MAP 4170 Name:_______________________

Test 2 Date: March 4, 2014

Show sufficient work and clearly mark your answers. Each problem is worth 10 points.

1. An annuity has payments of 100 at the beginning of every 2 years for 20 years. Determine the accumulated value of the annuity one year after the final payment using an annual effective interest rate of 6%.

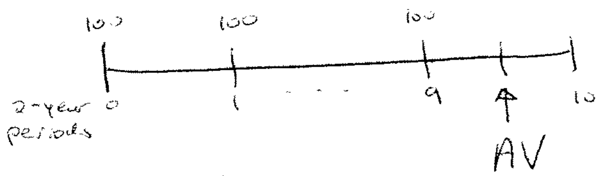
(A) 1395

(B) 1785

(C) 1895

(D) 2005

(E) 2125



aeir = 6%

baeir = 12.36%

$AV = 100\, s_{\overline{10}|.1236}(1.06) \doteq 1892.85$

2. A 5-year annuity immediate with semiannual payments has an initial payment of 30. Subsequent payments are 3 less than their preceding payment. Determine the present value of the annuity six months before the first payment using an annual effective interest rate of 5%.

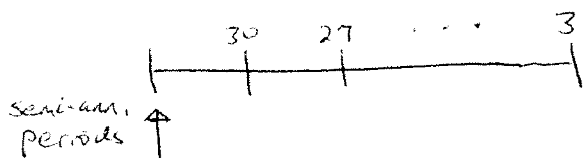
(A) 135

(B) 140

(C) 145

(D) 150

(E) 155



aeir = .05

Seir $= (1.05)^{1/2} - 1 = j$

$PV = 3(Da)_{\overline{10}|j} \doteq 149.92$

3. A perpetuity due has annual payments that form a geometric progression with common ratio 1.04. The initial payment 10. The present value of the perpetuity is 1050 using an annual effective interest rate of i. Determine i.

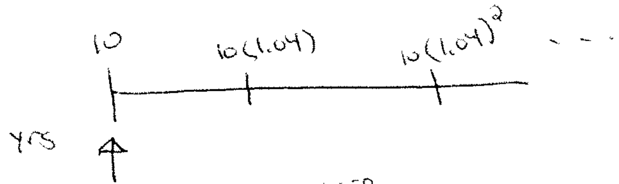
(A) 4.90%

(B) 4.95%

(C) 5.00%

(D) 5.05%

(E) 5.10%



$$PV = 1050 \overset{VEP}{=} 10 + 10(1.04)v_i + \cdots$$

$$\therefore\ 1050 = \frac{10}{1 - \frac{1.04}{1+i}}$$

$$\Longrightarrow i = .05$$

4. Jason deposits of 100 into an account at the end of each year for 20 years. Interest on the deposits is paid annually using an annual effective interest rate of i. The interest payments are reinvested into another account that pays an annual effective interest rate of 6%.

Chris deposits of 100 into an account at the end of each year for 20 years. Interest on the deposits is paid annually using an annual effective interest rate of i + .02. The interest payments are reinvested into another account that pays an annual effective interest rate of 6%.

Determine how much more Chris has than Jason at the end of 20 years.

