MAP 4170 Name: Key
Test 2 Date: June 18, 2015

Show sufficient work and clearly mark your answers. Each problem is worth 10 points.

1. Perpetuity A is perpetuity immediate with annual payments. The first payment is 3 and subsequent payments are 3 more than their preceding payments. Perpetuity B is a level payment perpetuity immediate with annual payments of 3. Using an annual effective interest rate of 5%, the present value of Perpetuity A is X, and the present value of Perpetuity B is Y. Determine the ratio $\frac{X}{Y}$

(A) 18.4

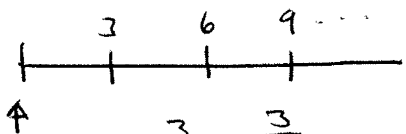
(B) 19.1

(C) 19.8

(D) 20.3

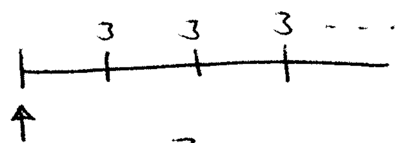
(E) 21.0

A:



$PV = X = \frac{3}{.05} + \frac{3}{(.05)^2} = 1260$

B:



$PV = Y = \frac{3}{.05} = 60$

$\therefore \frac{X}{Y} = 21$

2. The initial payment of a 20-year annuity immediate with semiannual payments is 30. Subsequent payments are 5% more than each preceding payment. Determine the present value of the annuity, immediately before the first payment, using an interest rate of 10.25% compounded annually.

(A) 575

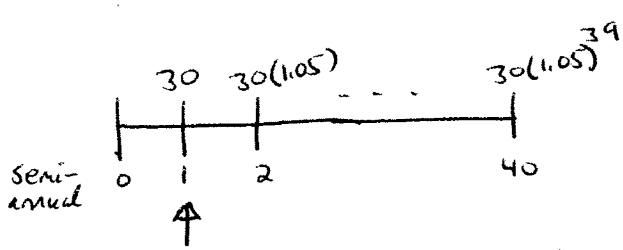
(B) 600

(C) 1100

(D) 1150

(E) 1200

aeir $= .1025 \Rightarrow$ seir $= .05 \Rightarrow$ sdf $= v = \frac{1}{1.05}$



$PV = 30 + 30\underbrace{\cancel{(1.05)v}}_{=1} + \cdots$ (40 terms)

$= 30(40) = 1200$

3. A 20-year annuity due with annual payments has a first payment of 1000 and each subsequent payment is 30 less than its preceding payment. Calculate the accumulated value of this annuity at a nominal interest rate of 4% compounded annually for the first 10 years, and a 3% annual effective interest rate thereafter.

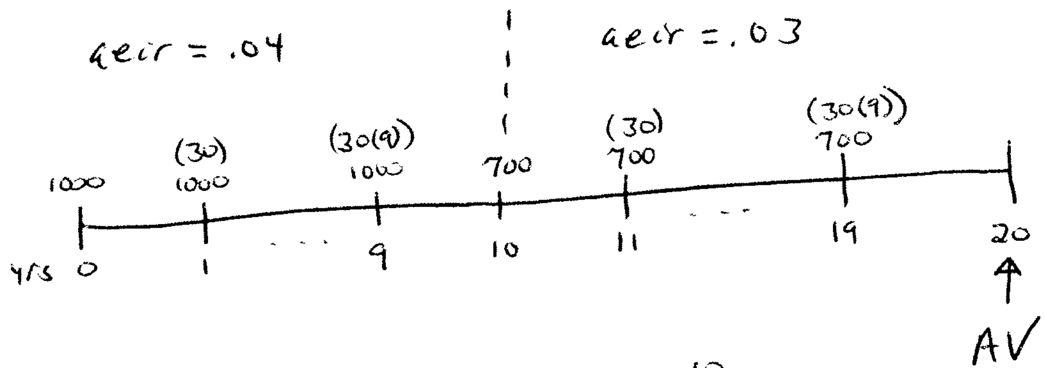
(A) 21035

(B) 21235

(C) 21435

(D) 21635

(E) 21835



$$AV = \left[1000\,\ddot{s}_{\overline{10}|.04} - 30\,(I\ddot{s})_{\overline{9}|.04}\right](1.03)^{10}$$

$$+\ 700\,\ddot{s}_{\overline{10}|.03} - 30\,(I\ddot{s})_{\overline{9}|.03}$$

$$\therefore AV = 21435.36$$

4. Greg buys a 30-year annuity immediate with level annual payments for 6000. This price is the present value of the annuity using an annual effective interest rate of 4%. Immediately before receiving the 7th payment, Greg sells the annuity to Linda. The price Linda pays, *P*, is the present value at the time she purchases the annuity, of the remaining future payments, using an annual effective interest rate of 3%. Determine *P*.

