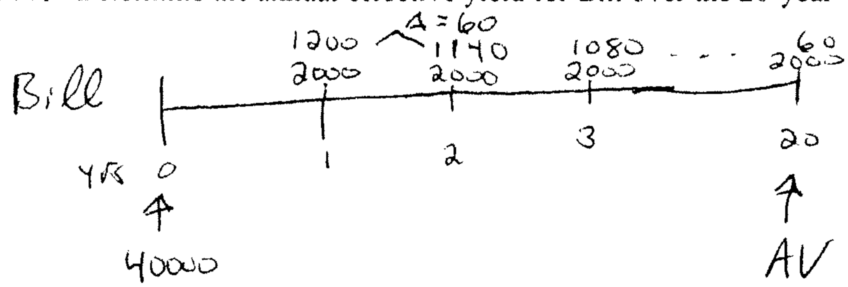
MAP 4170 Name:______________________________
Test 3 Date: July 15, 2013

Show sufficient work and clearly mark your answers. Each problem is worth 10 points.

1. Bill loans Ted 40000 over a 20-year period. Ted repays bill with annual payments of 2000 plus interest on the unpaid balance at an annual effective interest rate of 3%. Each payment Bill receives is invested in an account that pays an annual effective interest rate of 5%. Determine the annual effective yield for Bill over the 20-year period.



(A) 3.8%

(B) 3.9%

(C) 4.0%

(D) 4.1%

(E) 4.2%

$$AV = 2000\, s_{\overline{20}|.05} + 60(Ds)_{\overline{20}|.05} \doteq 90131.91$$

$$\therefore\ 40000(1+i)^{20} = 90\,131.91$$

$$\Rightarrow i \doteq 4.1\%$$

2. A loan of 100,000 is repaid with monthly deposits of 1000 as long as necessary, plus a final deposit of X, $0 < X < 1000$, payable at the same time as the last regular payment of 1000. Interest on the loan is charged using a nominal interest rate of 6%, compounded monthly. Determine X.

(A) 950

(B) 960

(C) 970

(D) 980

(E) 990

$$100\,000 = 1000\, a_{\overline{n}|.005} \Rightarrow n = 138+$$

$$\therefore\ 100000 = 1000\, a_{\overline{138}|.005} + X v^{138}_{.005}$$

$$\Rightarrow X \doteq 970.93$$

3. A 20-year bond with semiannual coupons is bought to yield 4% annual effective over the first ten years and 6% annual effective thereafter. The accumulation of discount for the 3[rd] installment is 5 and the accumulation of discount for the 38[th] installment is 21.96. Determine the total write-up on the bond from the end of the 5[th] year to the end of the 15[th] year.

$i = \text{seir} = 1.04^{1/2} - 1 \quad j = 1.06^{1/2} - 1 = \text{seir}$

(A) 198

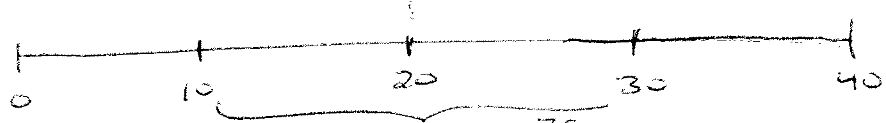
(B) 209

(C) 217

(D) 226

(E) 234

semi annual periods



$\sum_{11}^{30} P_K = \sum_{11}^{20} P_K + \sum_{21}^{30} P_K$

$\sum_{11}^{20} P_K = P_{11} + P_{12} + \cdots + P_{20} = P_{11}\left(1 + (1+i) + \cdots + (1+i)^9\right) = P_{11}\, s_{\overline{10}|i}$

Likewise $\sum_{21}^{30} P_K = P_{21} \cdot s_{\overline{10}|j}$

$P_3 = -5 \Rightarrow P_{11} = -5(1+i)^8 \qquad P_{38} = -21.96 \Rightarrow P_{21} = -21.96\, v_j^{17}$

$\therefore \sum_{11}^{30} P_K = -5(1+i)^8 s_{\overline{10}|i} + -21.96\, v_j^{17} s_{\overline{10}|j} = -217.10$

Answer $= 217$

4. Ed repays a 10-year loan of 100000 using the sinking fund method, making monthly payments. His total monthly payment, including interest to the lender and deposit into the sinking fund, is 950. The sinking fund interest rate is 9% compounded monthly. At the end of 10 years, Ed finds that he must make an additional payment of 3242.86 to repay the loan. Determine the nominal rate of interest compounded monthly that the lender is charging Ed for the loan.

