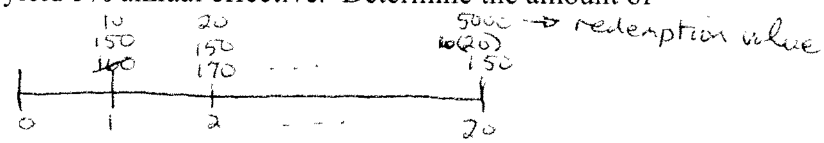
MAP 4170 | Name:______________________________
Test 3 | Date: April 3, 2014

Show sufficient work and clearly mark your answers. Each problem is worth 10 points.

1. A 20-year bond with annual coupons has an initial coupon of 160. Each subsequent coupon is 10 more than its preceding coupon. The redemption value of the bond is 5000 and the bond is bought to yield 5% annual effective. Determine the amount of premium or discount.




$A$ $\quad i = .05$

(A) discount of 135

(B) discount of 85

(C) bought at par

(D) premium of 85

(E) premium of 135

$$P = 150\, a_{\overline{20}|} + 5000\, v^{20} + 10\, (Ia)_{\overline{20}|}$$

$$\therefore P \doteq 4863.29$$

$$C = 5000$$

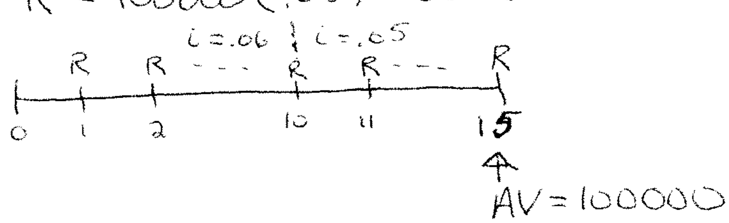
$\therefore$ the bond is bought at a discount of

$$5000 - 4863.29 = 136.71$$

2. Joan borrows 100000 for 15 years and will repay the loan using a sinking fund with level annual deposits. Annual interest payments to the lender are determined using 5% annual effective. The sinking fund interest rate is 6% annual effective for the first 10 years and 5% annual effective thereafter. Determine the net amount of interest Joan pays during the 11th year.

$$R^I = 100000(.05) = 5000$$

SF:




(A) 1195

(B) 1460

(C) 1755

(D) 2050

(E) 2345

$$\therefore 100000 = R\, s_{\overline{10}|.06}(1.05)^5 + R\, s_{\overline{5}|.05}$$

$$\Longrightarrow R \doteq 4474.67$$

$B_{10}^{SF} = R\, s_{\overline{10}|,06} \Longrightarrow$ interest earned in SF during year 11

$$= .05\, B_{10}^{SF} = .05 \cdot R \cdot s_{\overline{10}|.06} \doteq 2948.98$$

$\therefore$ net amount of interest paid during year $11 = 5000 - 2948.98 = 2051.02$

3. A loan at 5% annual effective is repaid with level annual payments at the end of each year for $2n$ years. The amount of principal repaid in the $n$th payment is 171. The balance immediately after the next to last payment is 476. Determine the amount borrowed.

(A) 6400

(B) 7000

(C) 7600

(D) 8200

(E) 8700

$P_n = 171 \qquad B_{2n-1} = 476$

$B_{2n} = 0 = B_{2n-1} - P_{2n} \Rightarrow P_{2n} = 476$

$P_n(1+i)^n = P_{2n} \Rightarrow (1+i)^n = \frac{476}{171} \Rightarrow v^n = \frac{171}{476}$

Also $B_{2n} = B_{2n-1}(1+i) - R \Rightarrow R = 476(1.05)$

$$L = R\,a_{\overline{2n}|.05} = 476(1.05)\,\frac{1-\left(\frac{171}{476}\right)^2}{.05}$$

$$L \doteq 8705.96$$

4. A loan of 100000 at 5% annual effective is repaid with annual payments. The first payment is 4000 and each subsequent payment is 5% more than its preceding payment. There is a smaller final payment, due one year after the last payment in this pattern, that pays the remaining balance of the loan. Determine the amount of the smaller final payment.

