MAP 4170 Name:________________________________
Test 2 Date: October 16, 2014

Show sufficient work and clearly mark your answers. Each problem is worth 10 points.

1. A perpetuity immediate has quarterly payments that repeat each year. For each year, the first payment is 1, the second is 5, the third is 3, and the fourth is 7. Determine the present value of this perpetuity using a nominal discount rate of 12% compounded quarterly.

$d^{(4)} = .12 \Rightarrow$ qedr $= .03 \Rightarrow v = .97 =$ qdf

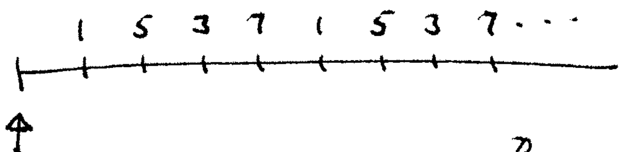
(A) 122.50

(B) 125.00

(C) 127.50

(D) 130.00

(E) 132.50



$$PV \overset{VEP}{=} v + v^5 + \cdots \longrightarrow \frac{v}{1-v^4}$$
$$+ 5v^2 + 5v^6 + \cdots \longrightarrow \frac{5v^2}{1-v^4}$$
$$+ 3v^3 + 3v^7 + \cdots \longrightarrow \frac{3v^3}{1-v^4}$$
$$+ 7v^4 + 7v^8 + \cdots \longrightarrow \frac{7v^4}{1-v^4}$$

$$\therefore \; PV = \frac{v + 5v^2 + 3v^3 + 7v^4}{1 - v^4} \overset{v = .97}{=\!=} 127.36$$

2. Determine the present value of a perpetuity due with annual payments of 250, using an annual effective interest rate of 6% for the first 10 years, 5% for the following 10 years, and 4% thereafter.

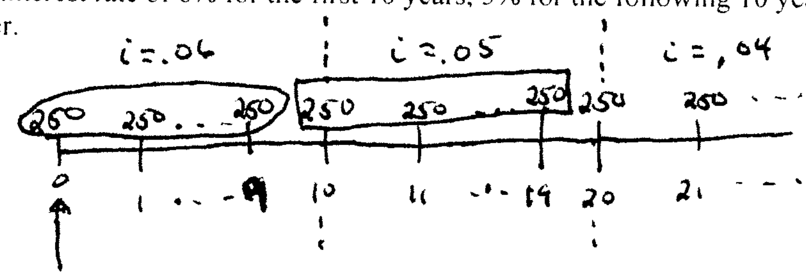
(A) 5060

(B) 5145

(C) 5225

(D) 5310

(E) 5405



$$PV = 250\,\ddot{a}_{\overline{10}|.06} + \boxed{250\,\ddot{a}_{\overline{10}|.05} \cdot v_{.06}^{10}} + \frac{250}{.04}(1.04)\,v_{.05}^{10}\,v_{.06}^{10}$$

$$\doteq 5310.51$$

3. A 10-year annuity immediate has bi-monthly (every two months) payments of 100 during the first year. The second year's bi-monthly payments are 150, the third year's bi-monthly payments are 200, and so on, with each year's level bi-monthly payments being 50 more than the previous year's payments. Determine the present value of this annuity immediately before the first payment, using a 6% annual effective interest rate.

   (A) Less than 13600

   (B) Greater than or equal to 13600, but less than 13700

   (C) Greater than or equal to 13700, but less than 13800

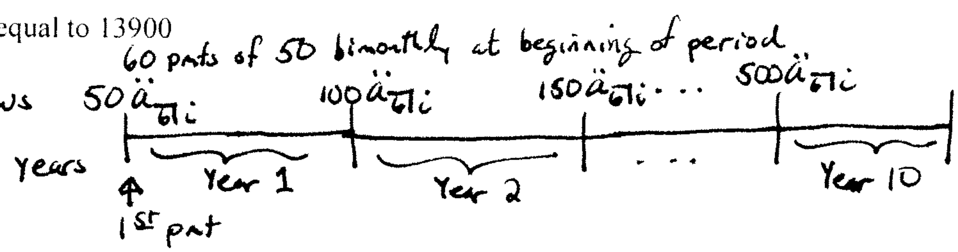
   (D) Greater than or equal to 13800, but less than 13900

   (E) Greater than or equal to 13900

i = 2-month eir

$(1+i)^6 = 1.06$

Rewrite as follows



$$PV = 50\ddot{a}_{\overline{60}|i} + 50\ddot{a}_{\overline{6}|i}(I\ddot{a})_{\overline{10}|.06}$$

$$\doteq 13758.37$$

4. Joe will receive 20 annual payments of 25000, with the first payment on his 65th birthday. Determine the present value, on Joe's 40th birthday, of this deferred annuity, using an 8% annual effective interest rate.