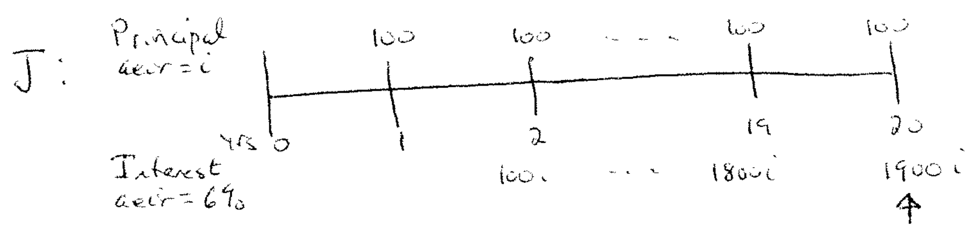
(A) 185

(B) 195

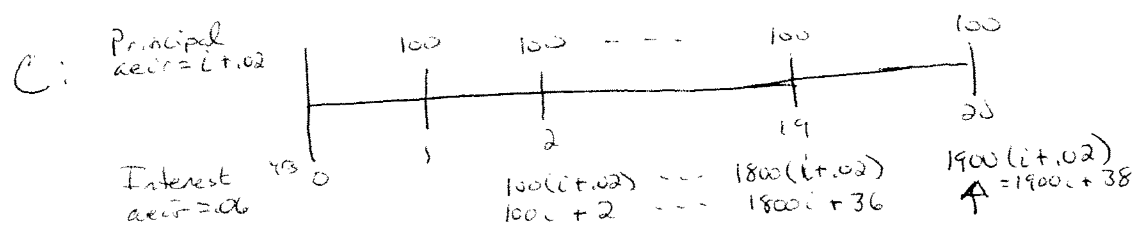
(C) 440

(D) 560

(E) 630



$$AV_J = 100(20) + 100i\,(Is)_{\overline{19|}.06}$$



$$AV_C = 100(20) + 100i\,(Is)_{\overline{19|}.06} + 2(Is)_{\overline{19|}.06}$$

$$\therefore AV_C - AV_J = 2(Is)_{\overline{19|}.06} = 559.52$$

5. An arithmetically increasing perpetuity due with semiannual payments has a present value of 4680 using a nominal interest rate 8%, compounded semiannually. Determine the amount of the 3rd payment, given the amount of the 8th payment is 54.

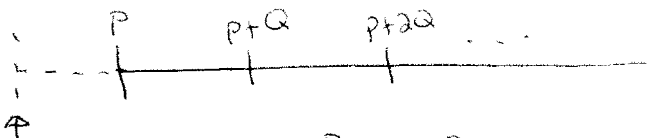
(A) 15

(B) 16

(C) 17

(D) 18

(E) 19



$\text{semi } r = .04$

$3^{rd}\text{ pmt} = P + 2Q$

$8^{th}\text{ pmt} = P + 7Q$

$PV = \frac{4680}{1.04} = \frac{P}{.04} + \frac{Q}{(.04)^2}$

$\therefore \begin{cases} 4500 = 25P + 625Q \\ 54 = P + 7Q \end{cases}$

$\Longrightarrow P = 5 \;\&\; Q = 7$

$\therefore 3^{rd}\text{ pmt} = 5 + 2(7) = 19$

6. An annual payment annuity has an initial payment of 3. Subsequent payments are 8 more than their preceding payment until reaching a payment of 115, after which subsequent payments are 8 less than their preceding payment until reaching a final payment of 3. Determine the present value of this annuity one year before the first payment using an annual effective interest rate of 6%.

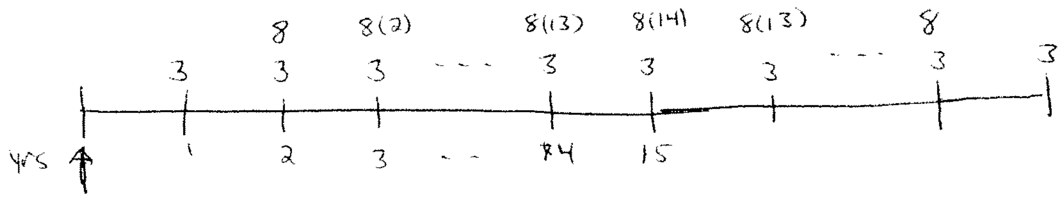
(A) 730

(B) 775

(C) 795

(D) 870

(E) 920

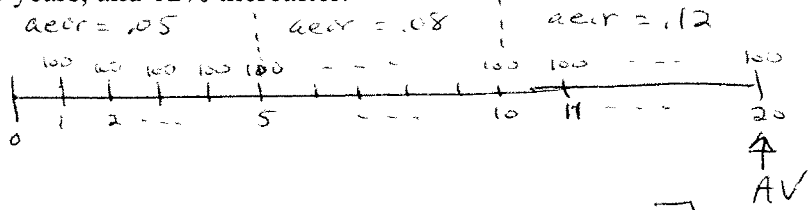


$PV = 3a_{\overline{29}|.06} + 8\left(\ddot{a}_{\overline{14}|.06}\right)^2 \cdot v_{.06}^2$

$= 3a_{\overline{29}|} + 8\left(a_{\overline{14}|}\right)^2$

$\doteq 731.95$

7. Determine the accumulated value of a 20-year annuity immediate with annual payments of 100 using an annual effective interest rate of 5% for the first five years, 8% for the next five years, and 12% thereafter.