(A) 2.4%

(B) 5.4%

(C) 6.0%

(D) 8.4%

(E) 17.4%

$R = 950 = R^I + R^{SF} \qquad R^I = 100000\, i$

$i = \text{meir (lender)}$

$\therefore R^{SF} = 950 - 100000\, i$

$\Rightarrow (950 - 100000\, i)\, s_{\overline{10}|.0075} = 10000 - 3242.86$

$\Rightarrow i \doteq .0045$

$\Rightarrow i^{(12)} = 12 i = .054$

5. Sue purchases a 1000 par value 20-year bond with 8% semiannual coupons and redemption value of 1200 at a price to yield 6% compounded semiannually. Sue invests each coupon received in an account that pays 10% compounded semiannually. Immediately after receiving the 12th coupon, Sue sells the bond to a buyer at a price that yields the buyer 8% compounded semiannually. Determine Sue's nominal yield, compounded semiannually, over the time in which he owns the bond.

(A) 4.65%

(B) 4.70%

(C) 4.75%

(D) 4.80%

(E) 4.85%

$$P(\text{buy}) = 40\,a_{\overline{40}|.03} + 1200\,v_{.03}^{40} \doteq 1292.46$$

$$P(\text{sells}) = 40\,a_{\overline{28}|.04} + 1200\,v_{.04}^{28} \doteq 1066.70$$

$$AV(\text{coupons}) = 40\,s_{\overline{12}|.05} \doteq 636.69$$

$$\therefore\ 1292.46\left(1+\frac{i^{(2)}}{2}\right)^{12} = 1066.70 + 636.69$$

$$\Longrightarrow i^{(2)} \doteq .0465$$

6. A special 30-year bond with annual coupons has an initial coupon of 50 and each subsequent coupon is 2% greater than its previous coupon. The redemption value is 1000. Determine the amortization of premium for year 14 if the bond is bought to yield 2% annual effective.

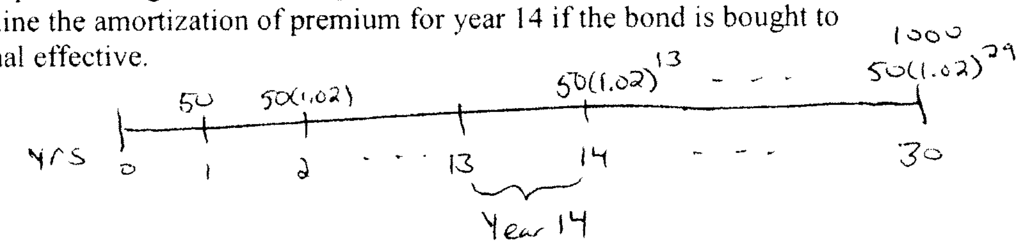
(A) 27.03

(B) 28.84

(C) 30.70

(D) 32.68

(E) 34.82



$$P_{14} = R_{14} - I_{14} = R_{14} - .02\,B_{13}$$

$$B_{13} = 50(1.02)^{13}\cdot v_{.02}\left[1 + \frac{1.02}{1.02} + \cdots (17\text{ terms})\right] + 1000\,v_{.02}^{17}$$

$$\therefore\ B_{13} = 50(1.02)^{12}(17) + 1000\,v_{.02}^{17} \doteq 1792.17$$

$$\therefore\ P_{14} \doteq 50(1.02)^{13} - .02(1792.17) \doteq 28.84$$

Alternate Solution: $B_{14} = B_{13} - P_{14}$

$$\Longrightarrow P_{14} = B_{13} - B_{14}$$

7. A 20-year loan is repaid with annual payments, the first payment being made one year after loan inception. Interest on the loan is 8% annual effective. The first payment equals 1000 and each subsequent payment is 100 more than its preceding payment. Determine the outstanding balance on the loan immediately after the 5th payment.

