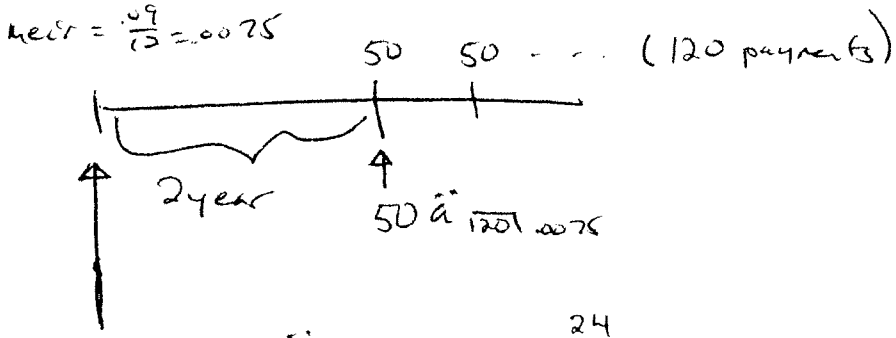
MAP 4170 Name: _____
Test 2 Date: October 17, 2013

Show sufficient work and clearly mark your answers. Each problem is worth 10 points.

1. Using a nominal interest rate of 9% compounded monthly, determine the present value of a 2-year deferred, 10-year annuity due with monthly payments of 50.

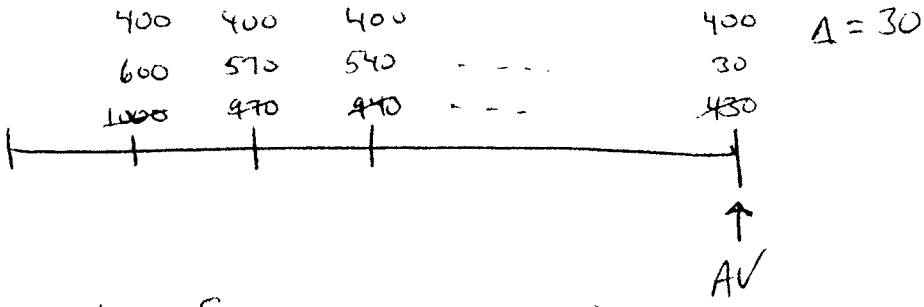
(A) 3300
(B) 3325
(C) 3350
(D) 3375
(E) 3400



$$PV = 50\ddot{a}_{\overline{120}|.0075} \cdot v^{24}_{.0075} = 3323.84$$

2. A 20-year annuity immediate with annual payments has an initial payment of 1000. Each subsequent payment decreases by 30. Determine the accumulated value of this annuity using an annual effective interest rate of 4%.

(A) 21580
(B) 21915
(C) 22230
(D) 22445
(E) 23345

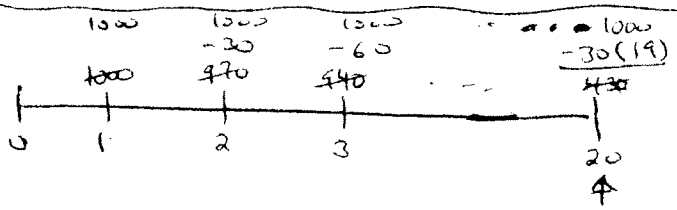


$$AV = 400\, s_{\overline{20}|.04} + 30\,(Ds)_{\overline{20}|.04}$$

$$= 400\, s_{\overline{20}|.04} + 30\left(\frac{20 - a_{\overline{20}|.04}}{.04}\right)(1.04)^{20}$$

$$\doteq 22444.52$$

(OR)



$$AV = 1000\, s_{\overline{20}|.04} - 30(Is)_{\overline{19}|.04}$$

3. The accumulated value of a $2n$-year annuity immediate with annual payments of K is 807 when determined using an annual effective interest rate of i. Using the same interest rate, the accumulated value of a $3n$-year annuity immediate with annual payments of $3K$ is 5649, and the present value of an n-year annuity immediate with annual payments of 100 is 1345. Determine i.

