MAP 4170 Name:____________________

Test 2 Date: June 16, 2016

Show sufficient work and clearly mark your answers. Each problem is worth 10 points.

1. A perpetuity due has annual payments of 200. Determine the present value of the perpetuity using an interest rate of 4%, compounded semiannually.

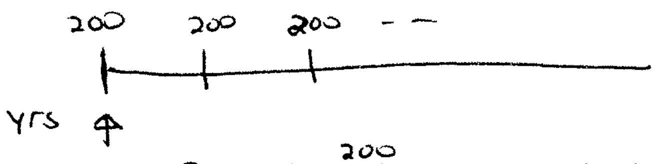
(A) 4950

(B) 5050

(C) 5150

(D) 5250

(E) 5350



$PV = 200 + \frac{200}{a}$ $\quad a = a_{eir} = (1.02)^2 - 1$

$\therefore PV \doteq 5150.50$

2. A 10-year annuity with semiannual payments has a first payment of 100. The next two payments are also 100, and then subsequent payments increase by 50 over their previous payment. Determine the present value of the annuity immediately before the first payment, using an interest rate of 10%, compounded semiannually.

$s = s_{eir} = .05$

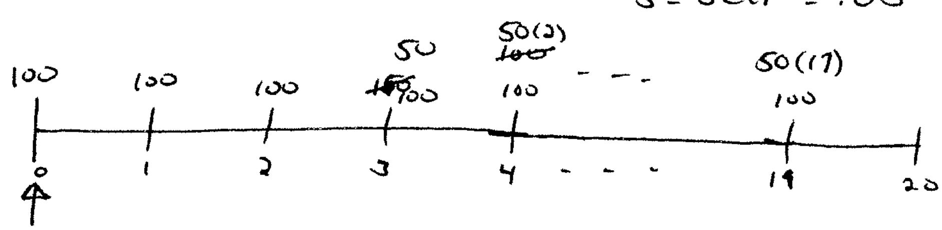
(A) 4750

(B) 4980

(C) 5125

(D) 5320

(E) 5550



$PV = 100\,\ddot{a}_{\overline{20}|.05} + 50\,(I\ddot{a})_{\overline{17}|.05}\, v^2_{.05}$

$= 5318.26$

3. An annuity has payments of 100 at the end of each three-year period for 30 years. Determine the accumulated value of this annuity, one year after the final payment, using an annual effective interest rate of 6%.

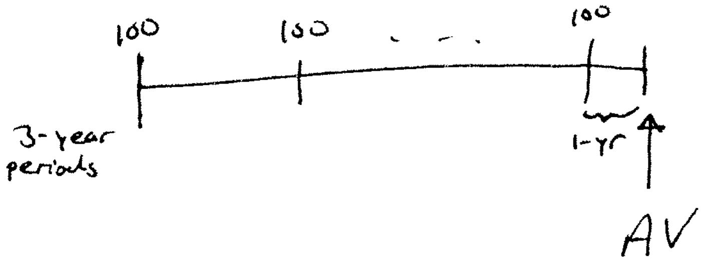
(A) 2480

(B) 2630

(C) 2780

(D) 2930

(E) 3080



$$AV = 100\, s_{\overline{10}|t} \cdot (1.06) \qquad t = t_{aeir} = (1.06)^3 - 1$$

$$\Rightarrow AV = 2632.29$$

4. A perpetuity immediate has annual payments 5, 10, 15, …, 65, 70, 65, … , 30, 30, 30, … Determine the present value of this perpetuity using an 8% annual effective interest rate.

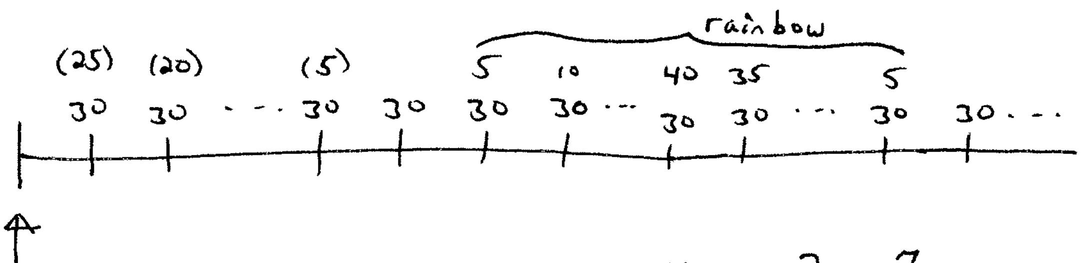
(A) 415

(B) 425

(C) 435

(D) 445

(E) 455



$$PV = \frac{30}{.08} - 5 \cdot (Da)_{\overline{5}|.08} + 5 \cdot \left(\ddot{a}_{\overline{8}|.08}\right)^2 \cdot v_{.08}^{7}$$

