MAP 4170 Name:______________________________
Test 2 Date: June 22, 2017

Show sufficient work and clearly mark your answers. Each problem is worth 10 points.

1. A perpetuity immediate has annual payments of 350 for the first 10 years and 500 thereafter. Determine the present value of the perpetuity using a discount rate of 6%, compounded semiannually.

(A) 6870

(B) 6940

(C) 7030

(D) 7110

(E) 7230

$sedr = .03 \Rightarrow aeir = (.97)^{-2} - 1 = a$

$PV = 350\, a_{\overline{10|}a} + \frac{500}{a} \cdot v_a^{10} = 6870.785\ldots$

2. Cindy plans for retirement by depositing 5500 into a retirement account on her 21st birthday. On each subsequent birthday until her 50th birthday, she deposits 5500. On her 50th birthday, she deposits 6500 and continues to deposit 6500 on each subsequent birthday up to and including her 64th birthday. Determine the amount in Cindy's retirement account, immediately before her 65th birthday, using an annual effective interest rate of 8%.

(A) 1.99 million

(B) 2.07 million

(C) 2.15 million

(D) 2.23 million

(E) 2.31 million

$AV = 5500\, \ddot{s}_{\overline{44|}.08} + 1000\, \ddot{s}_{\overline{15|}.08} = 2,149,605.179$

3. A 30-year annuity immediate with annual payments has an initial payment of 500. Subsequent payments increase by 50 over their preceding payment. Determine the present value of this annuity using an interest rate of 10%, compounded annually, for the first 10 years, and an annual effective interest rate of 6% thereafter.

$\Delta = 50$

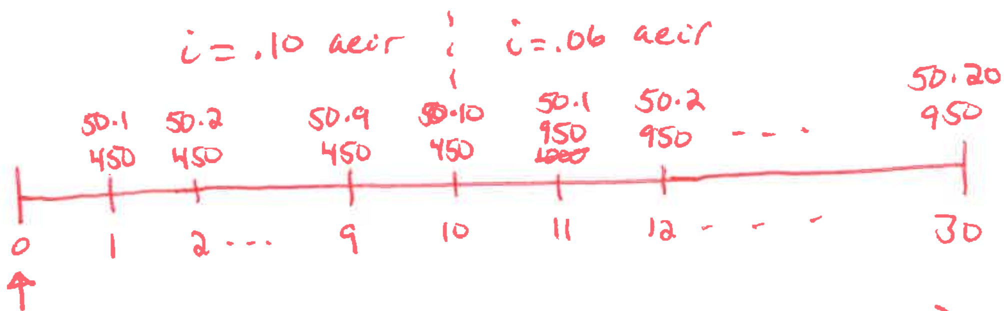
(A) 9,400

(B) 9,630

(C) 9,860

(D) 10,090

(E) 10,320



$$PV = 50(Ia)_{\overline{10|}.1} + 450\, a_{\overline{10|}.1} + \left(50(Ia)_{\overline{20|}.06} + 950\, a_{\overline{20|}.06}\right)\cdot v_{.1}^{10}$$

$$= 10320.557\ldots$$

4. On 1/1/2000, Judy began making payments of 500 at the beginning of each month into an account that pays an interest rate of 9%, compounded monthly. She misses making the monthly payments during calendar year 2009. In 2010, she resumes the regular payments of 500 at the beginning of each month, and in order to make up for the missed payments, she pays an additional amount with each monthly payment such that on 12/31/2016, she has the exact same amount as if she would have just continued making the regular monthly payments of 500 during 2009 through 2016. Determine the accumulated value on 12/31/2016 of the payments Judy made during calendar years 2010 through 2016.

$$i^{(12)} = .09 \Rightarrow m = meir = \frac{.09}{12} = .0075$$

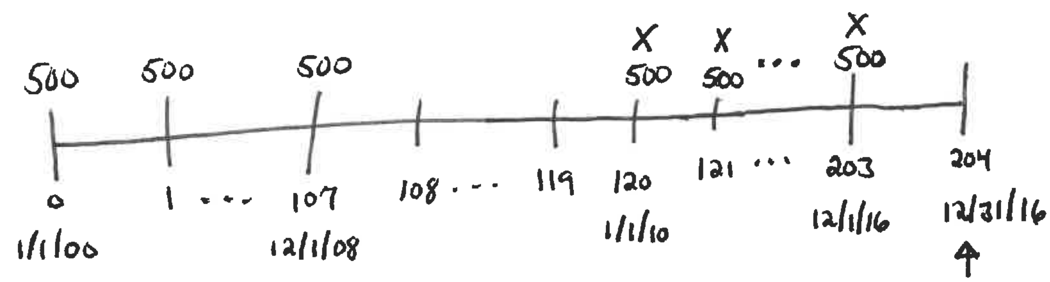
(A) 68,890

(B) 69,410

(C) 69,930

(D) 70,450

(E) 70,970



We seek the value of $(500+X)\cdot \ddot{s}_{\overline{84|}.0075}$

$$500\,\ddot{s}_{\overline{204|}.0075} = 500\,\ddot{s}_{\overline{108|}.0075}\cdot(1.0075)^{96} + (500+X)\cdot\ddot{s}_{\overline{84|}.0075}$$