(A) 5350

(B) 5525

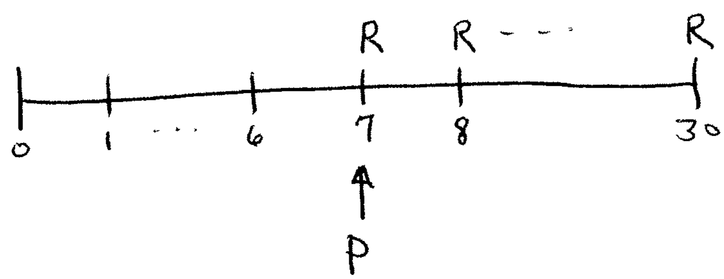
(C) 5700

(D) 5875

(E) 6050

R = level annual payments

$$\therefore\ 6000 = R\,a_{\overline{30}|.04} \implies R \doteq 346.98$$



$$P = R \cdot \ddot{a}_{\overline{24}|.03} = 6052.59$$

5. The present value of a $4n$-year annuity due with annual payments of R is 737.62, and the present value of a $2n$-year annuity due with annual payments of $2R$ is 1173.86, with both annuity values determined using an annual effective discount rate, d. Using the same d, the accumulated value of a $3n$-year annuity due with annual payments of 3 is 688.78. Determine d.

(A) 2.8%

(B) 2.9%

(C) 3.0%

(D) 3.1%

(E) 3.2%

$$737.62 = R\ddot{a}_{\overline{4n}|} \qquad \ddot{a}_{\overline{4n}|} = \ddot{a}_{\overline{2n}|}\left(1+v^{2n}\right)$$

$$1173.86 = 2\underline{R\ddot{a}_{\overline{2n}|}}$$

$$\therefore 737.62 = \underline{R\ddot{a}_{\overline{2n}|}}\left(1+v^{2n}\right) = \frac{1173.86}{2}\left(1+v^{2n}\right)$$

$$\Rightarrow v^{2n} = \frac{737.62(2)}{1173.86} - 1$$

$$688.78 = 3\ddot{s}_{\overline{3n}|} = 3\,\frac{(1+i)^{3n}-1}{d} = 3\cdot\frac{v^{-3n}-1}{d} = 3\,\frac{\left(v^{2n}\right)^{-\frac{3}{2}}-1}{d}$$

$$\therefore d \doteq .029125$$

6. Ed invests 1000 in an account that pays interest annually at an annual effective interest rate of 5%. Each interest payment Ed receives, he reinvests in an account that pays an annual effective interest rate equal to i. Just after the interest payment at the end of 10 years, Ed has accumulated interest of 659.

Nancy invests 100 at the end of each year in an account that pays interest annually at an annual effective interest rate of 5%. Each interest payment Nancy receives, she also reinvests in an account that pays an annual effective interest rate equal to i.

Determine the amount of accumulated interest Nancy has just after the interest payment at the end of 10 years.

(A) 208

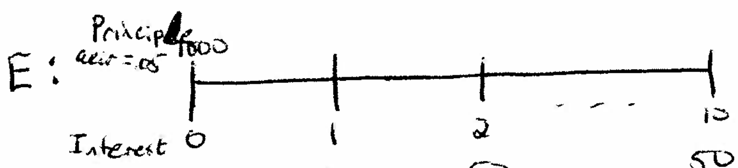
(B) 265

(C) 323

(D) 377

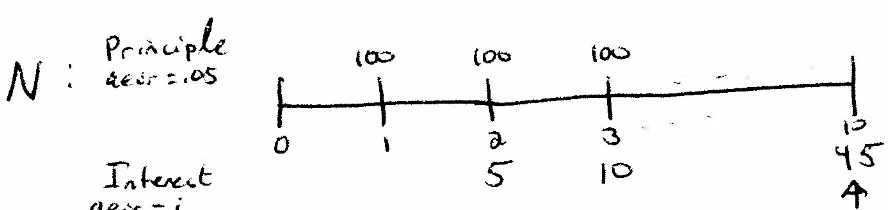
(E) 420

AI = Accumulated Interest



$$AI = 50\,s_{\overline{10}|i} \overset{\text{given}}{=} 659$$

$$\therefore i \doteq .06$$



$$AI = 5(Is)_{\overline{9}|i} \doteq 265.06$$

7. A 20-year annuity has level payments at the end of each month during each year. The first year's monthly payments are 100 each. Subsequent years' monthly payments are 3% less than the previous year's monthly payments. Using an annual effective interest rate of 3%, determine the accumulated value of this annuity immediately after the last payment.

