MAP 4170
Test 3

Name:_____
November 15, 2012

Each problem is worth 10 points. Show sufficient work for full credit.

1. A loan is repaid with monthly payments of 350. The outstanding balance just after the 10th payment is 18,030 and the outstanding balance just after the 35th payment is 12,360. Determine the annual effective interest rate charged by the lender.

(A) 9.2% (B) 9.4% (C) 9.6% (D) 9.8% (E) 10.0%

$$18030 = 350\, a_{\overline{25}|j} + 12360\, v_j^{25} \qquad j = \text{mer}$$

$$j \doteq 0.8\%$$

$$i - \text{aeir} \Rightarrow (1+j)^{12} = 1+i$$

$$\Rightarrow i \doteq 10.0\%$$

2. A 20-year 1000 face value bond with annual coupons, redeemable at 1200, is bought to yield 4% annual effective. The amortization of premium during the 5th year is 6.41. Determine the price of the bond.

(A) 1163 (B) 1211 (C) 1256 (D) 1363 (E) 1411

$$P_5 = 6.41$$

$$\text{Amount of premium} = \sum_{k=1}^{20} P_k = P_1 + P_1(1.04) + \cdots \text{ 20 terms}$$

$$= P_1 \cdot s_{\overline{20}|.04}$$

$$P_1 = P_5 \cdot v_{.04}^4 = 6.41\,(1.04)^{-4} \doteq 5.48$$

$$\text{Price} = 1200 + P_1\, s_{\overline{20}|.04} \doteq 1363$$

3. Bill and Ted each take out 10-year loans of 25,000 with the lender charging 8% annual effective on the loans. Bill repays his loan using the sinking fund method with annual payments. Bill's payments into the sinking fund earn 10% annual effective. Ted repays his loan using the amortization method with level annual payments. Let X and Y denote the total amounts of interest that Bill and Ted pay, respectively. Determine $X - Y$.

(A) -3420 (B) -1570 (C) 420 (D) 1570 (E) 3420

Bill: $R^I = 25000(.08) = 2000$

$R^{SF} \cdot s_{\overline{10}|.10} = 25000 \Rightarrow R^{SF} \doteq 1568.63$

X = Total Bill pays in interest $= 10(2000 + 1568.63) - 25000 \doteq 10686$

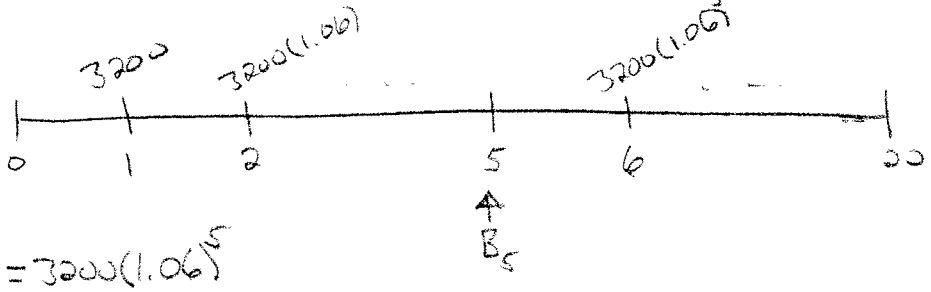
Ted: $R a_{\overline{10}|.08} = 25000 \Rightarrow R = 3725.74$

Y = Total Ted pays in interest $= 10(3725.74) - 25000 \doteq 12257$

$\therefore X - Y \doteq -1570$

4. A 20-year loan at an 8% annual effective interest rate is repaid with increasing annual payments. The first payment is 3200 and each subsequent payment is 6% greater than its preceding payment. Determine the amount of principal repaid in the 6th payment.

(A) 94 (B) 583 (C) 981 (D) 1573 (E) 1838



$R_6 = 3200(1.06)^5$

$B_5 \overset{\text{VEP}}{\underset{\text{Pro}}{=}} 3200(1.06)^5 \cdot v_{.08} + 3200(1.06)^6 \cdot v_{.08}^2 + \cdots$ 15 terms

$= \frac{3200(1.06)^5}{1.08}\left[1 + \frac{1.06}{1.08} + \cdots\right] = \frac{3200(1.06)^5}{1.08} \cdot \ddot{a}_{\overline{15}|\left(\frac{1.08}{1.06}-1\right)}$

$I_6 = B_5 \cdot (.08) \doteq 4188.19$

$P_6 = R_6 - I_6 \doteq 94.13$

5. A 25-year 1000 face value callable bond with annual coupons of 50 has the following redemption schedule:

Redemption value = 1000 if the bond is called at the end of years 11 through 15
Redemption value = 1100 if the bond is called at the end of years 16 through 20
Redemption value = 1200 if the bond is called at the end of years 21 through 25

Determine the maximum price that can be paid in order to guarantee an annual yield rate of at least 5%.

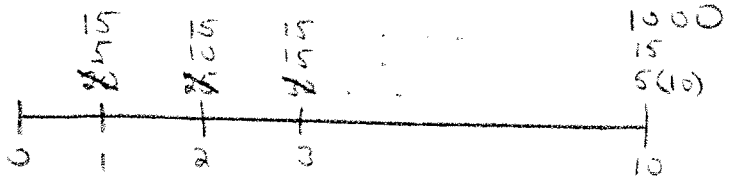
(A) 979 (B) 1000 (C) 1021 (D) 1072 (E) 1103

n	P(.05)
11	1000
15	1000
16	1046
20	1038
21	1072
25	1059

Use $P = 1000$

6. A special 10-year bond with annual coupons has a first coupon equal to 20 and each successive coupon is 5 more than its preceding coupon. The redemption value of the bond is 1000. Determine the price to yield 6% annual effective.