(A) 5800

(B) 5875

(C) 5950

(D) 6025

(E) 6100

$$AV = \left[100\, s_{\overline{5|}.05}(1.08)^5 + 100\, s_{\overline{5|}.08}\right](1.12)^{10} + 100\, s_{\overline{10|}.12}$$

$$= 6098.58$$

8. For the first 15 years, a perpetuity immediate with annual payments, and initial payment of 7, has subsequent payments that are 17.81% more than their preceding payments. Thereafter, payments are 2% more than their preceding payments. Using an annual effective interest rate of 7.1%, determine the present value of the perpetuity.

(A) Less than 650

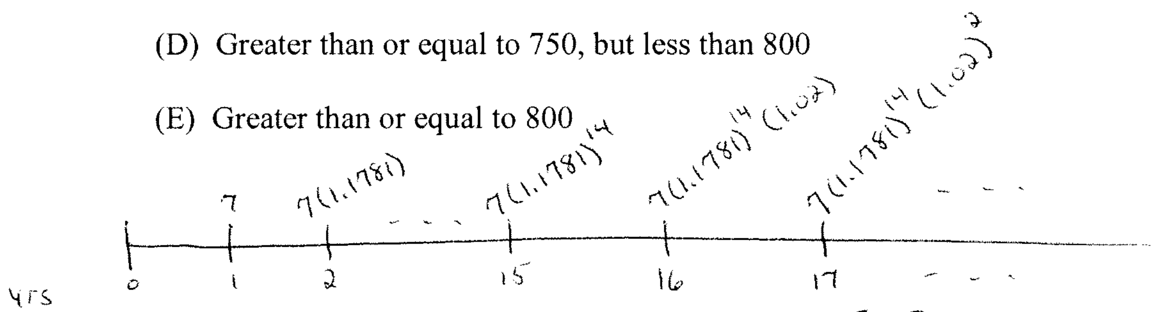
(B) Greater than or equal to 650, but less than 700

(C) Greater than or equal to 700, but less than 750

(D) Greater than or equal to 750, but less than 800

(E) Greater than or equal to 800

$v = \frac{1}{1.071}$



$$PV = \left[7v + 7(1.1781)v^2 + \cdots + 7(1.1781)^{14}v^{15}\right] + \left[7(1.1781)^{14}(1.02)v^{16} + 7(1.1781)^{14}(1.02)^2 v^{17} + \cdots\right]$$

$$= \left[\frac{7}{1.071}\left(1 + \underbrace{\frac{1.1781}{1.071}}_{=1.10} + \cdots + \left(\frac{1.1781}{1.071}\right)^{14}\right)\right] + \left[\frac{7(1.1781)^{14}(1.02)(1.071)^{-16}}{1 - \frac{1.02}{1.071}}\right]$$

$$= \frac{7}{1.071} \cdot s_{\overline{15|}.10} + \left[\frac{7(1.1781)^{14}(1.02)(1.071)^{-16}}{1 - \frac{1.02}{1.071}}\right]$$

$$= 704.07$$

9. Using an annual effective interest rate of i, the accumulated value of a $3n$-year annuity immediate with annual payments of $3K$ is triple the accumulated value of a $2n$-year annuity immediate with annual payments of $2K$. Also using i, the present value of an n-year annuity immediate with annual payments of 1 is 7.64. Determine i.

(A) 5.0%

(B) 5.1%

(C) 5.2%

(D) 5.3%

(E) 5.4%

$$\cancel{3}K s_{\overline{3n}|} = \cancel{3} \cdot 2K s_{\overline{2n}|}$$

$$\cancel{s_{\overline{n}|}}\left((1+i)^{2n} + (1+i)^n + 1\right) = 2\cancel{s_{\overline{n}|}}\left((1+i)^n + 1\right)$$

$$\therefore (1+i)^{2n} - (1+i)^n - 1 = 0$$

$$\Longrightarrow (1+i)^n = \frac{1 \pm \sqrt{1+4}}{2} = \frac{1+\sqrt{5}}{2}$$

$$7.64 = a_{\overline{n}|} = \frac{1-v^n}{i} = \frac{1-\left(\frac{2}{1+\sqrt{5}}\right)}{i}$$

$$\Longrightarrow i \doteq .05$$

10. Rounded to the nearest dollar, determine the present value of a 10-year annuity due with monthly payments of 100 using an annual effective interest rate of 6%.

(A) 9007

(B) 9052

(C) 9072

(D) 9117

(E) None of the above

$$aeir = .06 \Longrightarrow meir = (1.06)^{1/12} - 1 = j$$

$$PV = 100\, \ddot{a}_{\overline{120}|j}$$

$$\doteq 9117$$