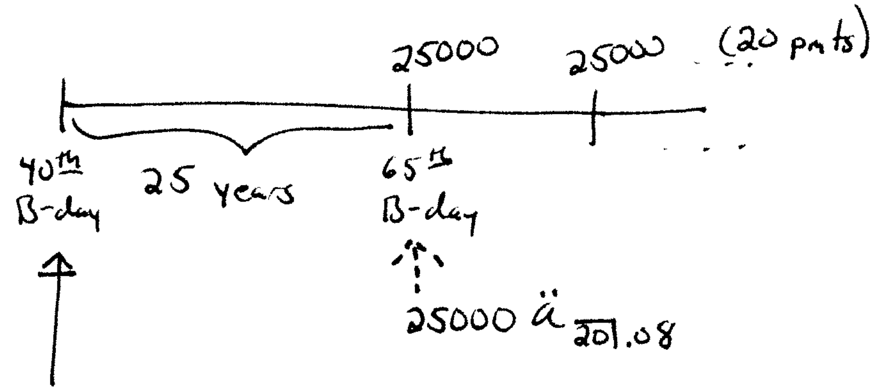
   (A) 35840

   (B) 38710

   (C) 44480

   (D) 52660

   (E) 56870



$$PV = 25000\left({}_{25|}\ddot{a}_{\overline{20}|.08}\right) \rightarrow \text{deferred notation}$$

$$= 25000\ddot{a}_{\overline{20}|.08} \cdot v^{25}_{.08}$$

$$\doteq 38707.88$$

5. Perpetuity A is a perpetuity due with monthly payments and an initial payment of 7. Subsequent payments are 3 more than their preceding payments. Perpetuity B is a perpetuity immediate with level monthly payments of X. Using the same annual effective interest rate, i, the present value of Perpetuity A is 5207 and the present value of Perpetuity B is 5760. Determine X.

$j = \text{meir}$

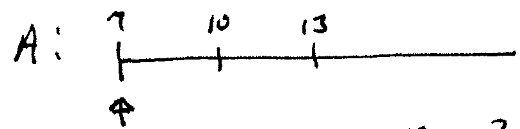
(A) 100

(B) 121

(C) 144

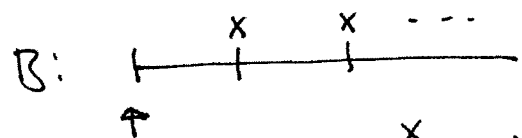
(D) 169

(E) 196



$$PV = 5207 = 7 + \frac{10}{i} + \frac{3}{i^2} \Rightarrow 5200i^2 - 10i - 3 = 0$$

$$\Rightarrow i = .025$$



$$PV = 5760 = \frac{X}{i} \Rightarrow X = 5760(.025) = 144$$

6. An annuity with semiannual payments has an initial payment of 1. Each subsequent payment is 1 more than its preceding payment until reaching a payment of 25. Following the payment of 25, the next seven payments are each equal to 26. Following the last payment of 26, payments decrease by 1 each semiannual period until reaching a final payment of 1. Determine the accumulated value of the annuity immediately after the last payment, using an annual effective interest rate of 10%.

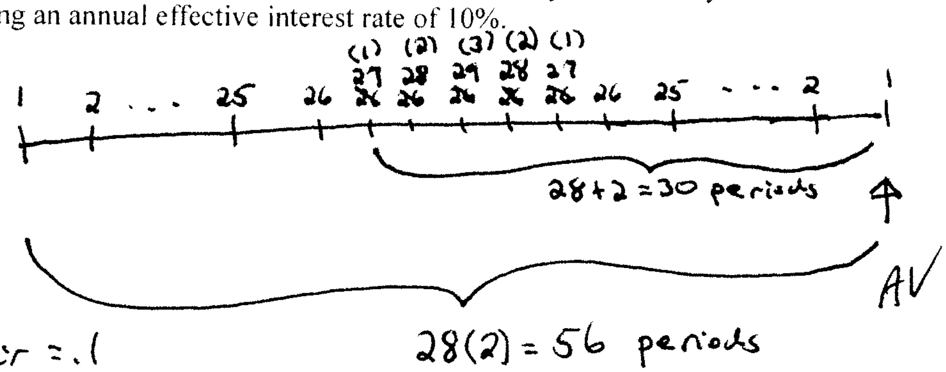
(A) 3520

(B) 3580

(C) 3640

(D) 3700

(E) 3760



$\text{aeir} = .1$

$i = \text{seir} = (1.1)^{1/2} - 1$

$$AV = \left(\ddot{a}_{\overline{29}|i}\right)^2 (1+i)^{56} - \left(\ddot{a}_{\overline{3}|i}\right)^2 (1+i)^{30}$$

$$\doteq 3700.56$$

7. A 20-year geometric annuity immediate with annual payments has an initial payment of 20. The sum of the payments is 739.6785. Determine the present value of the annuity using a 5% annual effective interest rate.

