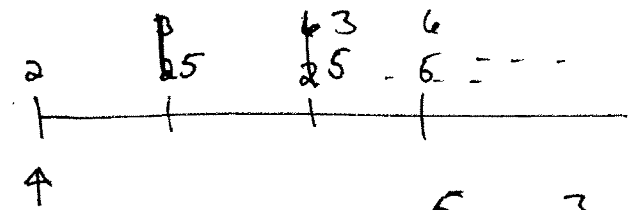
MAP 4170 Name:______________________________

Test 2 October 23, 2012

Each problem is worth 10 points. Show sufficient work for full credit.

1. A perpetuity due with annual payments has an initial payment of 2 and each subsequent payment is 3 more than its preceding payment. The present value is 7752 using an annual effective discount rate of d, $d > 0$. Determine d.

(A) 1.92% (B) 1.96% (C) 2.00% (D) 2.04% (E) 2.08%



$$PV = 7752 = 2 + \frac{5}{i} + \frac{3}{i^2}$$

$$\Rightarrow 7750 i^2 - 5i - 3 = 0 \Rightarrow i = .02$$

$$\Rightarrow d = \frac{i}{1+i} = \frac{.02}{1.02} \doteq .0196 \quad B$$

2. You are given that the present value of a level annuity immediate with $2n$ monthly payments of R is 1000. Given that $v^n = 0.8$, where v is the monthly discount factor, determine the present value of an annuity immediate with $4n$ level monthly payments of R, using the same interest rate.

(A) 1250 (B) 1320 (C) 1480 (D) 1560 (E) 1640

$$1000 = R\, a_{\overline{2n}|}$$

$$PV = R\, a_{\overline{4n}|} = R\, a_{\overline{2n}|} + R\, a_{\overline{2n}|} \cdot v^{2n}$$

$$= 1000 + 1000(0.8)^2 = 1640$$

$$1000 = R\,\frac{1-v^{2n}}{i} = \frac{R}{i}\left(1-(.8)^2\right) = \frac{R}{i}(.36)$$

$$PV = R\, a_{\overline{4n}|} = R \cdot \frac{1-v^{4n}}{i} = \frac{R}{i}\left(1-(.8)^4\right)$$

$$= \frac{1000}{.36}\left(1-(.8)^4\right) = 1640$$

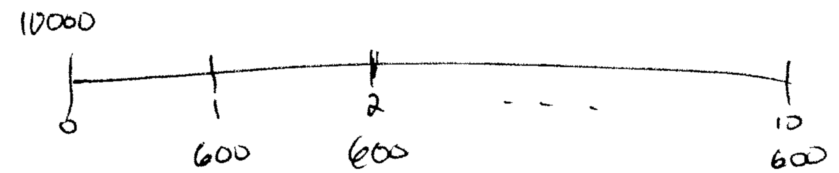
3. Al invests 10000 in an account in which interest is paid annually at an interest rate of 6% per annum. When received, Al reinvests the interest payments in an account that pays interest at 5% annual effective. Let X denote the amount Al has accumulated after 10 years.

Bob invests 1000 at the end of each year for 10 years in an account that pays interest annually at an interest rate of 6% per annum. Like Al, Bob reinvests the interest payments in an account that pays interest at 5% annual effective. Let Y denote the amount Bob has accumulated after 10 years.

Determine $X + Y$.

(A) 30460 (B) 30520 (C) 30580 (D) 30640 (E) 30700

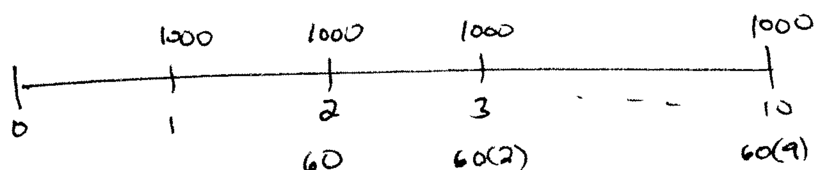
Al:



$X = AV = 10000 + 600\, s_{\overline{10}|.05}$

$\doteq 17547$

Bob:



$Y = AV = 10000 + 60 (Is)_{\overline{9}|.05}$

$= 13093$

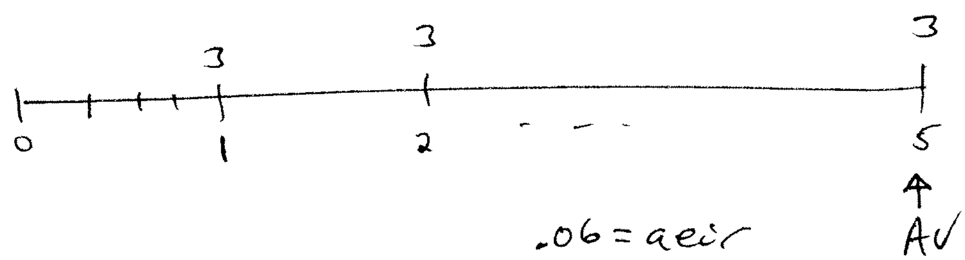
$\therefore X + Y = 30640$

(D)

4. Determine the accumulated value of an annuity immediate with level payments of 3 made every 4 year period for 20 years, using an interest rate of 6% compounded annually.

