MAP 4170 Name:____________________________

Test 2 Date: June 19, 2013

Show sufficient work and clearly mark your answers. Each problem is worth 10 points.

1. Bill deposits 50 at the beginning of every month for 10 years into an account that pays interest at a 2% quarterly effective interest rate. Determine the accumulated value in Bill's account 5 years after the last deposit.

(A) 13550

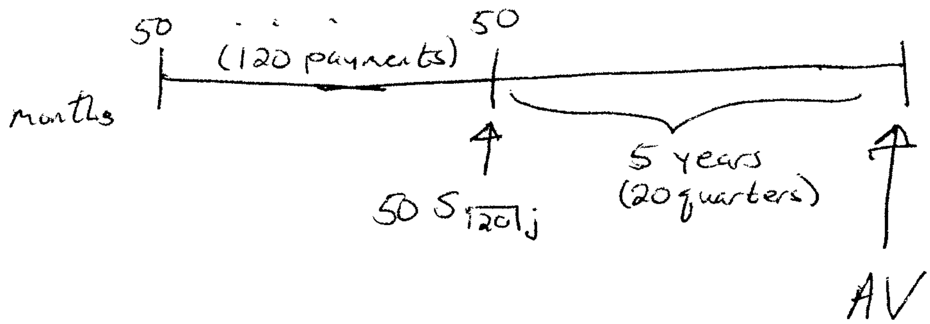
(B) 13595

(C) 13640

(D) 13685

(E) 13730

$.02 = \text{qeir} \qquad j = \text{meir} = (1.02)^{1/3} - 1$



$$AV = 50\, s_{\overline{120}|j}\,(1.02)^{20} \doteq 13552.48$$

2. A 20-year annuity immediate with annual payments has an initial payment of 1000. Each subsequent payment is 7.12% greater than its preceding payment. Determine the present value of this annuity using an annual effective interest rate of 4%.

(A) 14305

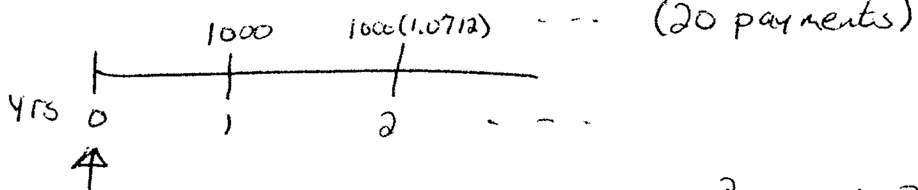
(B) 14734

(C) 15898

(D) 25837

(E) 26612

$\text{aeir} = .04$



$$PV \overset{\underline{VEP}}{=} 1000 v_{.04} + 1000(1.0712) v_{.04}^2 + \cdots \left(\begin{matrix}20\\ \text{payments}\end{matrix}\right)$$

$$= \frac{1000}{1.04}\left[1 + \underbrace{\frac{1.0712}{1.04}}_{=1.03} + \cdots \text{(20 payments)}\right.$$

$$\therefore PV = \frac{1000}{1.04}\, s_{\overline{20}|.03} \doteq 25836.90$$

3. A perpetuity due with annual payments has an initial payment of 75 and each subsequent payment is r % greater than its preceding payment. Using an annual effective discount rate of 5%, the present value of the perpetuity is 30,000. Determine r.

$aedr = .05 \Rightarrow v = .95 = adf$

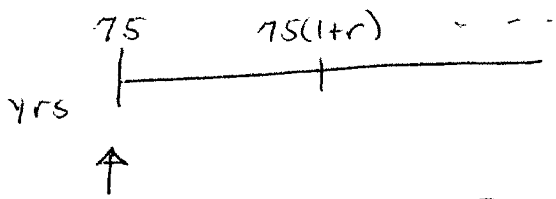
(A) 4.00

(B) 4.25

(C) 4.50

(D) 4.75

(E) 5.00



$PV = 30000 = 75 + 75(1+r)v + \cdots$

geometric with common ratio $= (1+r)v$

$= \frac{75}{1-(1+r)v} = \frac{75}{1-.95(1+r)}$

$\therefore\ 30000 = \frac{75}{1-.95(1+r)} \Rightarrow r = .05$

4. Sonny deposits 10,000 into an account that pays interest at the end of each year using an annual effective interest rate of 8%. The interest payments are reinvested in an account that pays an annual effective interest rate of 6%. The total accumulated value that Sonny has at the end of 10 years is X.

Cherry deposits 1000 at the beginning of each year for 10 years into an account that pays interest at the end of each year using an annual effective interest rate of 8%. The interest payments are reinvested in an account that pays an annual effective interest rate of 6%. The total accumulated value that Cherry has at the end of 10 years is Y.

Determine $X - Y$.

