MAP 4170 Name:______________________________
Test 3 Date: November 13, 2014

Show sufficient work and clearly mark your answers. Each problem is worth 10 points.

1. A special 10-year bond, redeemable at 1000, has increasing annual coupons whereby each coupon is 3% more than its preceding coupon. The initial coupon is 100. Determine the price of the bond to yield 5% annual effective.

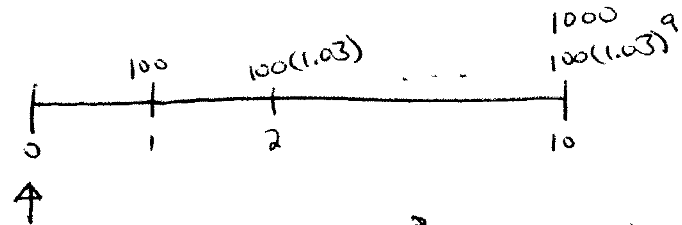
   (A) 1480

   (B) 1490

   (C) 1500

   (D) 1510

   (E) 1520



$v = \frac{1}{1.05}$

$$P = 100v + 100(1.03)v^2 + \cdots (10 \text{ terms}) + 1000v^{10}$$

$$= \frac{100}{1.05} \cdot \ddot{a}_{\overline{10}|\left(\frac{1.05}{1.03}-1\right)} + 1000\, v^{10}_{.05}$$

$$\therefore\ P \doteq 1488.67$$

2. A 20-year 1000 face value callable bond with 6% semiannual coupons is redeemable at the end of any year starting with year 16. The redemption value is 1200 for all years except year 18, in which the redemption value is 1150. Determine the maximum price an investor should pay in order to guarantee a semiannual effective yield of at least 4%.

   (A) 833

   (B) 843

   (C) 847

   (D) 868

   (E) 878

n	$P(.04)$
32	$P = 30\, a_{\overline{32}\|.04} + 1200\, v^{32}_{.04} \doteq 878.28$
34	$P = 30\, a_{\overline{34}\|.04} + 1200\, v^{34}_{.04} \doteq 868.60$
36	$P = 30\, a_{\overline{36}\|.04} + 1150\, v^{36}_{.04} \doteq 847.47$
38	$P = 30\, a_{\overline{38}\|.04} + 1200\, v^{38}_{.04} \doteq 851.38$
40	$P = 30\, a_{\overline{40}\|.04} + 1200\, v^{40}_{.04} \doteq \boxed{843.73}$

not needed

$$\therefore\ \text{answer} = 843$$

3. You are given the following information on Bonds A and B:

	Bond A	Bond B
Term	n years	$2n$ years
Annual coupon amount	K	K
Annual effective yield	i	i
Redemption Value	C	(See below)
Price	10000	P

Bond B is such that C will be redeemed at the end of year n, and another C will be redeemed at the end of year $2n$. Given $(1+i)^n = 2.2$, determine the price P of Bond B.

(A) Less than 10000

(B) Greater than or equal to 10000, but less than 12000

(C) Greater than or equal to 12000, but less than 14000

(D) Greater than or equal to 14000, but less than 16000

(E) Greater than or equal to 16000

Bond A: $10000 = Ka_{\overline{n}|} + Cv^n$

Bond B: $P = Ka_{\overline{2n}|} + Cv^n + Cv^{2n} = Ka_{\overline{n}|}(1+v^n) + Cv^n + Cv^{2n}$

$= Ka_{\overline{n}|} + Ka_{\overline{n}|}\cdot v^n + Cv^n + Cv^{2n} = \underbrace{Ka_{\overline{n}|} + Cv^n}_{=10000} + \underbrace{v^n}_{=\frac{1}{2.2}}\underbrace{(Ka_{\overline{n}|} + Cv^n)}_{=10000} \doteq 14545$

4. A 30-year loan is repaid with level annual payments of 5500. The balance on the loan immediately before the 29th payment is 10500. Determine the amount of principal repaid with the 15th payment.

(A) 1200

(B) 1280

(C) 1315

(D) 1400

(E) 1435

$B_{29}^{\text{before}} = 10500 \Rightarrow B_{29} = 10500 - 5500 = 5000$

$B_{29}(1+i) - R = B_{30}$

$\Rightarrow 5000(1+i) - 5500 = 0 \Rightarrow i = .1$

We seek $P_{15} = R - I_{15} = R - i\cdot B_{14} = 5500 - .1B_{14}$

$B_{14} = Ra_{\overline{15}|} + B_{29}\cdot v^{15} = 5500a_{\overline{15}|} + 5000v^{15}$

$\doteq 43030.40$

$\Rightarrow P_{15} \doteq 1196.96$

5. A lender charges an annual effective interest rate of 6% on a 100000 loan. Payments are annual. The first 15 payments are 150% of the interest due at the time of the payment. The next 15 payments are level payments of 6000. Immediately after the 30^{th} payment, the borrower still owes X. Determine X.

