MAP 4170 Name:________________________
Test 2 Date: June 18, 2014

Show sufficient work and clearly mark your answers. Each problem is worth 10 points.

1. Determine the accumulated value of a 15-year annuity due with annual payments of 1000, using an annual effective interest rate of 4% for the first 10 years and an annual effective interest rate of 6% thereafter.

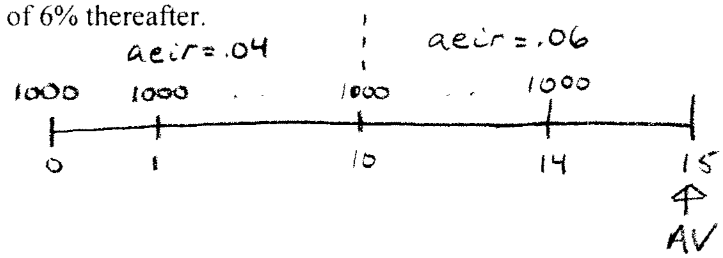
(A) 21400

(B) 21705

(C) 22165

(D) 22685

(E) 23010



$$AV = 1000\,\ddot{s}_{\overline{10}|.04}(1.06)^5 + 1000\,\ddot{s}_{\overline{5}|.06}$$

$$\doteq 22684.87$$

2. Perpetuity A is perpetuity due with semiannual payments. The first payment is 4 and subsequent payments are X more than their preceding payments. Perpetuity B is a level perpetuity immediate with semiannual payments of X. Using an annual effective interest rate equal to i, the present value of Perpetuity A is 4284. Using the same interest rate, the present value of Perpetuity B is 200. Determine $1000X$.

(A) 9980

(B) 9990

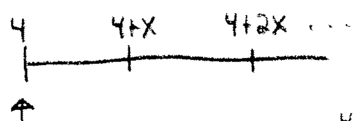
(C) 10000

(D) 10010

(E) 10020

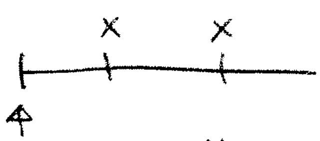
$j = \text{seir}$

A:



$$PV = 4284 = 4 + \frac{4+X}{j} + \frac{X}{j^2}$$

B:



$$PV = 200 = \frac{X}{j}$$

$$\therefore\ 4284 = 4 + \frac{4}{j} + \frac{X}{j} + \frac{X/j}{j} = 4 + \frac{4}{j} + 200 + \frac{200}{j}$$

$$\Rightarrow j = .05 \Rightarrow X = 200(.05) = 10$$

$$\Rightarrow 1000X = 10{,}000$$

3. John deposits 5000 into an account that pays interest quarterly using an interest rate of 8% compounded quarterly. Upon receipt of each interest payment, John reinvests the payment in an account that pays a quarterly effective interest rate of 3%. John's accumulated value at the end of 5 years is X.

   Jane deposits Y at the end of each quarter into an account that pays interest quarterly using an interest rate of 8% compounded quarterly. Upon receipt of each interest payment, Jane reinvests the payment in an account that pays a quarterly effective interest rate of 3%. Jane's accumulated value at the end of 5 years is also X.

   Determine Y to the nearest dollar.