(A) 2950

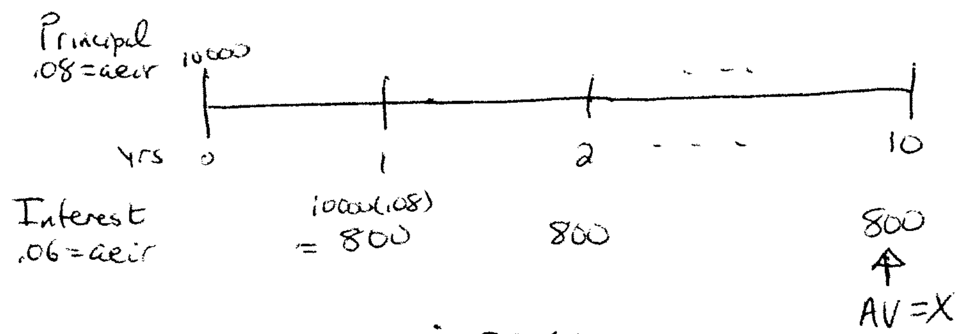
(B) 4100

(C) 5250

(D) 7600

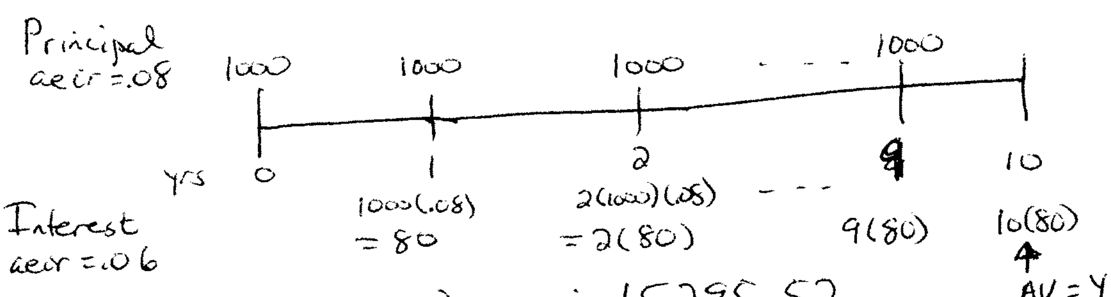
(E) 8200

S:



$X = 10000 + 800\, s_{\overline{10}|.06} \doteq 20544.64$

C:



$Y = 10(1000) + 80\,(Is)_{\overline{10}|.06} \doteq 15295.52$

$\therefore\ X - Y \doteq 5250$

5. Linda is due to receive payments at the end of each month for 5 years. The monthly payments are level during each year, but at the end of each year, the next year's monthly payments increase by 25. The first year's monthly payments are 100 each. Determine the present value of the payments one month before the first payment of 100, using a 6% annual effective interest rate. $i = .06 = \text{aeir} \Rightarrow j = \text{meir} = (1.06)^{1/12} - 1$

(A) 7520

(B) 7560

(C) 7600

(D) 7640

(E) 7680

100 @ EOM	125 @ EOM	. . .
0 — 1	1 — 2	2 . . . 5

Replace with
75 @ EOM for 5 years, plus

25 @ EOM	50 @ EOM	. . .
0 — 1	1 — 2	2 . . . 5

0
↑
PV

$$PV = 75 a_{\overline{60}|j} + 25 s_{\overline{12}|j} \cdot v_i + 50 s_{\overline{12}|j} \cdot v_i^2 + \cdots$$

$$= 75 a_{\overline{60}|j} + 25 s_{\overline{12}|j} \left(v_i + 2v_i^2 + \cdots + 5v_i^5 \right)$$

$$= 75 a_{\overline{60}|j} + 25 s_{\overline{12}|j} (Ia)_{\overline{5}|i} \doteq 7637.52$$

6. A perpetuity immediate has semiannual payments as follows: 1, 4, 7, 1, 4, 7, etc. Determine the present value of this perpetuity using an annual effective interest rate of 6.09%.

$\text{aeir} = .0609 \Rightarrow \text{seir} = (1.0609)^{1/2} - 1 = .03$

$\Rightarrow v = \frac{1}{1.03} = \text{sdr}$

(A) 125

(B) 130

(C) 135

(D) 140

(E) 145

Semi-annual periods

1	4	7	1	4	7	. . .

↑
PV

$$PV \overset{VEP}{=} v + v^4 + v^7 + \cdots \longrightarrow \frac{v}{1-v^3}$$

$$+ 4v^2 + 4v^5 + 4v^8 + \cdots \longrightarrow \frac{4v^2}{1-v^3}$$

$$+ 7v^3 + 7v^6 + 7v^9 + \cdots \longrightarrow \frac{7v^3}{1-v^3}$$

$$\therefore PV = \frac{v + 4v^2 + 7v^3}{1 - v^3} \doteq 131.36$$

$$(Da)_{\overline{n|}} = \frac{n - a_{\overline{n|}}}{i} \qquad (Ds)_{\overline{n|}} = \frac{n(1+i)^n - s_{\overline{n|}}}{i}$$

7. Which expression represents the present value of an n-year decreasing annuity due with annual payments of n, $n-1$, …, 2, 1. That is, which expression is equal to $(D\ddot{a})_{\overline{n|}}$.