(A) 21 (B) 23 (C) 25 (D) 27 (E) 29

4-year periods



$.06 = aeir$

$AV = 3\, s_{\overline{5}|j}$

$j = 4$ year eir

$\therefore 1 + j = 1.06^4$

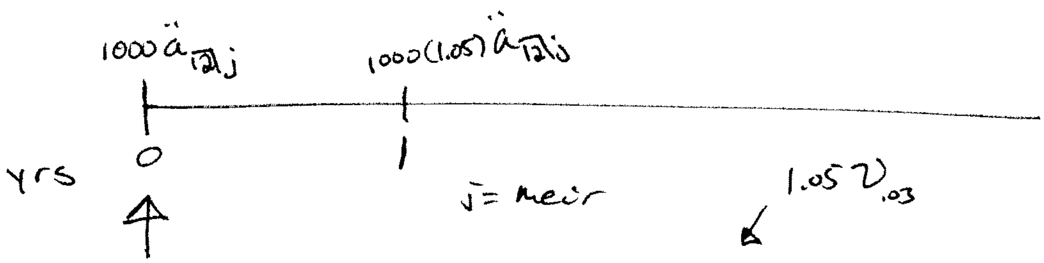
$\Longrightarrow AV \doteq 25.23$ (C)

5. For 10 years the monthly payments from a lottery are level each year, but increase from year to year. For the first year the monthly payments are 1000 each. Each subsequent year, the monthly payments increase by 5% over the previous year's monthly payments. Using an annual effective interest rate of 3%, determine the present value of the payments at the time of, and including, the first payment.

.03 = aeir

(A) 129300 (B) 129600 (C) 129900 (D) 130200 (E) 130500

Replace original set of payments with the following



j = meir

$1.05\,v_{.03}$

$(1+j)^{12} = 1.03$

$$PV = 1000\,\ddot{a}_{\overline{12}|j}\left(1 + \frac{1.05}{1.03} + \cdots \text{(10 terms)}\right)$$

$$= 1000\,\ddot{a}_{\overline{12}|j} \cdot \ddot{s}_{\overline{10}|\left(\frac{1.05}{1.03}-1\right)}$$

$$= 129288.50$$

6. A 15000 loan at an annual effective interest rate of 6% is to be repaid with monthly payments of 100 for as long as necessary, plus a final smaller payment one month after the last regular payment of 100. Determine the amount of last payment.

(A) 71 (B) 73 (C) 75 (D) 77 (E) 79

$$15000 = 100\,a_{\overline{n}|j}$$

j = meir

$(1+j)^{12} = 1.06$

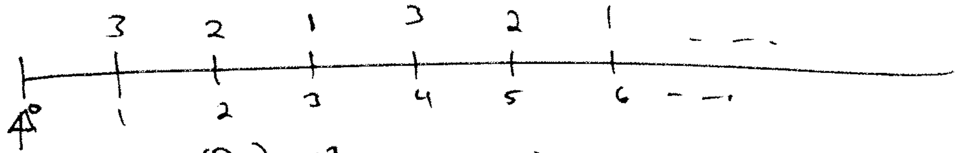
$$\Longrightarrow n = 269+$$

$$15000 = 100\,a_{\overline{269}|j} + X\,v_j^{270}$$

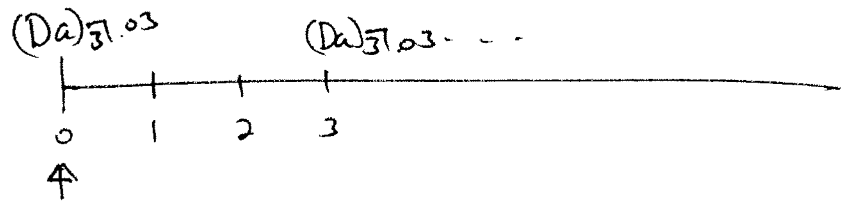
$$X = 74.80$$

7. Using an annual effective interest rate of 3%, determine the present value of a perpetuity immediate with annual payment pattern equal to 3, 2, 1, 3, 2, 1, …