$$\Rightarrow (500+X)\,\ddot{s}_{\overline{84|}.0075} = 500\,\ddot{s}_{\overline{204|}.0075} - 500\,\ddot{s}_{\overline{108|}.0075}(1.0075)^{96} = 70{,}452.5425$$

5. A 10-year annuity has payments at the end of each month. During the first year, the monthly payments are 100 each. Beginning with the second year, the monthly payments increase by $x\%$ over the previous year's monthly payments. The accumulated value of the payments immediately after the last payment is 29,570 when using an annual effective interest rate of 10%. Determine the amount of the monthly payments during the 7^{th} year.

$m = meir = (1.1)^{1/12} - 1$

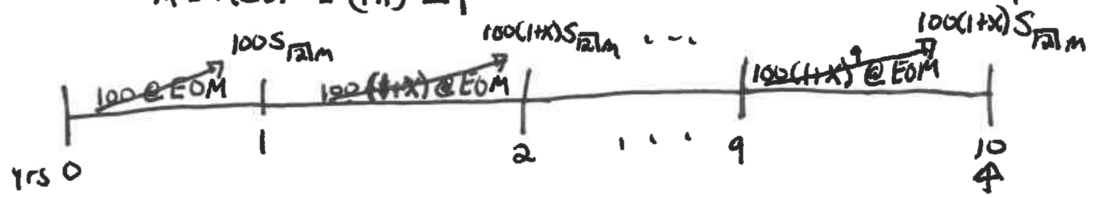
(A) 161

(B) 167

(C) 177

(D) 182

(E) 195



$AV = 29,570$

$$29,570 = 100 s_{\overline{12}|m}(1.1)^9 + 100(1+x)s_{\overline{12}|m}\cdot(1.1)^8 + \cdots \text{ (10 terms)}$$

$$\therefore\ 29,570 = 100 s_{\overline{12}|m}(1.1)^9\left(1 + \frac{1+x}{1.1} + \cdots \text{ (10 terms)}\right)$$

$$\Rightarrow x = .1$$

$$\therefore \text{ monthly payments during year } 7 = 100(1+x)^6 = 177.1561$$

6. A 10-year annuity-due with quarterly payments has a first payment of 1000. The next five payments are also 1000, and then subsequent payments increase by 5.06% over their previous payment. Determine the present value of the annuity using an interest rate of 8%, compounded quarterly.

$i^{(4)} = .08 \Rightarrow q = qeir = \frac{.08}{4} = .02$

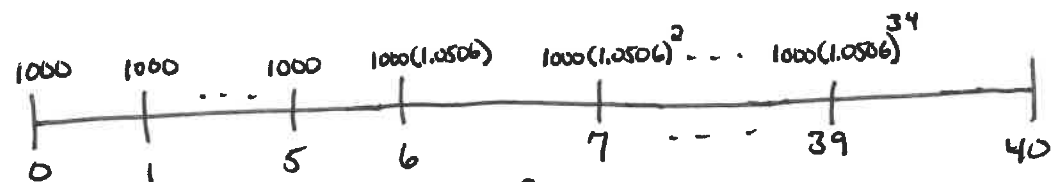
(A) 58,295

(B) 59,570

(C) 60,845

(D) 62,120

(E) 63,395



$v = \frac{1}{1.02} = qdf$

$$PV = 1000\ddot{a}_{\overline{6}|} + 1000(1.0506)v^6 + 1000(1.0506)^2 v^7 + \cdots \text{ (34 terms)}$$

$$= 1000\ddot{a}_{\overline{6}|} + \frac{1000(1.0506)}{(1.02)^6}\cdot\left(1 + \frac{\overbrace{1.0506}^{=1.03}}{\cancel{1.02}} + \cdots \text{ (34 terms)}\right)$$

$$\therefore\ PV = 1000\ddot{a}_{\overline{6}|.02} + \frac{1000(1.0506)}{(1.02)^6}\cdot s_{\overline{34}|.03} = 59,570.099\ldots$$

7. A perpetuity due with annual payments has an initial payment of X and each subsequent payment is Y more than its preceding payment. The present value of the perpetuity is 671 when determined using an annual effective interest rate of 10%, whereas the present value of the perpetuity is 2121 when determined using an annual effective interest rate of 5%. Determine the value of the 11th payment.