(A) 0.017

(B) 0.027

(C) 0.037

(D) 0.047

(E) 0.057

$$807 = K s_{\overline{2n}|} = K s_{\overline{n}|}\left((1+i)^n + 1\right)$$

$$5649 = 3K s_{\overline{3n}|} = 3K s_{\overline{n}|}\left((1+i)^{2n} + (1+i)^n + 1\right)$$

$$1345 = 100\, a_{\overline{n}|}$$

$$\frac{1883}{3} = \frac{5649}{3} = 3\left(\frac{807}{(1+i)^n + 1}\right)\left((1+i)^{2n} + (1+i)^n + 1\right)$$

$$\Rightarrow 1883\left[(1+i)^n + 1\right] = 807\left[(1+i)^{2n} + (1+i)^n + 1\right]$$ quadratic in $(1+i)^n$

$$\Rightarrow 807(1+i)^{2n} - 1076(1+i)^n - 1076 = 0$$

$$\Rightarrow (1+i)^n = \frac{1076 \pm \sqrt{(-1076)^2 - 4(807)(-1076)}}{2(807)} = 2 \Rightarrow v^n = 1/2$$

$$\therefore\ 1345 = 100\,\frac{1 - v^n}{i} \Rightarrow i = \frac{50}{1345} \doteq .037$$

4. Judy deposits 500 at the end of each year for 20 years into an account that pays interest at the end of each year using an annual effective interest rate of i. The interest payments are reinvested in an account that pays an annual effective interest rate of 8%. At the end of 20 years, Judy has a total accumulated value of 19258.21. Determine i.

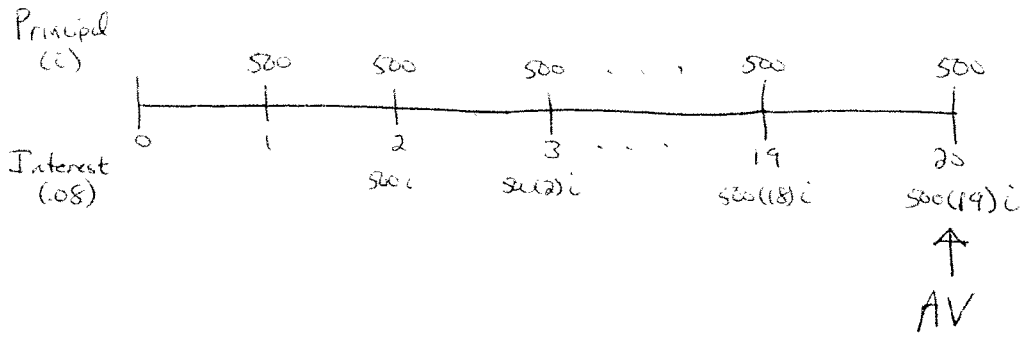
(A) 4.75%

(B) 5.00%

(C) 5.25%

(D) 5.50%

(E) 5.75%



$$AV = 19258.21 = 500(20) + 500i\,(I_s)_{\overline{19}|.08}$$

$$\Rightarrow 925821 = 500i\,\frac{s_{\overline{19}|.08} - 19}{.08}$$

$$\Rightarrow i \doteq .0575$$

5. Ed is to receive payments at the beginning of each quarter for 10 years. The quarterly payments are level during each year, but at the end of each year, the next year's quarterly payments increase by 50. The first year's quarterly payments are 300 each. Determine the accumulated value of the payments, immediately after the final payment, using a 6% annual effective interest rate.