$$= 424.42$$

5. A 20-year annuity has level payments at the end of each month during each year. The first year's monthly payments are 100 each. Subsequent years' monthly payments are 3 less than the previous year's monthly payments. Using an annual effective interest rate of 3%, determine the accumulated value of this annuity immediately after the last payment.

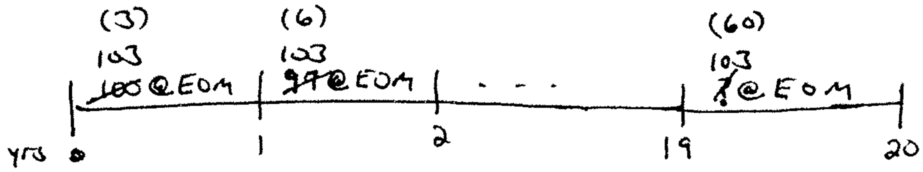
(A) 24,150

(B) 24,210

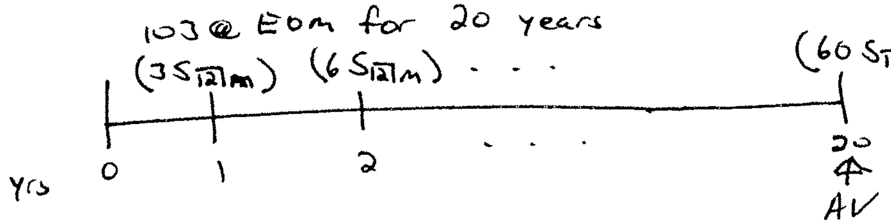
(C) 24,270

(D) 24,330

(E) 24,390



Replace with the following timeline (equivalent)



$m = $ m.e.i.r $= (1.03)^{1/12} - 1$

$$AV = 103 s_{\overline{240}|m} - 3 s_{\overline{12}|m} \cdot (Is)_{\overline{20}|.03}$$

$$= 24328.24$$

6. During each year, a perpetuity due has quarterly payments of X, 20, 60, 50. The present value of the perpetuity is 2500 using a discount rate of 8%, compounded quarterly. Determine the present value of the perpetuity using an interest rate of 8%, compounded quarterly.

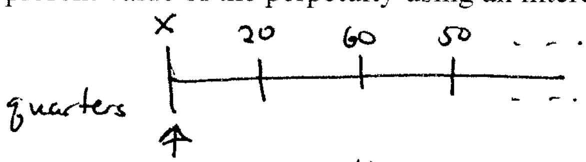
(A) 2510

(B) 2520

(C) 2530

(D) 2540

(E) 2550



$$PV = X + Xv^4 + \cdots \longrightarrow \frac{X}{1-v^4}$$
$$+ 20v + 20v^5 + \cdots \longrightarrow \frac{20v}{1-v^4}$$
$$+ 60v^2 + 60v^6 + \cdots \longrightarrow \frac{60v^2}{1-v^4}$$
$$+ 50v^3 + 50v^7 + \cdots \longrightarrow \frac{50v^3}{1-v^4}$$

$v = $ q.d.f

$$\therefore PV = \frac{X + 20v + 60v^2 + 50v^3}{1-v^4}$$

$d^{(4)} = .08 \Rightarrow$ q.e.d.r $= .02 \Rightarrow v = .98$ gives $PV = 2500$

$\therefore X = 69.796$

$\therefore$ using $i^{(4)} = .08$, q.e.i.r $= .02 \Rightarrow v = \frac{1}{1.02} \Rightarrow PV = 2549.95$

7. A perpetuity has semi-annual payments that follow an arithmetic progression. The third payment is 125 and the sixth payment is 275. Determine the present value immediately before the first payment using 5% annual effective.

