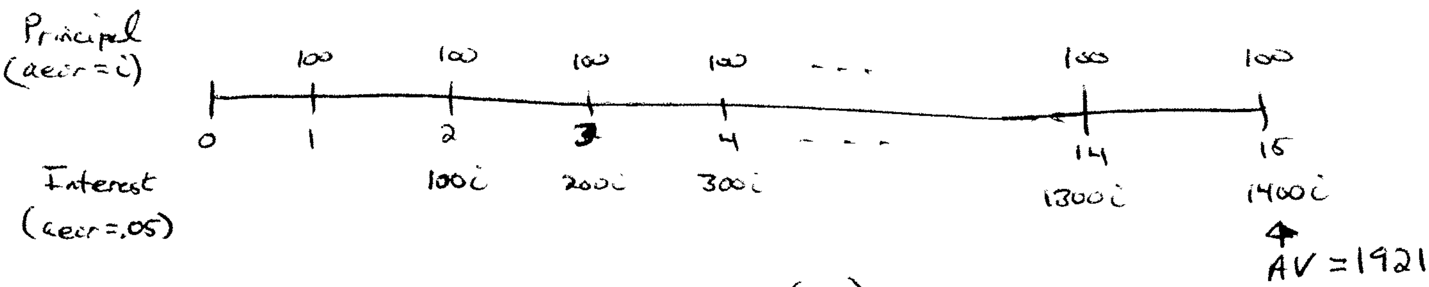
$$PV = 100\left(\ddot{a}_{\overline{5}|.02}\right)^2 + 100\, v^8_{.02} + \frac{200}{.02} \cdot v^8_{.02}$$

$$\doteq 10931.68$$

D

7. Sue invests 100 at the end of each year for 15 years into an account that pays interest annually at an annual effective interest rate of i. The interest payments are reinvested at an annual effective interest rate of 5%. At the end of the 15 year period, Sue has a total accumulated value of 1921. Determine i.

(A) 0.032 (B) 0.034 (C) 0.036 (D) 0.038 (E) 0.040



$$\therefore\ 1921 = 100(15) + 100i \cdot (Is)_{\overline{14|}.05}$$

$$\Longrightarrow\ i \doteq 0.032 \qquad A$$

8. At an annual effective interest rate i, both of the following annuities have a present value of X.

(i) a 10-year annuity due with annual payments of 15
(ii) a 15-year annuity due with annual payments of 10 for the first 5 years, 20 for the second 5 years, and 30 for the last five years

Determine X.

(A) 54.25 (B) 67.60 (C) 72.30 (D) 74.80 (E) 88.15

$$PV_i = 15\,\ddot{a}_{\overline{10|}} \quad \left(= 15\,\ddot{a}_{\overline{5|}} + 15\,\ddot{a}_{\overline{5|}}\,v^5\right)$$

$$PV_{ii} = 10\,\ddot{a}_{\overline{5|}} + 20\,\ddot{a}_{\overline{5|}}\,v^5 + 30\,\ddot{a}_{\overline{5|}}\,v^{10}$$

$$PV_i = PV_{ii} \Longrightarrow \cancel{\ddot{a}_{\overline{5|}}}\left(15 + 15v^5\right) = \cancel{\ddot{a}_{\overline{5|}}}\left(10 + 20v^5 + 30v^{10}\right)$$

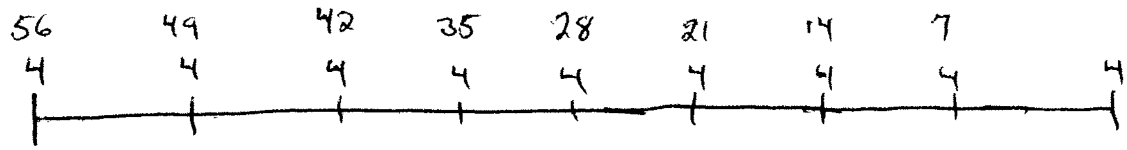
$$\Longrightarrow 30v^{10} + 5v^5 - 5 = 0$$

$$\Longrightarrow v^5 = \frac{-5 \pm \sqrt{625}}{2(30)} = \frac{1}{3} \Longrightarrow i = 3^{1/5} - 1$$

$$\therefore\ X = 15\,\ddot{a}_{\overline{10|}i} \doteq 67.6 \qquad B$$

9. An annuity due with semiannual payments has an initial payment of 60 and each subsequent payment decreases by 7 until reaching a final payment of 4. Determine the present value of the annuity using an annual effective interest rate of 12.36%.

(A) 220 (B) 230 (C) 240 (D) 250 (E) 260



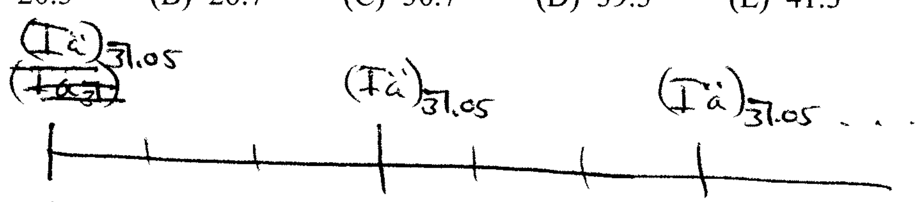
$\text{seir} = \sqrt{1.1236} - 1 = .06$

$$PV = 4\,\ddot{a}_{\overline{9}|.06} + 7(D\ddot{a})_{\overline{8}|.06} \doteq 250.23$$

D

10. A perpetuity due with annual payments has the following payment pattern:
1, 2, 3, 1, 2, 3, …
Determine the present value of the perpetuity at an annual effective interest rate of 5%.

(A) 20.3 (B) 26.7 (C) 30.7 (D) 39.3 (E) 41.3



$j = \text{taeir} = 1.05^3 - 1$

$$PV = \frac{(I\ddot{a})_{\overline{3}|.05}}{j}(1+j) \doteq 41.3$$

E