

MAP 4170
Test 3

Name: _____
Date: November 22, 2011

Show sufficient work and clearly mark your answers. Each problem is worth 10 points.

1. Bob takes out a loan of 1000 at an annual effective interest rate of i . You are given:

- (i) The first payment is made at the end of year 6.
- (ii) Ten equal annual payments are made to repay the loan in full at the end of 15 years
- (iii) The outstanding principal after the payment made at the end of year 10 is 908.91.

Calculate the outstanding principal at the end of year 5.

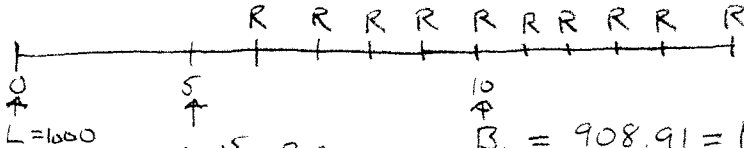
(A) 1490

(B) 1510

(C) 1530

(D) 1550

(E) 1570



$$L = 1000$$

$$B_5 = L(1+i)^5 = Ra_{\overline{10}|i}$$

$$B_{10} = 908.91 = Ra_{\overline{5}|i}$$

$$Ra_{\overline{10}|i} = Ra_{\overline{5}|i}(1+i)^5$$

$$L = 1000 = Ra_{\overline{10}|i} \cdot v^5 = Ra_{\overline{5}|i}(1+i)^5 \cdot v^5 = 908.91(1+i)^5 \cdot v^5$$

$$\therefore v^{10} + v^5 - \frac{1000}{908.91} = 0 \Rightarrow v^5 = \frac{-1 \pm \sqrt{1 - 4(1)(-\frac{1000}{908.91})}}{2}$$

$$B_5 = 1000(1+i)^5 = 1000(v^5)^{-1} = 1000 \frac{2}{-1 + \sqrt{1 + 4(\frac{1000}{908.91})}} \approx 1510$$

2. A 20-year loan of 250,000 is being repaid by the sinking fund method with annual payments. The lender charges an interest rate of 4% annual effective and payments into the sinking fund earn 5% annual effective. Determine the net amount of interest paid during the 10th year; i.e. the difference between the amount of interest paid to the lender and the amount of interest earned in the sinking fund.

(A) 5830

(B) 5860

(C) 5890

(D) 5920

(E) 5950

$$R^I = .04(250000) = 10000$$

$$R^{SF} \cdot S_{\overline{20}|.05} = 250000 \Rightarrow R^{SF} \approx 7560.65$$

$$B_9^{SF} = R^{SF} \cdot S_{\overline{9}|.05} \approx 83367.96$$

$\therefore$ the amount of interest earned in the sinking fund during the 10th year is $.05 \cdot B_9^{SF} \approx 4168.40$

$\therefore$ the net amount of interest paid during the 10th year is $10000 - 4168.40 = 5831.60$

3. A loan of 1000 is being repaid in 10 years by semiannual installments of 50, plus interest on the unpaid balance at 4% per annum compounded semiannually. The installments and interest payments are reinvested at 5% per annum compounded semiannually. Calculate the annual effective yield rate of the loan. ($i = a \text{ yr}$)

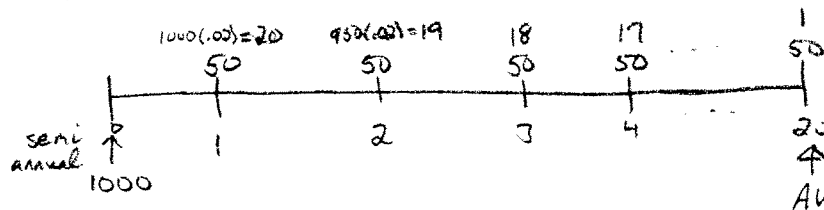
(A) 4.4% Loan interest rate = .02 seir
payments investment rate = .025 seir

(B) 4.6%

(C) 4.8%

(D) 5.0%

(E) 5.2%



$$AV = 50 S_{\overline{20}|.025} + (DS)_{\overline{20}|.025} = 50 S_{\overline{20}|.025} + \frac{20(1.025)^{20} - S_{\overline{20}|.025}}{.025} = 1566.34$$

$$\therefore 1000(1+i)^{10} = 1566.34 \Rightarrow i \approx 4.6\%$$

4. A 10,000 loan is amortized using an annual effective interest rate of 6%. The payments are increasing with the first payment of 1000 due one year after loan inception and each subsequent payment is 100 more than its preceding payment. Determine the balance of the loan immediately before the 5th payment.

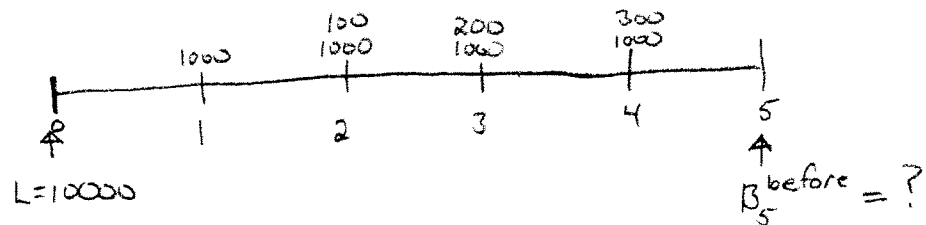
(A) 6685

(B) 7625

(C) 8085

(D) 8425

(E) 8880



$$B_5^{\text{before}} = B_4 (1.06)$$

$$B_4 \stackrel{\text{Retro}}{=} 10000(1.06)^4 - 1000 S_{\overline{4}|.06} - 100(I\ddot{S})_{\overline{4}|.06} \\ = 10000(1.06)^4 - 1000 S_{\overline{4}|.06} - 100 \frac{S_{\overline{4}|.06} - 3}{.06} \approx 7625.79$$

$$\therefore B_5^{\text{before}} = B_4 (1.06) \approx 8083.34$$

5. A 1000 loan is repaid with equal payments at the end of each year for 20 years.

The principal portion of the 13th payment is 1.5 times the principal portion of the 5th payment.