$j = \text{qear} = (1.06)^{1/4} - 1$

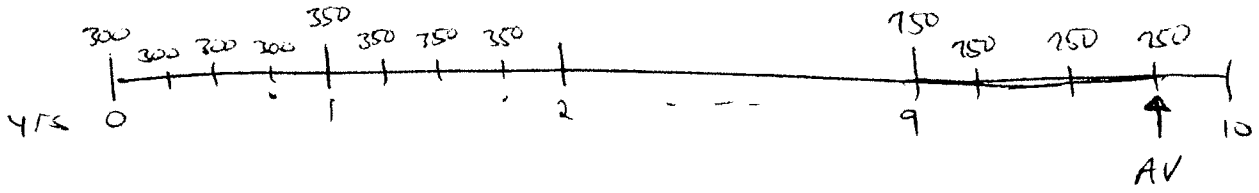
(A) Less than 26,000

(B) Greater than or equal to 26,000, but less than 26,400

(C) Greater than or equal to 26,400, but less than 26,800

(D) Greater than or equal to 26,800, but less than 27,200

(E) Greater than or equal to 27,200



$AV = 250\, s_{\overline{40}|j} + 50\, s_{\overline{4}|j} \cdot (Is)_{\overline{10}|.06} = 27066.83$

6. A perpetuity immediate has quarterly payments as follows: 2, 5, 8, 2, 5, 8, etc. Determine the present value of this perpetuity using a nominal discount rate of 8% compounded quarterly.

$d^{(4)} = .08 \Rightarrow d = .02 = \text{qear}$

$v = .98 = 1 - d \quad \left(= \frac{1}{1+i}\right)$

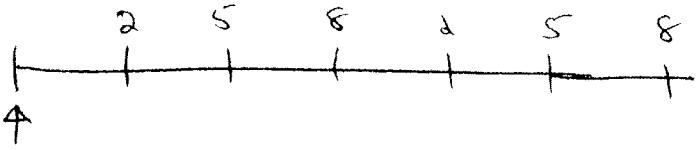
(A) 233

(B) 238

(C) 243

(D) 248

(E) 253



$PV = 2v + 2v^4 + \cdots + 5v^2 + 5v^5 + \cdots + 8v^3 + 8v^6 + \cdots$

$= \frac{2v + 5v^2 + 8v^3}{1 - v^3} \overset{v=.98}{=\!=} 243$

7. Pancho and Lefty each receive an inheritance of 10000. Pancho uses his 10000 to purchase an annual payment increasing perpetuity immediate with an initial payment of 100 and each subsequent payment being 20 more than its preceding payment. Using the same interest rate implied by Pancho's perpetuity, Lefty uses his 10000 to purchase an annual payment perpetuity immediate with an initial payment of 100 and each subsequent payment being r % more than its preceding payment. Determine r.

(A) 3.85

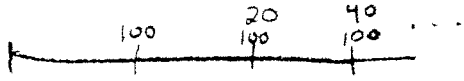
(B) 3.90

(C) 3.95

(D) 4.00

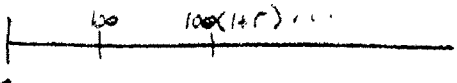
(E) 4.05

P:



$$10000 = \frac{100}{i} + \frac{20}{i^2} \Rightarrow i = .05$$

L:



$$10000 \overset{VEP}{=} 100v + 100(1+r)v^2 + \cdots$$

$$\therefore 10000 = \frac{(100/1.05)}{1 - \frac{1+r}{1.05}} \Rightarrow r = .04$$

8. An arithmetically increasing perpetuity due with payments every 2 years has a present value of 13310 using an annual effective interest rate of 10%. The amount of the sixth payment is 2415. Determine the amount of the third payment.

(A) 882

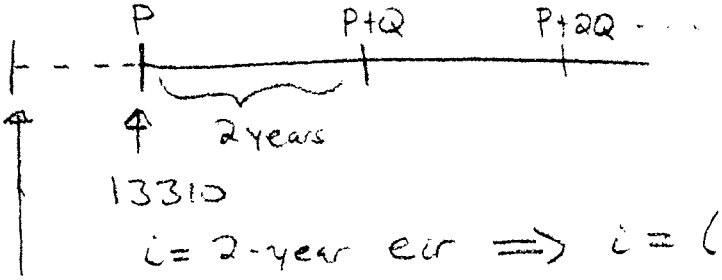
(B) 1092

(C) 1276

(D) 1408

(E) 1533

aeir = .1



i = 2-year eir $\Rightarrow i = (1.1)^2 - 1 = .21$

$$\therefore \frac{13310}{1.21} = \frac{P}{.21} + \frac{Q}{(.21)^2}$$

$$\Rightarrow \begin{cases} \left[11000 = \frac{P}{.21} + \frac{Q}{.0441}\right](.0441) \\ 2415 = P + 5Q \end{cases}$$