   $5000(.02) = 100$

   (A) 109

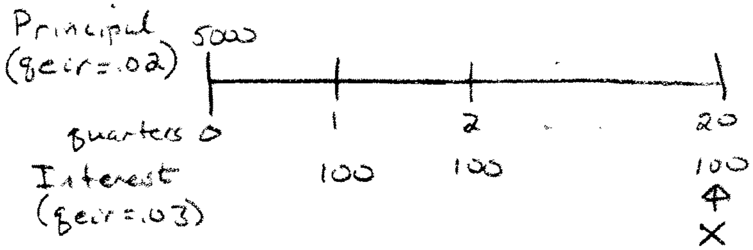
   (B) 306

   (C) 313

   (D) 578

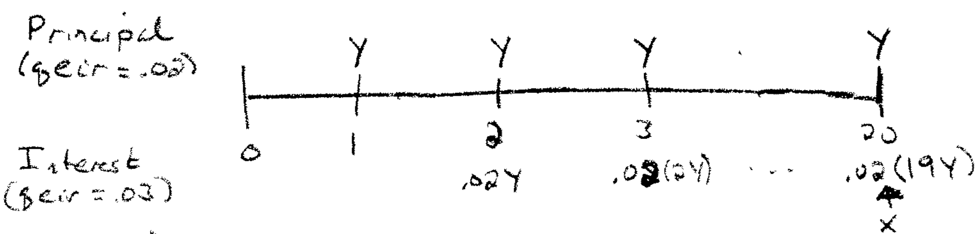
   (E) 587

John:



$$X = 5000 + 100\, s_{\overline{20}|.03}$$

Jane:



$$X = 20Y + .02Y\,(Is)_{\overline{19}|.03} = 5000 + 100\, s_{\overline{20}|.03} \Rightarrow Y \doteq 312.73$$

4. The initial payment of a perpetuity with semiannual payments is 200. The next payment is 400 and payments continue to alternate between 200 and 400. Using a discount rate of 12% compounded quarterly, determine the present value of the perpetuity one year before the first payment.

   (A) 4450

   (B) 4550

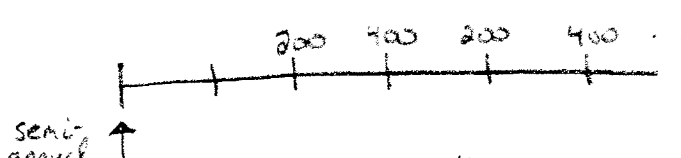
   (C) 4650

   (D) 4750

   (E) 4850

$d^{(4)} = .12 \Rightarrow d = \frac{.12}{4} = .03 = \text{qedr} \Rightarrow 1 - .03 = .97 = \text{qdf}$

$\Rightarrow v = (.97)^2 = .9409 = \text{sdf}$



$$PV = 200v^2 + 200v^4 + \cdots \longrightarrow \frac{200v^2}{1-v^2}$$
$$+\ 400v^3 + 400v^5 + \cdots \longrightarrow \frac{400v^3}{1-v^2}$$

$$\therefore\ PV = \frac{200v^2 + 400v^3}{1 - v^2} \ \underset{v=.9409}{=} \ 4448.26$$

5. An annuity immediate with annual payments has an initial payment of 2. Subsequent payments increase by 2 until reaching a payment of 20. The payment following the payment of 20 is also 20, and then subsequent payments decrease by 2 until reaching a final payment of 2. The present value of this annuity is 180 using an annual effective interest rate of i. Determine i.

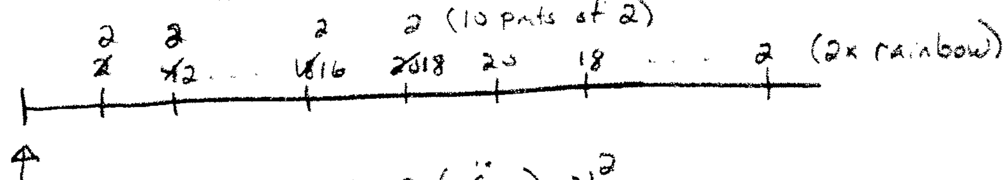
(A) 1.75%

(B) 2.00%

(C) 2.25%

(D) 2.50%

(E) 2.75%



$$PV = 180 = 2a_{\overline{10}|} + 2\underbrace{(\ddot{a}_{\overline{10}|})\cdot v^2}_{=(a_{\overline{10}|})^2}$$

$$\therefore\ 180 = 2a_{\overline{10}|} + 2(a_{\overline{10}|})^2 \quad (\text{quadratic in } a_{\overline{10}|})$$

$a = 2$
$b = 2$
$c = -180$

$$\Longrightarrow a_{\overline{10}|} = \frac{-2 \pm \sqrt{4 - 4(2)(-180)}}{2(2)} = 9$$

$$\Longrightarrow i \doteq 1.963\%$$

6. The initial payment of a perpetuity with annual payments is 30. Subsequent payments are 5% more than each preceding payment. Determine the present value of the perpetuity, one year before the first payment, using an interest rate of 8% compounded annually.