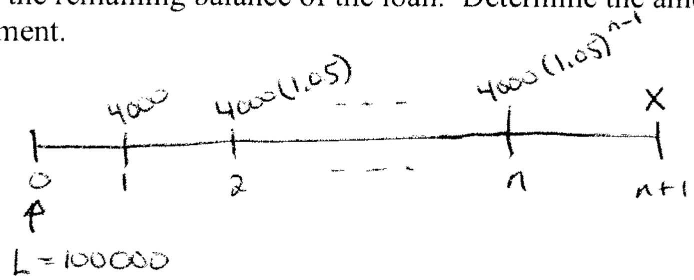
(A) 3225

(B) 3385

(C) 3555

(D) 3735

(E) 3920




$100000 = 4000v + 4000(1.05)v^2 + \cdots (n \text{ terms})$

$+ Xv^{n+1}$ ignore for the moment

$v = \frac{1}{1.05}$

$$\therefore\ 100000 = \frac{4000}{1.05}\big(\overbrace{1+1+\cdots+1}^{n \text{ terms}}\big) \Rightarrow n = 26.25$$

∴ 26 full payments and a smaller payment at time 27

$$\therefore\ 100000 = \frac{4000}{1.05}(26) + Xv^{27} \Rightarrow X \doteq 3555.67$$

5. An $n$-year annual coupon bond, redeemable at par, can be bought at a price equal to 73.15% of its face value. At the same yield, a $2n$-year bond, redeemable at par, with the same coupon rate, can be bought at a price equal to 63.12% of its face value. At the same yield, determine the percentage of face value that a $3n$-year bond, redeemable at par, with the same coupon rate, can be bought.

$ra_{\overline{n}|} = .7315 - v^n$

(A) 58.8 $.7315\cancel{F} = \cancel{F} r a_{\overline{n}|} + \cancel{F} v^n \Rightarrow \cancel{ra_{\overline{n}|} = v^n = .7315}$

(B) 59.1 $.6312 = r a_{\overline{2n}|} + v^{2n}$

(C) 59.4 $= r a_{\overline{n}|}(1+v^n) + v^{2n}$

(D) 59.7 $\therefore .6312 = (.7315 - v^n)(1+v^n) + v^{2n}$

(E) 60.0 $\Rightarrow v^n = \frac{1003}{2685}$ & $r a_{\overline{n}|} = .7315 - \frac{1003}{2685}$

3n-year bond: $P = F r a_{\overline{3n}|} + F v^{3n} = F[r a_{\overline{n}|}(1+v^n+v^{2n}) + v^{3n}]$

$\Rightarrow P = .5937F$

6. A 1000 face value 20-year bond, redeemable at 1200, with semiannual coupons is priced at 1385 to yield 4% annual effective for the first 10 years and 5% annual effective thereafter. Determine the nominal coupon rate, payable semiannually.

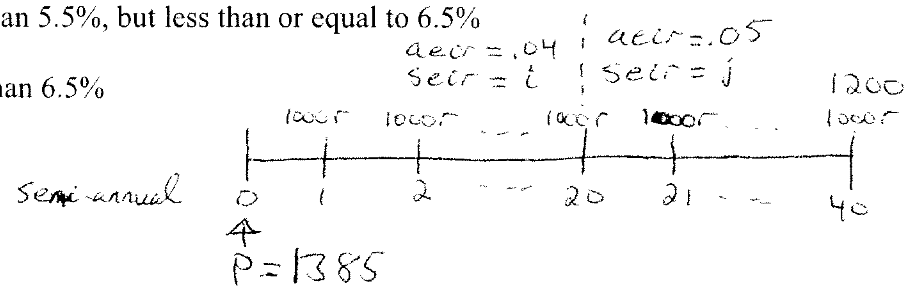
(A) less than or equal to 3.5%

(B) greater than 3.5%, but less than or equal to 4.5%

(C) greater than 4.5%, but less than or equal to 5.5%

(D) greater than 5.5%, but less than or equal to 6.5%

(E) greater than 6.5%




$\therefore 1385 = 1000 r a_{\overline{20}|i} + [1000 r a_{\overline{20}|j} + 1200 v_j^{20}] \cdot v_i^{20}$

$i = (1.04)^{1/2} - 1$ $j = (1.05)^{1/2} - 1$

$\Rightarrow r \doteq .03293$

$\Rightarrow$ nominal coupon rate $= 2r \doteq .0659$

7. A loan at an annual effective interest rate of $i$ is amortized with level payments of 1000 at the end of every 2-year period for 20 years. Immediately after the payment made at the end of year 10, the outstanding balance on the loan equals 60% of the original loan amount. Determine $i$.

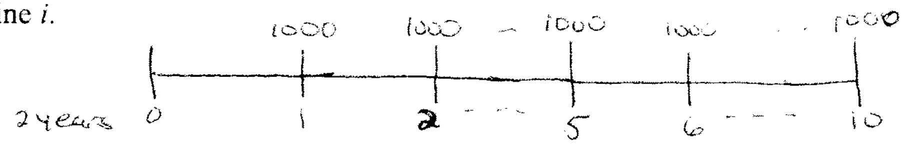
(A) 4.1%

(B) 4.2%

(C) 8.3%

(D) 8.4%

(E) 8.5%




$j$ = 2-year eir

$L = B_0 = 1000\, a_{\overline{10}|j}$ $\qquad B_5 \overset{\text{Pro}}{=} 1000\, a_{\overline{5}|j}$

