Show sufficient work and clearly mark your answers. Each problem is worth 10 points.

1. A loan of $L$ is repaid with 120 level monthly payments of 300 with the first payment due 2 years after the loan inception date. Interest is charged using an interest rate of 6% compounded monthly. Determine $L$.

   \( m = 5 \Rightarrow r = 0.005 \)

   (A) Less than 24,000
   (B) Greater than or equal to 24,000, but less than 24,200
   (C) Greater than or equal to 24,200, but less than 24,400
   (D) Greater than or equal to 24,400, but less than 24,600
   (E) Greater than or equal to 24,600

\[
L = 300 \cdot \frac{a_{50}^{2}}{1.005^{2}} \cdot \frac{1}{0.005} = 24093.43
\]

2. An $n$-year 1000 face value annual coupon bond, redeemable at 1200, is priced at 1100. A $2n$-year 10000 face value annual coupon bond is priced at 11000. The two bonds have the same coupon rate, are bought to yield the same rate, and are such that the annual effective yield equals the annual coupon rate. Determine the redemption value of the $2n$-year bond.

   \( r = \frac{c}{1-c} \)

   (A) 10000
   \[
   1100 = 10000 \cdot a_{n}^{5} + 1200 \cdot 2^n
   \]
   (B) 11000
   \[
   = 10000 (1 - 2^n) + 1200 \cdot 2^n
   \]
   (C) 12000
   \[
   \Rightarrow \quad 2^n = 0.5
   \]
   (D) 13000
   \[
   11000 = 10000 \cdot a_{n}^{5} + \frac{c}{1-c} \cdot 2^n
   \]
   \[
   \Rightarrow \quad 11000 = 10000 (1 - 2^n) + \frac{c}{1-c} \cdot 2^n
   \]
   \[
   2^n = (0.5)^2 = 0.25 \quad \Rightarrow \quad c = 14000
   \]
3. A special 30-year bond, redeemable at 1000, has increasing annual coupons whereby each coupon is $r\%$ more than its preceding coupon. The initial coupon is 100. At an annual effective yield rate of 6%, the price of the bond is 3004.30. Determine the amount of the 15th coupon.

\[
\begin{array}{cccccc}
\text{(A) 220} & \text{100} & \text{100(1.01r)} & \ldots & \text{100(1.01r)}^{14} & \text{1000} \\
\text{(B) 225} & \downarrow \\
\text{(C) 230} & \text{P \approx 3004.3} = 1000 \cdot 1.06^0 + 100 \cdot (1.01r)^{15} \ldots + 1000 \cdot 1.06^{30} \\
\text{(D) 235} & \text{\Rightarrow 3004.3} - 1000 \cdot 1.06^{30} = \frac{1000}{1.06} \left(1 + \frac{1.01r}{1.06} + \ldots (30 \text{ terms})\right) \\
\text{(E) 240} & \therefore 1 + \frac{1.01r}{1.06} + \ldots (30 \text{ terms}) = 30 \\
\text{\Rightarrow r = 6} & \Rightarrow F_{15} = 100(1.06)^{14} \approx 226 \end{array}
\]

4. A mortgage with level monthly payments of 2125 has an interest rate of 6%, compounded monthly. The amount of interest paid during the 150th month is 1290. Determine the amount of interest paid during the 100th month.

\[
\begin{array}{cccccc}
\text{(A) 1005} & me^{ir} = .005 \\
\text{(B) 1280} & I_{150} = 2125 \cdot B_{149} \Rightarrow B_{149} = \frac{1290}{.005} = 258000 \\
\text{(C) 1475} & I_{100} = .005 \cdot B_{99} \\
\text{(D) 1560} & \text{B}_{99} = 2125 \cdot a_{\overline{50|1.005}} + 258000 \cdot \frac{50}{.005} \\
\text{(E) 1655} & \therefore \text{B}_{99} = 294859.83 \\
\text{\Rightarrow I}_{100} = 1474.30 \end{array}
\]
5. The interest rate on a loan with level annual payments of 1000 is 5% annual effective for the first 10 years and 4% annual effective thereafter. The outstanding balance on the loan immediately after the 15th payment is 8000. Determine the amount of principal repaid with the 6th payment.

\[ \text{Ae} = 0.05 \]

\[ \text{Ae} = 0.04 \]

(A) 310

(B) 320

(C) 330

\[ B_{15} = 1000 \times A_{5.05} + 8000 \times V_{0.04} = 11027.24 \]