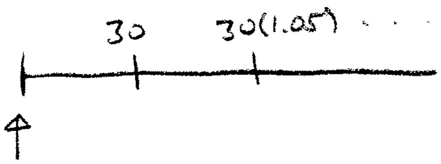
(A) 900

(B) 925

(C) 950

(D) 975

(E) 1000



$$PV = 30v_{.08} + 30(1.05)v_{.08}^2 + \cdots$$

$$= \frac{30v_{.08}}{1 - 1.05v_{.08}} = \frac{30/1.08}{1 - \frac{1.05}{1.08}} = 1000$$

7. Determine the present value of a perpetuity due with annual payments in which the first payment is 30, subsequent payments decrease by 3 until reaching a payment of 9, and then payments remain level at 9. Use an interest rate of 4% per annum.

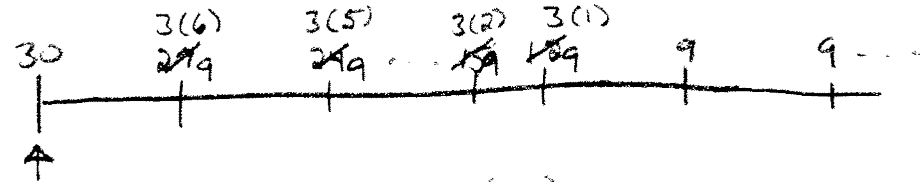
(A) 300

(B) 310

(C) 320

(D) 330

(E) 340



$$PV = 30 + \frac{9}{.04} + 3(Da)_{\overline{6|}.04}$$

$$= 30 + 225 + 3\left(\frac{6 - a_{\overline{6|}}}{.04}\right)$$

$$\doteq 311.84$$

8. A 20-year annuity has level payments at the end of each month during each year. The first year's monthly payments are 50 each. Subsequent years' monthly payments are 5% greater than the previous year's monthly payments. Using an annual effective interest rate of 3%, determine the accumulated value of this annuity immediately after the last payment.

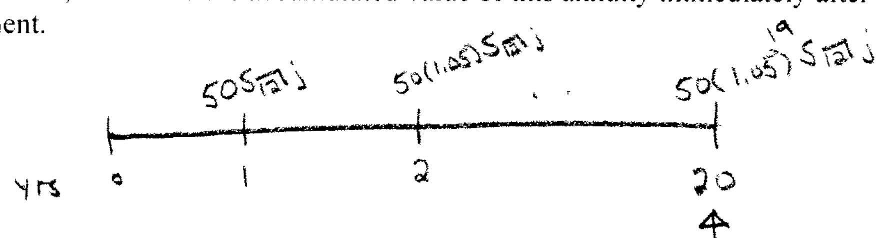
(A) 25120

(B) 25270

(C) 25760

(D) 37130

(E) 37850



AV

aecr = .03

$j = \text{mecr} = (1.03)^{1/12} - 1$

$$AV = 50(1.05)^{19} s_{\overline{12|}j} + 50(1.05)^{18} s_{\overline{12|}j}(1.03) + \cdots \text{ 20 terms}$$

$$= 50(1.05)^{19} s_{\overline{12|}j}\left(1 + \frac{1.03}{1.05} + \cdots\right)$$

$$= 50(1.05)^{19} s_{\overline{12|}j}\ \ddot{a}_{\overline{20|}\left(\frac{1.05}{1.03}-1\right)}$$

$$\doteq 25763.19$$

9. Beginning with a deposit on July 1, 2015, Wyly expects to make quarterly deposits of 1500 into an account, up to and including a final deposit on July 1, 2055. Unfortunately Wyly is unable to make any of the deposits during the years 2040 through 2045, but he makes all other deposits. Determine the accumulated value in Wyly's account immediately after the final deposit on July 1, 2055, using a nominal interest rate of 6% compounded quarterly.