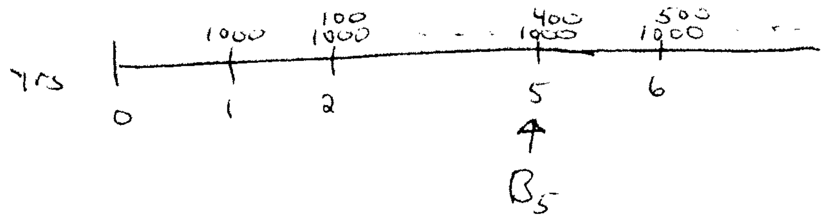
(A) 16,225

(B) 16,400

(C) 16,750

(D) 17,275

(E) 17,625



$$B_5 = 100(Ia)_{\overline{\cancel{20}}\,\overline{15}|.08} + 1400\, a_{\overline{\cancel{20}}\,\overline{15}|.08}$$

$$\therefore B_5 \doteq 17627.47$$

8. A 1000 face value 40-year callable bond with 5% annual coupons may be called at the end of any year beginning with the 20th year. If the bond is called before the 30th year, then the redemption value is 1400. If the bond is called during or after the 30th year, then the redemption value is 1200. If the bond is not called then it matures at a redemption value of 1100. The bond is bought at the maximum price that guarantees an annual yield of at least 3%. Determine the annual yield on the bond if the bond was never called.

(A) 3.00%

(B) 3.02%

(C) 3.04%

(D) 3.06%

(E) 3.08%

Redemption Time	$P(.03)$
20	$50\, a_{\overline{20}|} + 1400 v^{20} = 1519.01$
29	$50\, a_{\overline{29}|} + 1400 v^{29} = 1553.50$
30	$50\, a_{\overline{30}|} + 1200 v^{30} = 1474.40$
39	$50\, a_{\overline{39}|} + 1200 v^{39} = 1519.31$
40	$50\, a_{\overline{40}|} + 1100 v^{40} = 1492.95$

The bond was bought for 1474.40.

never called $\Rightarrow 1474.40 = 50\, a_{\overline{40}|i} + 1100 v_i^{40}$

$\Rightarrow i \doteq .0306$

9. A 5-year loan is repaid using the amortization method with equal monthly payments. The amount of interest paid in the 54^{th} payment equals two-thirds of the amount of interest paid in the 47^{th} payment. Determine the proportion of the 40^{th} payment that repays principal.

(A) 1/4

(B) 1/5

(C) 1/6

(D) 1/7

(E) 1/8

$$I_{54} = \frac{2}{3} I_{47}$$

$$I_{54} = i \cdot B_{53} = i \cdot R a_{\overline{7|}}$$

$$I_{47} = i\, B_{46} = i \cdot R\, a_{\overline{14|}} = i R a_{\overline{7|}}(1+v^7)$$

$$\therefore \cancel{iRa_{\overline{7|}}} = \frac{2}{3} \cdot \cancel{iRa_{\overline{7|}}}(1+v^7) \Rightarrow v^7 = \frac{1}{2}$$

$$P_{40} = R - I_{40} = R - i \cdot B_{39} = R - i \cdot R a_{\overline{21|}} = R - R(1-v^{21})$$

$$\therefore \quad P_{40} = R v^{21} = R\left(\tfrac{1}{2}\right)^3 = \tfrac{1}{8} R$$

$$\text{Answer} = \frac{1}{8}$$

10. Carol purchases an annual level coupon bond at a price to yield 5% annual effective. During the 12^{th} year, the amount of interest earned is 64.50 and the principal adjustment is -25.80. Determine the price Carol paid for the bond.

(A) 1075

(B) 1200

(C) 1325

(D) 1450

(E) 1575

$$I_{12} = 64.50 = i \cdot B_{11} \Rightarrow B_{11} = \frac{64.50}{.05} = 1290$$

$$P_{12} = -25.80$$

$$\therefore Fr = I_{12} + P_{12} = 38.70$$

$$P = B_0 = Fr\, a_{\overline{11|}.05} + B_{11} v^{11}_{.05}$$

$$= 38.7\, a_{\overline{11|}.05} + 1290\, v^{11}_{.05}$$

$$\therefore P = 1075.69$$