(A) 61

(B) 63

(C) 65

(D) 67

(E) 69

X X+Y X+2Y . . .

i	PV
.1	671
.05	2121

$$PV = X + \frac{X+Y}{i} + \frac{Y}{i^2}$$

$$671 = X + \frac{X+Y}{.1} + \frac{Y}{.1^2} = X + 10(X+Y) + 100Y = 11X + 110Y$$

$$2121 = X + \frac{X+Y}{.05} + \frac{Y}{.05^2} = X + 20(X+Y) + 400Y = 21X + 420Y$$

$$\therefore \left.\begin{array}{l}(671 = 11X + 110Y)\cdot(-21)\\(2121 = 21X + 420Y)\cdot(11)\end{array}\right\} \Rightarrow \begin{array}{rl}-14091 &= -231X - 2310Y\\ +\ 23331 &= 231X + 4620Y\\ \hline 9240 &= 2310Y \Rightarrow Y = 4 \Rightarrow X = 21\end{array}$$

value of 11^{th} payment $= X + 10Y = 21 + 10\cdot 4 = 61$

8. Using an annual effective interest of i, the present value of a 2n-year annuity due with semiannual payments of 1 is equal to 1.50 times the present value of an n-year annuity due with semiannual payments of 1. Using the same interest rate, the accumulated value after 3n years of a single deposit of 100 is X. Determine X.

(A) 770

(B) 780

(C) 790

(D) 800

(E) 810

S = seir

$$PV = \ddot{a}_{\overline{4n}|s} = 1.5\, \ddot{a}_{\overline{2n}|s}$$

$$\ddot{a}_{\overline{4n}|s} = \ddot{a}_{\overline{2n}|s}\left(1 + v_s^{2n}\right)$$

$$\therefore\ v_s^{2n} = .5$$

$$X = 100(1+i)^{3n} = 100(1+s)^{6n} = 100\, v_s^{-6n}$$

$$v_s^{2n} = .5 \Rightarrow v_s^{-6n} = (.5)^{-3} = 2^3 = 8$$

$$\therefore\ X = 800$$

9. Annuity A has annual payments $1, 2, \cdots, (n-1), n, n, (n-1), \cdots, 2, 1$ at the end of each year. Annuity B is an n-year annuity immediate with annual payments of 1. Using an annual effective interest rate of i, the present value of Annuity B is 12. Determine the present value of Annuity A using the same interest rate.

(A) 144

(B) 156

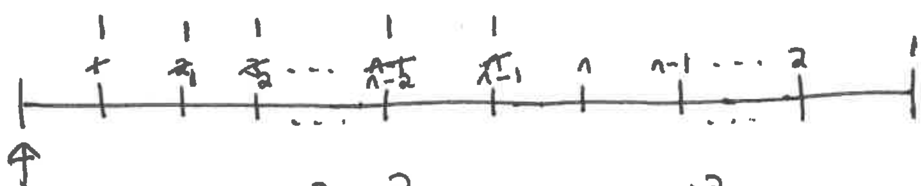
(C) 168

(D) 180

(E) 192

B: $12 = a_{\overline{n}|}$

A:



$$PV = a_{\overline{n}|} + (\ddot{a}_{\overline{n}|})^2 \cdot v^2 = a_{\overline{n}|} + (\ddot{a}_{\overline{n}|} \cdot v)^2 = a_{\overline{n}|} + (a_{\overline{n}|})^2$$

$$\therefore \; PV = 12 + 12^2 = 156$$

10. A 20-year annuity immediate with annual payments has an initial payment of 80. Subsequent payments are 4 less than their preceding payment. Using a nominal interest rate of 10%, compounded semiannually, determine the accumulated value of the annuity one year after the last payment.

(A) 3090

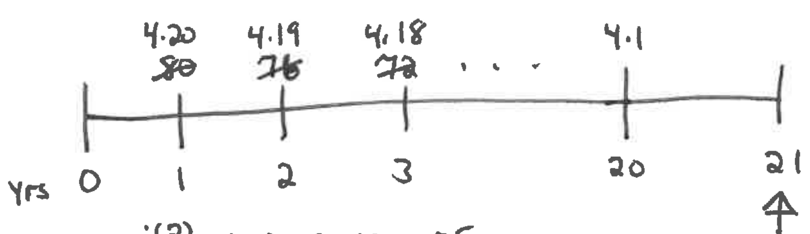
(B) 3195

(C) 3400

(D) 3525

(E) 3640

$\Delta = 4$



$i^{(2)} = .1 \Rightarrow seir = .05$

$a = aeir = 1.05^2 - 1 = .1025$

AV

$$AV = 4 \cdot (D\ddot{s})_{\overline{20}|, .1025} = 3522.538\ldots$$