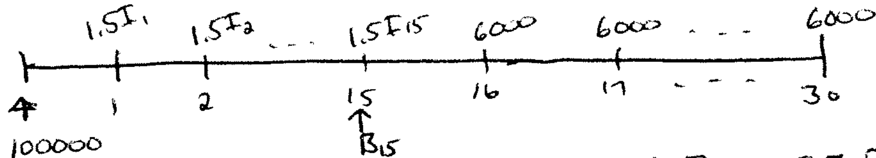
(A) 9810

(B) 10245

(C) 10970

(D) 11565

(E) 12105



$B_1 = B_0 - P_1 = B_0 - .5I_1 = B_0 - .5(.06)B_0 = .97\,B_0$

Likewise $B_2 = .97B_1 = (.97)^2 \cdot B_0$

$\therefore\ B_{15} = (.97)^{15} \cdot (100000) = 6000\,a_{\overline{15|}} + B_{30}\,v^{15}$

$\Rightarrow (.97)^{15}(100000) = 6000\,a_{\overline{15|}} + B_{30}\,v^{15}$

$\Rightarrow B_{30} \doteq 12106.51$

6. Billy buys a 1000 face value 20-year bond, redeemable at par, with 8% semiannual coupons at a price to yield 6% compounded semiannually. Each time Billy receives a coupon, he invests the coupon in an account that earns a nominal interest rate of i, compounded semiannually. Immediately after receiving the 7^{th} coupon, Billy sells the bond to Bob at a price to yield Bob 8% compounded semiannually. Billy's annual effective yield during the time that he owns the bond is 1.6%. Determine i.

(A) 2.5%

(B) 3.0%

(C) 5.0%

(D) 6.0%

(E) 10.0%

$P^{Billy} = 40\,a_{\overline{40|}.03} + 1000\,v^{40}_{.03} \doteq 1231.15$

$P^{Bob} = 40\,a_{\overline{33|}.04} + 1000\,v^{33}_{.04} = 1000$

$i = i^{(2)}$; let $j = \frac{i}{2}$ = seir

At time 7, Billy's accumulated coupons

$= 40\,s_{\overline{7|}j}$

$\therefore$ At time 7 (semiannual), Billy has $1000 + 40\,s_{\overline{7|}j}$

Billy's annual yield = .016 over 3.5 years

$\Rightarrow 1231.15\,(1.016)^{3.5} = 1000 + 40\,s_{\overline{7|}j} \Rightarrow j \doteq .0245$

$\Rightarrow i = 2j \doteq .049$

7. Smith lends Brown 10000. Brown repays the loan with 10 annual payments of 1000 plus interest on the unpaid balance at 8% annual effective. Each time Smith receives a payment, she invests the payment in an account that earns 6% annual effective. Determine Smith's annual yield on the loan.

(A) 4.5%

(B) 5.3%

(C) 6.1%

(D) 6.9%

(E) 7.7%

Payments

0	1	2	…	9	10
	$I_1 = 800$, $P_1 = 1000$	$I_2 = 720$, $P_2 = 1000$	…	$I_9 = 160$, $P_9 = 1000$	$I_{10} = 80$, $P_{10} = 1000$
					AV

$$AV = 1000\, s_{\overline{10}|.06} + 80(Ds)_{\overline{10}|.06}$$

$i =$ Smith's annual yield

$$\Rightarrow 10000(1+i)^{10} = AV = 19484.37$$

$$\Rightarrow i \doteq .069$$

8. A loan of L is to be repaid using the sinking fund method with annual payments for 10 years. The lender charges 10% annual effective and deposits into the sinking fund earn 12% annual effective. The total annual payment is 345 less than the payments would be if the borrower amortized the loan with level annual payments for 10 years at 10% annual effective. Determine L.

(A) Less than 61000

(B) Greater than or equal to 61000, but less than 63000

(C) Greater than or equal to 63000, but less than 65000

(D) Greater than or equal to 65000, but less than 67000

(E) Greater than or equal to 67000

Amortization

$$L = R\, a_{\overline{10}|.1}$$

$$\Rightarrow R^{Amort} = \frac{L}{a_{\overline{10}|.1}}$$

$$R^I = .1L \qquad R^{SF} \cdot s_{\overline{10}|.12} = L \Rightarrow R^{SF} = \frac{L}{s_{\overline{10}|.12}}$$

$$\text{Total Payment} = R^I + R^{SF} = R^{Amort} - 345$$

$$\Rightarrow .1L + \frac{L}{s_{\overline{10}|.12}} = \frac{L}{a_{\overline{10}|.1}} - 345$$

$$\Rightarrow L \doteq 59883.04$$

9. An n-year loan of L is amortized with level monthly payments of 1640. The amount of interest paid in the 4th month is 1240 and the amount of interest paid in the 17th month is 1140. Determine L.

(A) 72415

(B) 72560

(C) 72780

(D) 72965

(E) 73155

$$I_4 = 1240 \implies P_4 = 1640 - 1240 = 400$$

$$I_{17} = 1140 \implies P_{17} = 1640 - 1140 = 500$$

$$P_{17} = P_4 (1+i)^{13} \implies (1+i)^{13} = \frac{500}{400}$$

$$\implies i \doteq$$

Using I_4: $\overset{1240}{I_4} = i \cdot B_3 \implies B_3 = 71622.27$

$$\therefore \quad L = B_0 = 1640\, a_{\overline{3}|i} + B_3 \cdot v_i^3$$

$$\implies L \doteq 72781.88$$

10. An n-year bond with annual coupons of 50 is bought to yield 4% annual effective. The accumulation of discount in the 8th year is 8. Determine the price of the bond.

(A) 1100

(B) 1105

(C) 1250

(D) 1395

(E) 1400

$$\implies P_8 = -8 = Fr - I_8$$

$$Fr = 50 \implies I_8 = 58 = .04\, B_7$$

$$\implies B_7 = 1450$$

$$\therefore \quad P = B_0 = 50\, a_{\overline{7}|.04} + 1450\, v_{.04}^7$$

$$P \doteq 1401.98$$