(A) 61 (B) 63 (C) 65 (D) 67 (E) 69



Replace with

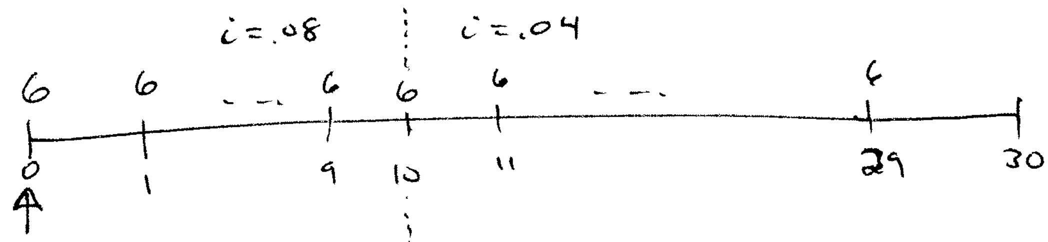


$$PV = \frac{(Da)_{\overline{3}|.03}}{j}\,(1+j) \qquad j = \text{3-year eir}, \quad 1+j = (1.03)^3$$

$$\doteq 67.32 \quad (D)$$

8. Determine the present value of a 30-year annuity due with level annual payments of 6, assuming an annual effective interest rate of 8% for the first 10 years and 4% thereafter.

(A) 81 (B) 83 (C) 85 (D) 87 (E) 89

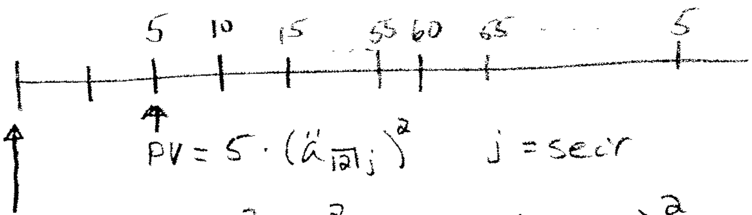


$$PV = 6\,\ddot{a}_{\overline{10}|.08} + 6\,\ddot{a}_{\overline{20}|.04} \cdot v^{10}_{.08} \doteq 82.76$$

$$PV = 6\,\ddot{a}_{\overline{11}|.08} + 6\,\ddot{a}_{\overline{19}|.04} \cdot v_{.04} \cdot v^{10}_{.08}$$

9. An annuity with semiannual payments has an initial payment of 5. Subsequent payments increase by 5 until reaching a payment of 60, at which time payments begin to decrease by 5 until reaching a final payment of 5. The present value of this annuity, one year before the first payment, is 450 when using an annual effective interest rate of i. Determine i.

(A) 3.8% (B) 4.8% (C) 5.8% (D) 6.8% (E) 7.8%



semiannual

$$PV = 5 \cdot (\ddot{a}_{\overline{12}|j})^2 \qquad j = \text{sec'r}$$

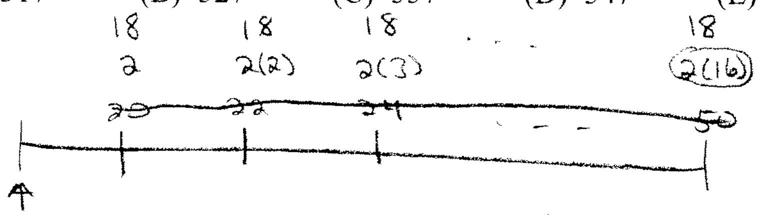
$$PV = 5(\ddot{a}_{\overline{12}|j})^2 \cdot v_j^2 = 5(a_{\overline{12}|j})^2 = 450$$

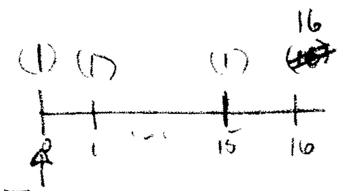
$$\Longrightarrow j \doteq .038$$

$$1+i = (1+j)^2 \Longrightarrow i = .078 \qquad \text{E}$$

10. An annuity with annual payments has an initial payment of 20 and each subsequent payment is 2 more than its previous payment until a final payment of 50. Using an annual effective interest rate of 5%, determine the present value of this annuity one year before the payment of 20.

(A) 317 (B) 327 (C) 337 (D) 347 (E) 357





$$PV = 18\, a_{\overline{16}|.05} + 2(Ia)_{\overline{16}|.05}$$

$$= 18\, a_{\overline{16}|.05} + 2 \cdot \frac{\ddot{a}_{\overline{16}|.05} - 16 v_{.05}^{16}}{.05}$$

$$\doteq 357.07$$

1-step: BGN
16 N
5 I/Y
-1 PMT
16 FV
CPT PV

$$PV = 20\, a_{\overline{16}|.05} + 2(Ia)_{\overline{15}|.05} \cdot v_{.05}$$