Calculate the total amount of interest paid on the loan during the first 10 years.

(A) 375

(B) 440

(C) 500

(D) 675

(E) 815

$$P_{13} = P_5 (1+i)^8 = 1.5 P_5 \Rightarrow (1+i)^8 = 1.5$$

$$\Rightarrow i \approx .052 \text{ \textit{aeir}}$$

$$L = 1000 = R a_{\overline{20}|i} \Rightarrow R \approx 81.60$$

$$\sum_{k=1}^{10} P_k = B_0 - B_{10} = 1000 - (R a_{\overline{10}|i}) \approx 375.94$$

$$\therefore \sum_{k=1}^{10} I_k = 10R - \sum_{k=1}^{10} P_k \approx 440$$

6. A 1000 face value 10% annual coupon bond is redeemable as follows:

1200 at the end of years 15, 16, or 17

1100 at the end of years 18, 19, or 20

Greg pays 1625 for the bond. Determine Greg's minimum yield rate on the bond.

(A) Less than or equal to 4.96%

(B) Greater than 4.96%, but less than or equal to 5.01%

(C) Greater than 5.01%, but less than or equal to 5.06%

(D) Greater than 5.06%, but less than or equal to 5.11%

(E) Greater than 5.11%

	N	$i; P=1625$
$C=1200$	15	4.936%
	16	5.053%
	17	5.155%
$C=1100$	18	5.006%
	19	5.107%
	20	5.197%

$\therefore$ the minimum yield rate Greg will earn is 4.936% and occurs if the bond is called after 15 years.

7. A 100-year bond with annual coupons is bought to yield an annual effective interest rate of 5%. The accumulation of discount in the 10th installment is 0.59. Determine the amount of write-up in the 40-year period consisting of installments 31 through 70. $i = .05$

- (A) 30 $A = P_{31} + P_{32} + \dots + P_{70} = P_{31} (1 + (1+i) + \dots + (1+i)^{39}) = P_{31} S_{\overline{40}|i}$
- (B) 70 $P_{31} = P_{10} (1+i)^{21} = 0.59 (1.05)^{21} \approx 1.64$
- (C) 115
- (D) 155 $\therefore A \approx 198.56$
- (E) 200

8. Amy buys a 20-year 1000 par value bond with 5% semiannual coupons at a price to yield 6% compounded semiannually. Each coupon Amy receives is deposited into an account that earns 5% interest compounded semiannually. Immediately after receiving the 9th coupon, Amy sells the bond to Brad at a price to yield Brad 4% compounded semiannually. Determine Amy's yield rate, as an annual effective interest rate, during the time in which she owned the bond.

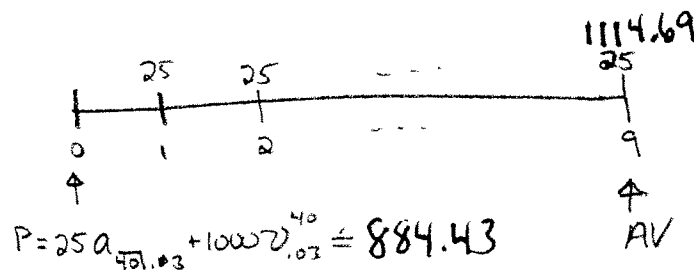
- (A) 6% Brad pays Amy $P = 25a_{\overline{37}|.02} + 1000v_{.02}^{37} \approx 1114.69$

- (B) 7%

- (C) 8% Amy's timeline over the 4.5 years is

- (D) 9%

- (E) 10%



$$AV = 25S_{\overline{9}|.025} + 1114.69 = 1363.55$$

$\therefore$ Amy's acyr, i , is found by solving

$$(1+i)^{4.5} = \frac{1363.55}{884.43}$$

$$\therefore i \approx 10\%$$

9. A 1000 par value 5-year bond with 8.0% semiannual coupons was bought to yield 7.5% convertible semiannually. Determine the amount of premium amortized in the 6th coupon payment.

(A) 1.85

(B) 1.93

(C) 2.00

(D) 2.08

(E) 2.15

$$P_6 = Fr - I_6 = 40 - .0375 BV_5$$

$$(OR) \quad BV_5 - P_6 = BV_6$$

$$P_6 = BV_5 - BV_6$$

$$BV_5 = 40 a_{\overline{5}|.0375} + 1000 v^5$$

$$BV_6 = 40 a_{\overline{4}|.0375} + 1000 v^4$$

10. A bond that is bought to yield 5% has coupons of 100. The amount of interest earned in the n^{th} installment is 109.42 and the amount of interest earned in the $(n+k)^{\text{th}}$ installment is 116.12. Determine k .

(A) 9

(B) 11

(C) 13

(D) 15

(E) 17

$$I_n = 109.42 = .05 \underline{BV_{n-1}} \quad \swarrow k \text{ periods}$$

$$I_{n+k} = 116.12 = .05 \underline{BV_{n+k-1}}$$

$$\overset{PV}{BV_{n-1}} = \overset{PMT}{100} a_{\overline{21}|.05} + \overset{FV}{BV_{n+k-1}} \cdot v_{.05}^k$$

CPT N problem

$$OR \quad P_n = \overset{100}{Fr} - I_n = -9.42$$

$$P_{n+k} = -16.12$$

$$-9.42 (1.05)^k = -16.12$$