MAP 4170 Name:______________________

Test 3 Date: November 14, 2013

Show sufficient work and clearly mark your answers. Each problem is worth 10 points.

1. A loan of L is repaid with 120 level monthly payments of 300 with the first payment due 2 years after the loan inception date. Interest is charged using an interest rate of 6% compounded monthly. Determine L.

meir = .005

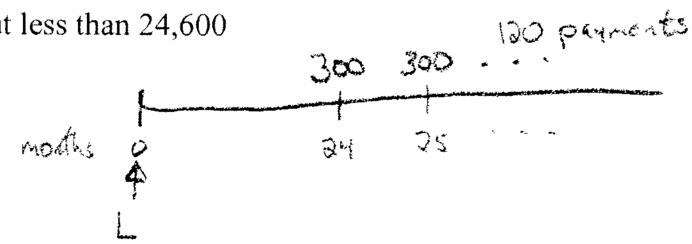
(A) Less than 24,000

(B) Greater than or equal to 24,000, but less than 24,200

(C) Greater than or equal to 24,200, but less than 24,400

(D) Greater than or equal to 24,400, but less than 24,600

(E) Greater than or equal to 24,600



$$\therefore L = 300\,\ddot{a}_{\overline{120}|.005} \cdot v^{24}_{.005} \doteq 24093.43$$

2. An n-year 1000 face value annual coupon bond, redeemable at 1200, is priced at 1100. A $2n$-year 10000 face value annual coupon bond is priced at 11000. The two bonds have the same coupon rate, are bought to yield the same rate, and are such that the annual effective yield equals the annual coupon rate. Determine the redemption value of the $2n$-year bond.

$r = i$

(A) 10000

(B) 11000

(C) 12000

(D) 13000

(E) 14000

$$1100 = 1000i \cdot a_{\overline{n}|} + 1200 v^n$$

$$= 1000(1 - v^n) + 1200 v^n$$

$$\Longrightarrow v_i^n = .5$$

$$11000 = 10000i\, a_{\overline{2n}|} + C v_i^{2n}$$

$$\Longrightarrow 11000 = 10000(1 - v^{2n}) + C v_i^{2n}$$

$$v_i^{2n} = (.5)^2 = .25 \Longrightarrow C = 14000$$

3. A special 30-year bond, redeemable at 1000, has increasing annual coupons whereby each coupon is r % more than its preceding coupon. The initial coupon is 100. At an annual effective yield rate of 6%, the price of the bond is 3004.30. Determine the amount of the 15th coupon.

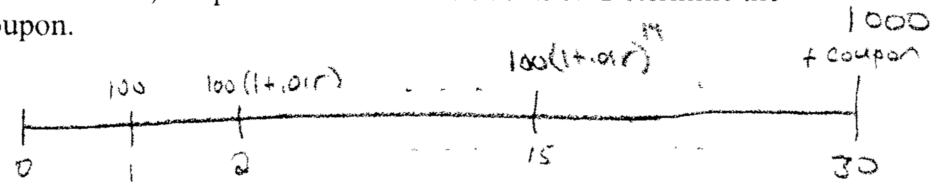
(A) 220

(B) 225

(C) 230

(D) 235

(E) 240



$$P = 3004.3 = 100v_{.06} + 100(1+.01r)v_{.06}^{2} + \cdots + 1000v_{.06}^{30}$$

$$\Rightarrow 3004.3 - 1000v_{.06}^{30} = \frac{100}{1.06}\left(1 + \frac{1+.01r}{1.06} + \cdots (30 \text{ terms})\right)$$

$$\therefore \quad 1 + \frac{1+.01r}{1.06} + \cdots (30 \text{ terms}) = 30$$

$$\Rightarrow r = 6 \quad \Rightarrow Fr_{15} = 100(1.06)^{14} \doteq 226$$

4. A mortgage with level monthly payments of 2125 has an interest rate of 6%, compounded monthly. The amount of interest paid during the 150th month is 1290. Determine the amount of interest paid during the 100th month.

(A) 1005

(B) 1280

(C) 1475

(D) 1560

(E) 1655

meir = .005

$$I_{150} = i \cdot B_{149} \Rightarrow B_{149} = \frac{1290}{.005} = 258000$$

$$I_{100} = .005\, B_{99}$$

$$B_{99} = 2125\, a_{\overline{50}|.005} + 258000\, v_{.005}^{50}$$

$$\Rightarrow B_{99} \doteq 294859.23$$

$$\Rightarrow I_{100} \doteq 1474.30$$

5. The interest rate on a loan with level annual payments of 1000 is 5% annual effective for the first 10 years and 4% annual effective thereafter. The outstanding balance on the loan immediately after the 15th payment is 8000. Determine the amount of principal repaid with the 6th payment.

(A) 310

(B) 320

(C) 330

(D) 340

(E) 350

aeir = .05 | aeir = .04

yrs 0 B_5 10 4

$B_{15} = 8000$

$$B_{10} = 1000\, a_{\overline{5}|.04} + 8000\, v_{.04}^{5} \doteq 11027.24$$

$$B_5 = 1000\, a_{\overline{5}|.05} + 11027.24\, v_{.05}^{5} \doteq 12969.61$$

$$\Rightarrow I_6 = .05\, B_5 \doteq 648.48$$

$$\Rightarrow P_6 = 1000 - I_6 \doteq 351.52$$

6. A 20-year 1000 face value callable bond with 6% semiannual coupons is redeemable at then end of any year starting with year 10. The redemption value is 1200 – 25k, where k = the number of years for which the bond is called early. (E.g., if the bond is called at the end of the 10th year, then $C = 1200 - 25(10) = 950$ is the redemption value, if the bond is called at the end of the 11th year, then $C = 1200 - 25(9) = 975$, and so on.) Determine the maximum price an investor is willing to pay to guarantee a semiannual yield of 4%.