(A) Less than 21,000

(B) Greater than or equal to 21000, but less than 42000

(C) Greater than or equal to 42000, but less than 63000

(D) Greater than or equal to 63000, but less than 84000

(E) Greater than or equal to 84000

$R_3 = 125$

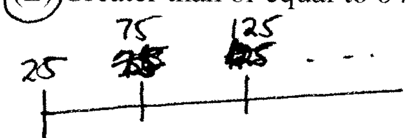
$R_4 = 125 + \Delta$

$R_5 = 125 + 2\Delta$

$R_6 = 125 + 3\Delta = 275$

$\Longrightarrow \Delta = 50$

$\therefore R_2 = 75$ and $R_1 = 25$



semiannual periods

$PV = 25 + \frac{75}{s} + \frac{50}{s^2}$ $\quad s = s e i r = (1.05)^{1/2} - 1$

$\Longrightarrow PV = 85049.85$

8. Annuity A and Annuity B are both annual payment 20-year annuities immediate. For Annuity A, the first 10 payments are 50, and the second 10 payments are 100. For Annuity B, the first 10 payments are 100, and the second 10 payments are 50. Using annual effective interest rates of *i* for the first 10 years and 6% thereafter, the present value of Annuity A is 690. Determine the present value of Annuity B.

(A) 850

(B) 900

(C) 950

(D) 1000

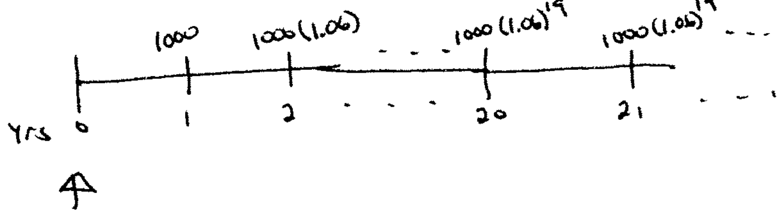
(E) 1050

$690 = 50 a_{\overline{10}|i} + 100 a_{\overline{10}|.06} \cdot v_i^{10}$

$$\left.\begin{array}{l} N = 10 \\ PV = 690 \\ PMT = -50 \\ FV = -100 a_{\overline{10}|.06} \end{array}\right\} \Longrightarrow i = .077129\cdots$$

$PV^B = 100 a_{\overline{10}|i} + 50 a_{\overline{10}|.06} \cdot v_i^{10} = 854.84$

9. A perpetuity with annual payments has an initial payment of 1000. The next 19 payments are 6% more than their preceding payment. Starting with the 21st payment, the rest of the payments are level and equal to the 20th payment. Determine the present value of this perpetuity, one year before the first payment, using an annual effective interest rate of 3%.



(A) 81,700

(B) 82,350

(C) 83,540

(D) 84,280

(E) 85,050

$$PV = \frac{1000}{1.03}\left(1 + \frac{1.06}{1.03} + \cdots (20 \text{ terms})\right) + \frac{1000(1.06)^{19}}{.03} \cdot v_{.03}^{20}$$

$$= \frac{1000}{1.03} \cdot s_{\overline{20}|\left(\frac{1.06}{1.03}-1\right)} + \frac{1000(1.06)^{19}}{.03}(1.03)^{-20}$$

$$= 81697.14$$

10. An annuity with quarterly payments has an initial payment of 1500. Subsequent payments are 50 less than their preceding payment until reaching a final payment of 1050. Determine the accumulated value immediately following the final payment, using a quarterly effective interest rate of 4%.

$q_{eir} = .04$

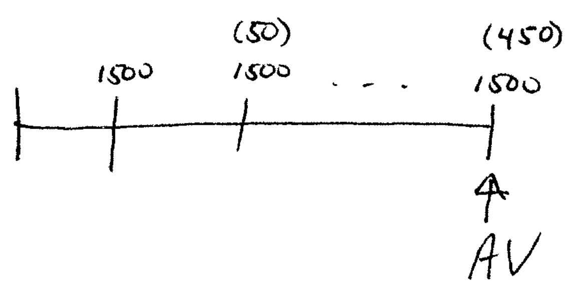
(A) 14,330

(B) 14,905

(C) 15,500

(D) 16,120

(E) 16,765



$$AV = 1500\, s_{\overline{10}|.04} - 50\,(Is)_{\overline{9}|.04}$$

$$= 15501.53$$