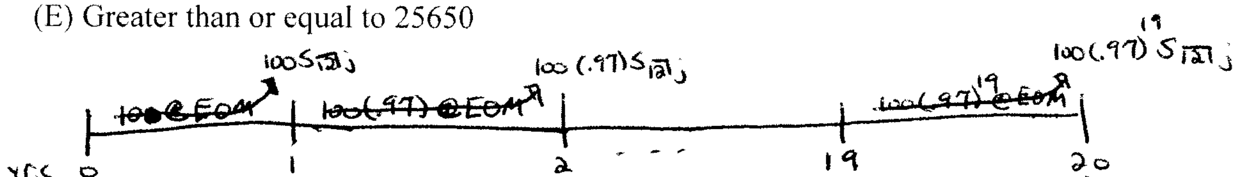
   (A) Less than 25500

   (B) Greater than or equal to 25500, but less than 25550

   (C) Greater than or equal to 25550, but less than 25600

   (D) Greater than or equal to 25600, but less than 25650

   (E) Greater than or equal to 25650



aeir = .03 $\quad j = \text{meir} = (1.03)^{1/12} - 1$ $\qquad \uparrow$ AV

$$AV = 100(.97)^{19} s_{\overline{12}|j} + 100(.97)^{18} s_{\overline{12}|j}(1.03) + \cdots \text{ (20 terms)}$$

$$= 100(.97)^{19} s_{\overline{12}|j}\Big(1 + \underbrace{\tfrac{1.03}{.97}}_{r>1} + \cdots\Big) = 100(.97)^{19} s_{\overline{12}|j} \cdot s_{\overline{20}|\left(\frac{1.03}{.97}-1\right)}$$

$$\therefore AV = 25591.62$$

8. The initial payment of a perpetuity with semiannual payments is 12. The next payment is 15 followed by a payment of 9. Payments continue in this pattern: 12, 15, 9, 12, 15, 9, … Using an annual effective discount rate of 19%, determine the present value of the perpetuity one year before the first payment.

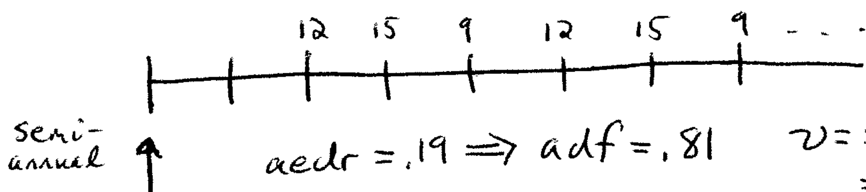
   (A) 47

   (B) 54

   (C) 73

   (D) 98

   (E) 122



aedr = .19 $\Rightarrow$ adf = .81 $\qquad v = \text{sdf} \Rightarrow v^2 = .81 \Rightarrow v = .9$

$$PV = 12v^2 + 12v^5 + \cdots \longrightarrow \frac{12v^2}{1-v^3}$$

$$+ 15v^3 + 15v^6 + \cdots \longrightarrow \frac{15v^3}{1-v^3}$$

$$+ 9v^4 + 9v^7 + \cdots \longrightarrow \frac{9v^4}{1-v^3}$$

$$\therefore PV = \frac{12v^2 + 15v^3 + 9v^4}{1-v^3} \overset{v=.9}{=\!=} 98$$

9. A perpetuity immediate with annual payments has an initial payment of 30. Subsequent payments decrease by 3 until reaching a payment of 3. The payment following the payment of 3 is also 3, and then subsequent payments increase by 3 until reaching a payment of 21. Payments then remain level at 21. Determine the present value of this perpetuity using an annual effective interest rate of 3%

aeir = .03

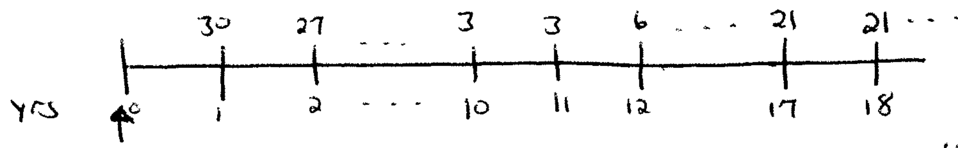
(A) 615

(B) 620

(C) 625

(D) 630

(E) 635



$$PV = 3(Da)_{\overline{10|}} + 3(Ia)_{\overline{7|}} \cdot v^{10} + \frac{21}{.03} \cdot v^{17}$$

$$\Rightarrow PV = 624.48$$

10. An annuity has payments of 1000 at the end of every 3 years for 30 years. Determine the present value of the annuity one year before the first payment using a nominal discount rate of 6% compounded monthly.

(A) 4765

(B) 4780

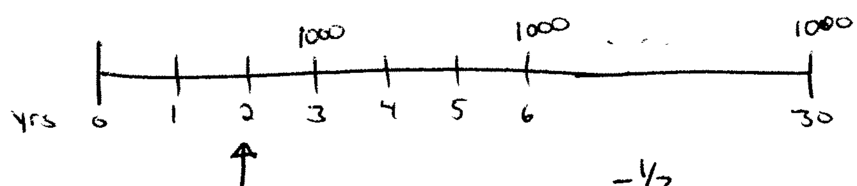
(C) 5040

(D) 5050

(E) 5060

$$d^{(12)} = .06 \Rightarrow medr = \frac{.06}{12} = .005$$

$$j = \text{3-year eir} \Rightarrow 1+j = (1-.005)^{-36} = (.995)^{-36}$$



$$PV = 1000\,\ddot{a}_{\overline{10|}j}\,(1+j)^{-1/3} = 1000\,\ddot{a}_{\overline{10|}j}\,(.995)^{12}$$

$$\Rightarrow PV = 4764.64$$