(A) 839

(B) 841

(C) 843

(D) 850

(E) 851

Call Time, n	Red. Value, C	$P(.04)$
20	950	$30\, a_{\overline{20}|} + 950\, v^{20} \doteq 841.28$
22	975	844.94
24	1000	847.53
26	1025	849.19
28	1050	850.04
30	1075	850.20
32	1100	849.77
34	1125	848.83
36	1150	847.47
38	1175	845.75
40	1200	843.73

Max. Price to guarantee 4% seir is 841.

7. A 30-year loan of 100000 is repaid with payments at the end of each year. The payments for the first 10 years consist of interest only. The payments for the next 10 years are equal to 150% of the amount of interest due at the time of the payment. The final 10 payments are level at X. Interest is charged using 8% annual effective. Determine X.

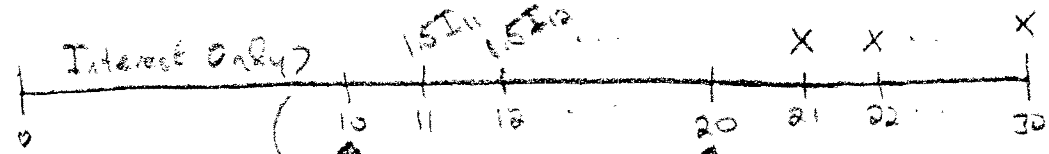
(A) 8970

(B) 8990

(C) 9910

(D) 9930

(E) 9950



$B_{10} = 100000$ $\qquad$ $B_{20} = X\, a_{\overline{10}|.08}$

$B_{11} = B_{10}(1.08) - 1.5I_{11}$

$= B_{10}(1.08) - 1.5\cdot(.08)B_{10} = .96B_{10}$

$B_{12} = B_{11}(1.08) - 1.5I_{12}$

$= B_{11}(1.08) - 1.5\cdot(.08)B_{11} = .96B_{11} = (.96)^2\cdot B_{10}$

$\vdots$

$\therefore B_{20} = (.96)^{10} B_{10} = 100000(.96)^{10} = X\, a_{\overline{10}|.08} \Rightarrow X \doteq 9908$

8. A loan of 500000 is repaid with annual increasing payments using the sinking fund method. The total amount of the first payment is 35000 and subsequent payments increase by 5000. The lender charges 6% annual effective on the loan and the sinking fund earns an annual effective interest rate of i. The net amount of interest paid during the 8th year is 23100. Determine the amount of interest earned in the sinking fund during the second year.

(A) 225

(B) 250

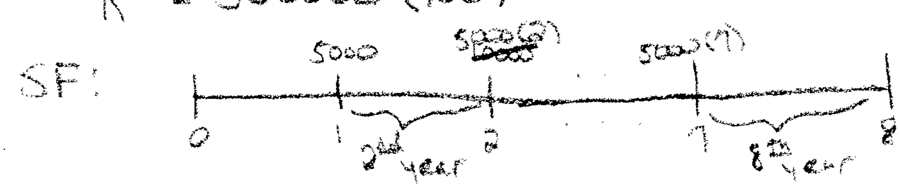
(C) 275

(D) 300

(E) 325

$R^I = 500000(.06) = 30000$

SF:



The amount of interest earned in the SF during the 8th year is $i\cdot B_7 = i\,(5000(Is)_{\overline{7}|i})$

$= 5000\, i\, \dfrac{\ddot{s}_{\overline{7}|} - 7}{i} = 5000\,(\ddot{s}_{\overline{7}|i} - 7)$

$\therefore 23100 = 30000 - 5000(\ddot{s}_{\overline{7}|i} - 7)$

$\Rightarrow i \doteq .045$

$\therefore I_2 = .045(5000) = 225$

9. A 20-year bond with annual coupons is such that the total write-up of the bond over the 10-year period from the end of year 5 to the end of year 15 is equal to 5/3 of the total write-up of the bond over the 5-year period from the end of year 10 to the end of year 15. Determine the annual yield rate at which this bond was purchased.

(A) 7.6%

(B) 8.0%

(C) 8.4%

(D) 8.8%

(E) 9.2%

$$P_6 + P_7 + \cdots + P_{15} = \frac{5}{3}(P_{11} + P_{12} + \cdots + P_{15})$$

$$P_6 \cdot s_{\overline{10|}i} = \frac{5}{3} P_{11} \cdot s_{\overline{5|}i}$$

$$s_{\overline{10|}} = s_{\overline{5|}}(1 + (1+i)^5)$$

$$P_{11} = P_6 (1+i)^5$$

$$\therefore \cancel{P_6} \cdot \cancel{s_{\overline{5|}}}(1 + (1+i)^5) = \frac{5}{3} \cdot \cancel{P_6}(1+i)^5 \cdot \cancel{s_{\overline{5|}}}$$

$$\Longrightarrow 1 + (1+i)^5 = \frac{5}{3}(1+i)^5$$

$$\Longrightarrow (1+i)^5 = \frac{3}{2} \Longrightarrow i \doteq .084$$

10. A bond with quarterly coupons of 150 was bought to yield 8.08% compounded semiannually. The book value of the bond, immediately after the 10th coupon is paid, equals 8880. Determine the price of the bond.

(A) 5085

(B) 7190

(C) 8615

(D) 8630

(E) 8650

$$i^{(2)} = 8.08\% \Longrightarrow i = .0404 = \text{secr}$$

$$j = \text{qecr} \Longrightarrow (1+j)^2 = 1.0404 \Longrightarrow j = .02$$

$$P = 150\, a_{\overline{10|}.02} + 8880\, v^{10}_{.02} \doteq 8632$$