(A) 830 (B) 840 (C) 850 (D) 860 (E) 870



$$P = P(.06) = 15\, a_{\overline{10}|.06} + 5 \cdot (Ia)_{\overline{10}|.06} + 1000\, v^{10}_{.06}$$

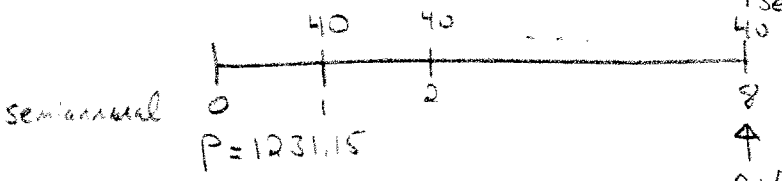
$$\doteq 853.06$$

7. Martin buys a 20-year 1000 par value bond with 8% semiannual coupons at a price to yield 6% compounded semiannually. Upon receiving each coupon, Martin invests the coupon in an account that earns 8% compounded semiannually. Immediately after receiving the 8th coupon, Martin sells the bond at a price that yields the new buyer 4% compounded semiannually. Determine Martin's annual yield rate over the period of time that he owned the bond.

(A) 5.1% (B) 7.2% (C) 8.3% (D) 9.4% (E) 10.5%

Martin Pays $P = 40\,a_{\overline{40}|.03} + 1000\,v^{40}_{.03} = 1231.15$

$P_{sell} = 40\,a_{\overline{32}|.02} + 1000\,v^{32}_{.02}$



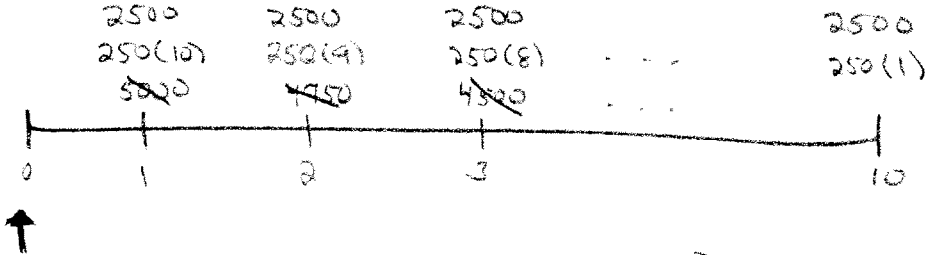
$AV = 40\,s_{\overline{8}|.04} + P_{sell}$

$\therefore$ i = annual yield rate

$1231.15(1+i)^4 = AV \Rightarrow i \doteq 10.5\%$

8. A lender charges 6% compounded annually on a 10-year loan. The loan is repaid with annual payments, the first of which is 5000 and each subsequent payment is 250 less than the previous year's payment. Determine the amount borrowed.

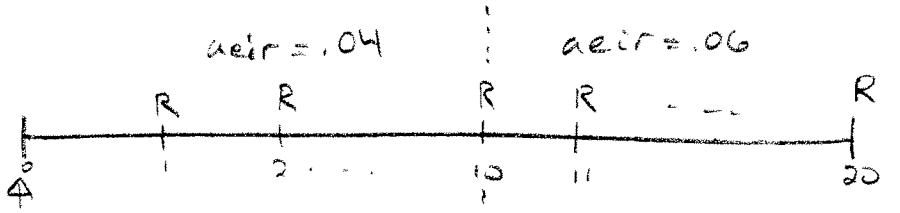
(A) 28500 (B) 28800 (C) 29100 (D) 29400 (E) 29700



$L = PV = 2500\,a_{\overline{10}|.06} + 250(Da)_{\overline{10}|.06} \doteq 29400$

9. A 100,000 loan is to be amortized over 20 years with level annual payments at the end of each year. The annual effective interest rate charged on the loan is 4% for the first 10 year, and 6% thereafter. Determine the amount of principal repaid in the 16th payment.

(A) 5380 (B) 5540 (C) 5710 (D) 5880 (E) 6050



$$L = 100000 = R a_{\overline{10}|.04} + R a_{\overline{10}|.06} \cdot v_{.04}^{10}$$

$$\Longrightarrow R \doteq 7643.44$$

$$P_{16} = R - I_{16} \qquad I_{16} = B_{15}(.06)$$

$$B_{15} \overset{\text{Pro}}{=\!=} R a_{\overline{5}|.06} = 7643.44\, a_{\overline{5}|.06}$$

$$\therefore P_{16} \doteq 5710$$

10. A 1000 face value n-year bond, redeemable at par, with $r\%$ semiannual coupons is bought to yield $2r\%$ semiannual effective. Given $v_{.02r}^{n} = 0.6$, where v is the semiannual discount factor, determine the price of the bond.

(A) 520 (B) 580 (C) 640 (D) 700 (E) 750

$$P = 1000\left(\frac{.01r}{2}\right) a_{\overline{2n}|.02r} + 1000\, v_{.02r}^{2n}$$

$$= 1000\left(\frac{.01r}{2}\right)\left(\frac{1 - v_{.02r}^{2n}}{\frac{.02r}{2}}\right) + 1000\, v_{.02r}^{2n}$$

$$= 250(1 - .36) + 1000(.36)$$

$$= 520$$