$q\,eir = \frac{.06}{4} = .015$

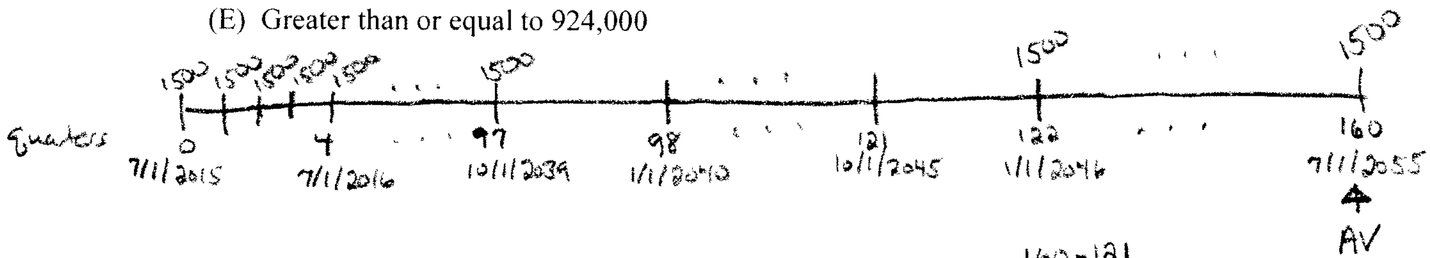
A) Less than 921,000

(B) Greater than or equal to 921,000, but less than 922,000

(C) Greater than or equal to 922,000, but less than 923,000

(D) Greater than or equal to 923,000, but less than 924,000

(E) Greater than or equal to 924,000



$$AV = 1500\, s_{\overline{161}|.015} - 1500\, s_{\overline{24}|.015}(1.015)^{160-121} \doteq 922{,}327.67$$

10. Amanda deposits 400 at the end of each 4-month period. The accumulated value of Amanda's deposits at the end of 10 years is half of the accumulated value of Amanda's deposits at the end of 15 years. Determine the accumulated value of Amanda's deposits at the end of 20 years. i = 4-month eir

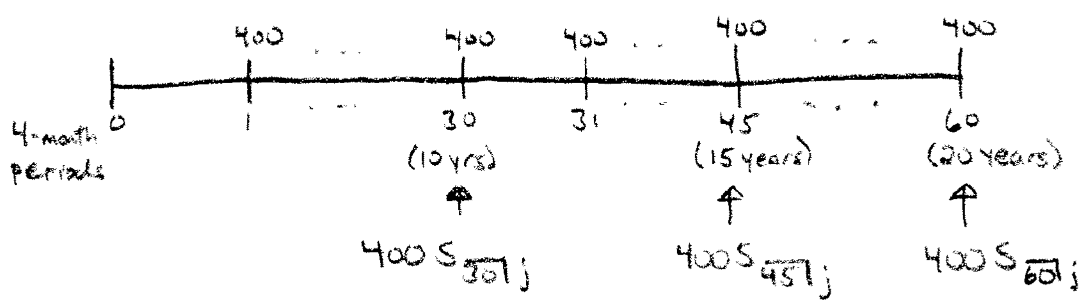
(A) 71825

(B) 72450

(C) 73260

(D) 74170

(E) 74890



$$\cancel{400}\, s_{\overline{30}|i} = \tfrac{1}{2}\cdot \cancel{400}\, s_{\overline{45}|i} \Longrightarrow s_{\overline{45}|} = 2\, s_{\overline{30}|}$$

$$s_{\overline{45}|} = s_{\overline{15}|}\left(1 + (1+i)^{15} + (1+i)^{30}\right) \qquad s_{\overline{30}|} = s_{\overline{15}|}\left(1 + (1+i)^{15}\right)$$

$$\therefore\ \cancel{s_{\overline{15}|}}\left(1 + (1+i)^{15} + (1+i)^{30}\right) = 2\cancel{s_{\overline{15}|}}\left(1 + (1+i)^{15}\right)$$

(quadratic in $(1+i)^{15}$; $a=1$ $b=-1$ $c=-1$)

$$\Rightarrow (1+i)^{15} = \frac{1 \pm \sqrt{1-4(1)(-1)}}{2} = \frac{1+\sqrt{5}}{2} \Longrightarrow i \doteq .0326$$

$$\therefore\ 400\, s_{\overline{60}|i} \doteq 71827.44$$