$\therefore\ 1000\, a_{\overline{5}|j} = .6\left(1000\, a_{\overline{10}|j}\right) = 600\, a_{\overline{5}|j}(1+v_j^5)$

$\Longrightarrow 1 + v_j^5 = \frac{1000}{600} \Longrightarrow v_j^5 = \frac{2}{3}$

$\Longrightarrow (1+j)^5 = \frac{3}{2}$ $\qquad i = \text{aeir} \Longrightarrow 1+j = (1+i)^2$

$\therefore\ (1+i)^{10} = \frac{3}{2} \Longrightarrow i \doteq .04138$

8. Herb borrows 20000 at an annual effective interest rate of 10%. Each year Herb owes interest on the unpaid balance on the loan. He originally plans to repay the loan by making 10 annual payments consisting of principal repayments of 2000 plus interest on the unpaid balance. After the 4th payment, Herb decides to increase his future annual payments to 4000 plus interest on the unpaid balance. Determine the amount of interest Herb saved by increasing his payments.

(A) 1200

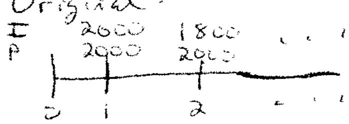
(B) 1400

(C) 1600

(D) 1800

(E) 2000

Original:

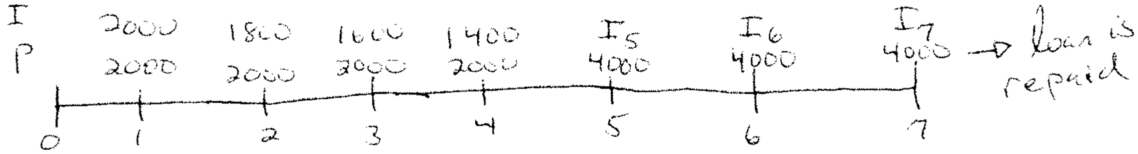



Total Interest $= 2000 + 1800 + \cdots + 200$

$= 200(10 + 9 + 8 + \cdots + 1)$

$= 200\left(\frac{10 \cdot 11}{2}\right) = 11000$

Modified:




$I_5 = B_4(.1) = 12000(.1) = 1200$

$I_6 = B_5(.1) = 8000(.1) = 800$

$I_7 = B_6(.1) = 4000(.1) = 400$

Total Interest $= 2000 + 1800 + 1600 + 1400 + 1200 + 800 + 400 = 9200$

Herb saved $11000 - 9200 = 1800$

9. An $n$-year bond with semiannual coupons of 50 was bought to yield 10% compounded semiannually. The accumulation of discount during the 12th year equals 18.40. Determine the price paid for the bond.

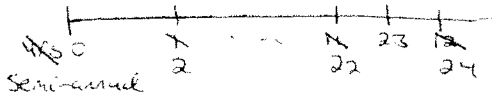
(A) 940

(B) 980

(C) 1020

(D) 1060

(E) 1100




$\underbrace{P_{23} + P_{24}}$ = accumulation of discount for year 12

$P_{24} = P_{23}(1+i) \qquad i = \frac{.10}{2} = .05$

$\therefore -18.40 = P_{23} + 1.05 P_{23} \Rightarrow P_{23} \doteq -8.9756$

↳ negative since bond was bought at a discount

$Fr = 50 = I_{23} + P_{23} \Rightarrow I_{23} \doteq 58.9756 = .05 B_{22}$

$\Rightarrow B_{22} \doteq 1179.51$

$\therefore P = 50 a_{\overline{22}|} + 1179.51 v^{22} \doteq 1061.37$

10. An $n$-year bond with semiannual coupons of 50 has a book value of 1292 at the end of 3 years and a book value of 1219 at the end of 5 years. Determine the price paid for the bond.

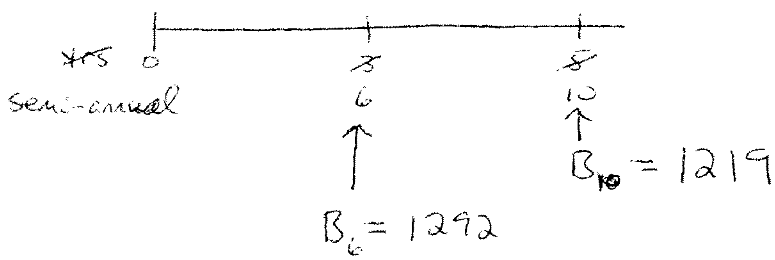
(A) 1380

(B) 1390

(C) 1400

(D) 1410

(E) 1420




$\therefore 1292 = 50 a_{\overline{4}|i} + 1219 v_i^4$

$\Rightarrow i \doteq .025095$

$\therefore P = 50 a_{\overline{10}|i} + 1219 v_i^{10} \doteq 1388.79$