$$\therefore \begin{cases} 485.1 = .21P + Q \\ 2415 = P + 5Q \end{cases} \Rightarrow \begin{matrix} P = 210 \\ Q = 441 \end{matrix}$$

$\therefore 3^{rd}$ payment $= P + 2Q = 1092$

9. An annuity due with annual payments has an initial payment of 3. Each subsequent payment increases by 2 over its preceding payment until reaching a payment of 31. After the payment of 31, the next payment is 29 and subsequent payments decrease by 2 until reaching a final payment of 3. Determine the present value of the annuity using an annual effective interest rate of 6%.

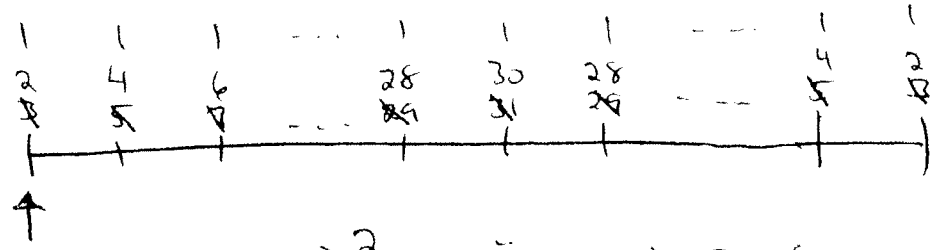
(A) 225

(B) 235

(C) 245

(D) 255

(E) 265



$$PV = 2\left(\ddot{a}_{\overline{15}|.06}\right)^2 + \ddot{a}_{\overline{29}|.06} \doteq 226.$$

10. A 20-year annuity due has annual payments of 5 for the first 5 years and 15 for the next 15 years. Using an annual effective interest rate of 4% for the first 5 years and i thereafter, the accumulated value of the annuity is 390. Determine i.

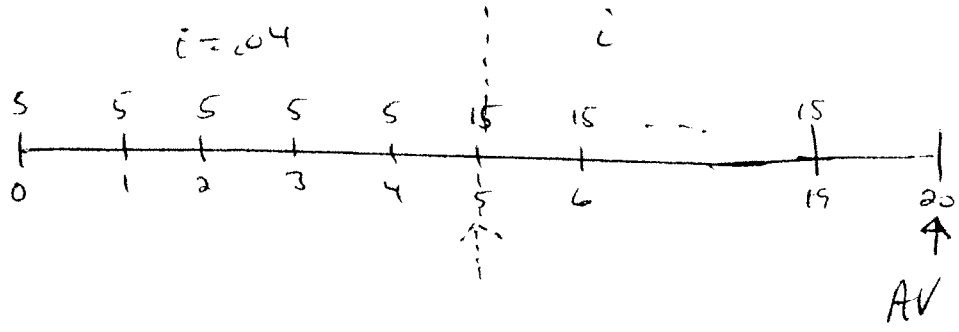
(A) 4.25%

(B) 4.50%

(C) 4.75%

(D) 5.00%

(E) 5.25%



$$AV = \underbrace{390}_{FV} = \underbrace{5\,\ddot{s}_{\overline{5}|.04}}_{PV}\,(1+i)^{\overbrace{15}^{n=15}} + \underbrace{15}_{PMT}\,\ddot{s}_{\overline{15}|i}$$

BGN 5 N 4 I/Y 0 PV 5 +/- PMT CPT FV +/- PV

15 N +/- PMT 390 FV CPT I/Y $i = .0477$