(A) 415

(B) 420

(C) 425

(D) 430

(E) 435

$$\begin{array}{cccccc} & 20 & 20r & \cdots & 20r^{19} \text{ (not needed)} \\ 0 & 1 & 2 & \cdots & 20 \end{array}$$

$$\Sigma = 739.6785 = 20(1 + r + \cdots \text{ (20 terms)})$$

$$r > 1 \implies 739.6785 = 20\, s_{\overline{20}|(r-1)}$$

$$\implies r \doteq 1.0605$$

$$\therefore PV = \frac{20}{1.05}\left(1 + \frac{1.0605}{1.05} + \cdots \text{ (20 terms)}\right)$$

$$= \frac{20}{1.05} \cdot s_{\overline{20}|\left(\frac{1.0605}{1.05} - 1\right)} \doteq 419.41$$

8. Using an annual effective interest rate of i, the accumulated value of an n-year annuity due with annual payments of 2 is 73.04, and the accumulated value of a $2n$-year annuity due with annual payments of 15 is 2996.46. Determine the present value of a $3n$-year annuity immediate with annual payments of 30 using i.

(A) 280

(B) 290

(C) 300

(D) 310

(E) 320

$$73.04 = 2\ddot{s}_{\overline{n}|} \implies \ddot{s}_{\overline{n}|} = 36.52$$

$$2996.46 = 15 \cdot \ddot{s}_{\overline{2n}|} = 15\, \ddot{s}_{\overline{n}|}\left(1 + (1+i)^n\right)$$

$$\implies (1+i)^n \doteq 4.469989$$

$$36.52 = \ddot{s}_{\overline{n}|} = \frac{(1+i)^n - 1}{d} \doteq \frac{4.469989 - 1}{d} \implies d \doteq .095016$$

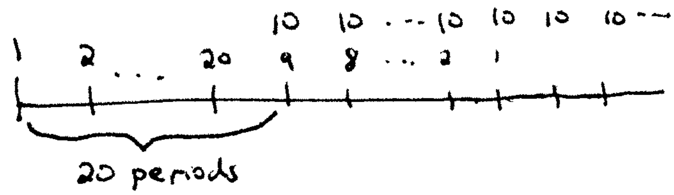
$$d = \frac{i}{1+i} \implies i = \frac{d}{1-d} \doteq .104992$$

$$(1+i)^n \doteq 4.469989 \implies n \doteq 15$$

$$PV = 30\, a_{\overline{3n}|i} = 30\, a_{\overline{45}|i} \doteq 282.54$$

9. A perpetuity due with annual payments has an initial payment of 1. Subsequent payments increase by 1 until reaching a payment of 20, at which time subsequent payments decrease by 1 until reaching a payment of 10. Payments then remain level at 10. Determine which expression represents the present value of the perpetuity, where d is the annual effective discount rate and v is the annual discount factor.

I. $(I\ddot{a})_{\overline{20}|} + (D\ddot{a})_{\overline{9}|} \cdot v^{20} + \frac{10}{d} \cdot v^{20}$

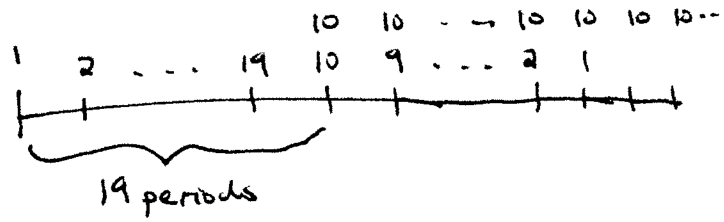


I. Yes

II. $(I\ddot{a})_{\overline{19}|} + (D\ddot{a})_{\overline{10}|} \cdot v^{19} + \frac{10}{d} \cdot v^{19}$

III. No
only values a finite # of payments

III. $\left(\ddot{a}_{\overline{20}|}\right)^2 + (I\ddot{a})_{\overline{10}|} \cdot v^{30}$



II. Yes

(A) I. and II.

(B) II. and III.

(C) I. and III.

(D) All of I., II., and III.

(E) None of the above

10. A 3-year annuity immediate with monthly payments has an initial payment of 200. Subsequent monthly payments are x% more than each preceding payment. Given that the amount of the 14[th] payment is 481.969, determine the present value of the annuity using a 9%, compounded monthly, interest rate. $i = .0075 = \text{meir}$

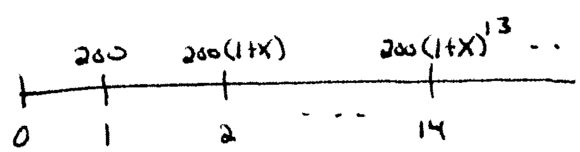
(A) 23750

(B) 24005

(C) 24255

(D) 24550

(E) 24735



$200(1+X)^{13} = 481.969 \Rightarrow X = .07$

$PV = \frac{200}{1.0075}\left(1 + \frac{1.07}{1.0075} + \cdots (36 \text{ terms})\right)$

$= \frac{200}{1.0075} \cdot s_{\overline{36}|\left(\frac{1.07}{1.0075} - 1\right)} \doteq 24734.70$