(D) 340

(E) 350

\[ B_5 = 1000 \times A_{5.05} + 11027.24 \times V_{0.05} = 12969.61 \]

\[ \Rightarrow I_6 = 0.05 B_5 = 648.48 \]

\[ \Rightarrow P_6 = 1000 - I_6 = 351.52 \]

6. A 20-year 1000 face value callable bond with 6% semiannual coupons is redeemable at then end of any year starting with year 10. The redemption value is 1200 – 25k, where \( k \) = the number of years for which the bond is called early. (E.g., if the bond is called at the end of the 10th year, then \( C = 1200 - 25(10) = 950 \) is the redemption value, if the bond is called at the end of the 11th year, then \( C = 1200 - 25(9) = 975 \), and so on.) Determine the maximum price an investor is willing to pay to guarantee a semiannual yield of 4%.

<table>
<thead>
<tr>
<th>( n )</th>
<th>Red. Value, ( C )</th>
<th>P(0.04)</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>950</td>
<td>( 30A_{50} + 950V_{50} = 841.28 )</td>
</tr>
<tr>
<td>22</td>
<td>975</td>
<td>844.94</td>
</tr>
<tr>
<td>24</td>
<td>1000</td>
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<td>26</td>
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<td>34</td>
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<td>38</td>
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</tr>
<tr>
<td>40</td>
<td>1200</td>
<td>843.73</td>
</tr>
</tbody>
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Max. Price to guarantee 4% semi is 841.
7. A 30-year loan of 100000 is repaid with payments at the end of each year. The payments for the first 10 years consist of interest only. The payments for the next 10 years are equal to 150% of the amount of interest due at the time of the payment. The final 10 payments are level at $X$. Interest is charged using 8% annual effective. Determine $X$.

\[
\begin{align*}
B_{10} &= 100000 \\
X &= \frac{B_{10}}{1.08^{20}}
\end{align*}
\]

8. A loan of 500000 is repaid with annual increasing payments using the sinking fund method. The total amount of the first payment is 35000 and subsequent payments increase by 5000. The lender charges 6% annual effective on the loan and the sinking fund earns an annual effective interest rate of $i$. The net amount of interest paid during the 8th year is 23100. Determine the amount of interest earned in the sinking fund during the second year.

\[
\begin{align*}
R^I &= 500000 \cdot (1.06) = 300000 \\
\text{SF:} & \quad \begin{array}{cccc}
0 & 5000 & 55000 & 55000 \\
& \text{year 0} & \text{year 1} & \text{year 2} \\
\end{array}
\end{align*}
\]
9. A 20-year bond with annual coupons is such that the total write-up of the bond over the 10-year period from the end of year 5 to the end of year 15 is equal to 5/3 of the total write-up of the bond over the 5-year period from the end of year 10 to the end of year 15. Determine the annual yield rate at which this bond was purchased.

(A) 7.6% \[ P_6 + P_7 + \ldots + P_{15} = \frac{5}{3} \left( P_{11} + P_{12} + \ldots + P_{15} \right) \]

(B) 8.0% \[ P_6 \cdot S_{15}^{11.2} = \frac{5}{3} \cdot P_{11} \cdot S_{15}^{11.2} \]

(C) 8.4% \[ S_{15}^{11.2} = S_{15}^{11.2} \left( 1 + (1 + i)^5 \right) \]

(D) 8.8% \[ P_{11} = P_6 (1 + i)^5 \]

(E) 9.2% \[ \therefore P_6 \cdot S_{15}^{11.2} \left( 1 + (1 + i)^5 \right) = \frac{5}{3} \cdot P_{11} \cdot S_{15}^{11.2} \]

\[ \Rightarrow 1 + (1 + i)^5 = \frac{5}{3} (1 + i)^5 \]

\[ \Rightarrow (1 + i)^5 = \frac{3}{2} \quad \Rightarrow \quad i = 0.084 \]

10. A bond with quarterly coupons of 150 was bought to yield 8.08% compounded semiannually. The book value of the bond, immediately after the 10th coupon is paid, equals 8880. Determine the price of the bond.

(A) 5085 \[ i = 8.08\% \quad \Rightarrow \quad i = 0.0404 = \text{semi-annual} \]

(B) 7190 \[ j = 0.0404 \quad \Rightarrow \quad (1+j)^2 = 1.0404 \quad \Rightarrow \quad j = 0.02 \]

(C) 8615

(D) 8630 \[ P = 150 \cdot A_{10\%}^{0.02} + 8880 \cdot A_{0.02}^{10} \approx 8632 \]

(E) 8650