(A) $\frac{n-a_{\overline{n|}}}{i}$

(B) $\frac{n-\ddot{a}_{\overline{n|}}}{i}$

(C) $\frac{n-\ddot{a}_{\overline{n|}}}{d}$

(D) $\frac{n(1+i)^n-s_{\overline{n|}}}{i}(1+i)^{n-1}$

(E) $\frac{n(1+i)^n-s_{\overline{n|}}}{i}(1+i)^{1-n}$

$$(D\ddot{a})_{\overline{n|}} = (Da)_{\overline{n|}} \cdot (1+i)$$
$$= (Ds)_{\overline{n|}} \cdot v^n \cdot (1+i)$$
$$= (Ds)_{\overline{n|}} \cdot (1+i)^{1-n}$$
$$= \frac{n(1+i)^n - s_{\overline{n|}}}{i}(1+i)^{1-n}$$

8. An arithmetically increasing perpetuity immediate with annual payments has a present value of 2400 using an annual effective interest rate of 10%. The amount of the third payment is 120. Determine the amount of the first payment.

(A) 20

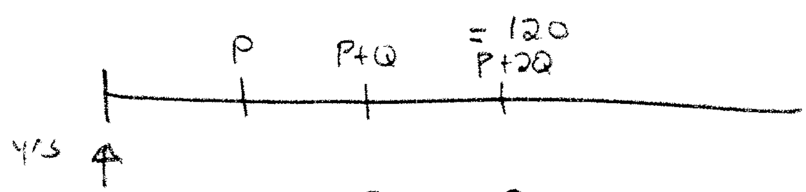
(B) 50

(C) 60

(D) 80

(E) 90

aeir = .1



$$PV = 2400 = \frac{P}{.1} + \frac{Q}{(.1)^2} = 10P + 100Q$$

$$\begin{cases} 10P + 100Q = 2400 \\ P + 2Q = 120 \end{cases}$$

$$\Longrightarrow P = 90$$

9. An annuity immediate with monthly payments has an initial payment of 5. Each subsequent payment increases by 1 over its preceding payment until reaching a payment of 15, at which time payments decrease by 1 until reaching a final payment of 5 again. Determine the accumulated value of this annuity immediately after the final payment of 5, using a nominal interest rate of 6% compounded monthly.

$i^{(12)} = .06$

meir $= .005$

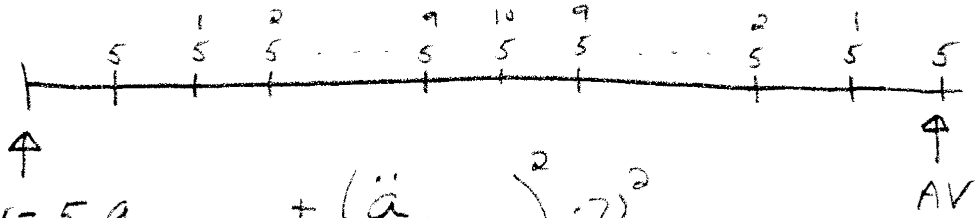
(A) 190

(B) 205

(C) 215

(D) 225

(E) 240



$$PV = 5\,a_{\overline{21}|.005} + \left(\ddot{a}_{\overline{10}|.005}\right)^2 \cdot v^2_{.005}$$

$$= 5\,a_{\overline{21}|.005} + \left(a_{\overline{10}|.005}\right)^2$$

$$AV = PV(1.005)^{21} \doteq 215.56$$

10. A 20-year annuity due has annual payments of X for the first 10 years and $2X$ for the next 10 years. Using an annual effective interest rate of 4% for the first 10 years and 6% thereafter, the present value of the annuity is 6831.51. Determine X.

aeir $= .04$ aeir $= .06$

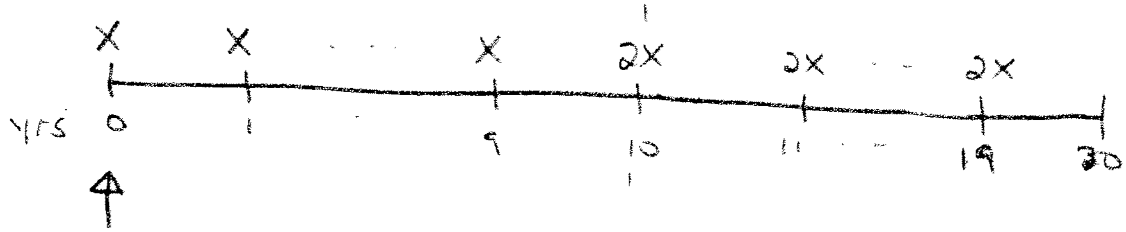
(A) 360

(B) 365

(C) 370

(D) 375

(E) 380



$$PV = 6831.51 = X\,\ddot{a}_{\overline{10}|.04} + 2X\,\ddot{a}_{\overline{10}|.06}\,v^{10}_{.04}$$

$$\Longrightarrow X \doteq 360$$