MAP 4170 Name:______________________________
Test 3 Date: July 8, 2014

Show sufficient work and clearly mark your answers. Each problem is worth 10 points.

1. A loan is to be repaid with annual payments over 20 years. The first payment is 1000 and subsequent payments are 7.12% greater than the previous payments. The loan interest rate is 3% annual effective. Determine the amount of principal repaid with the $16^{th}$ payment.

   (A) Less than 2400

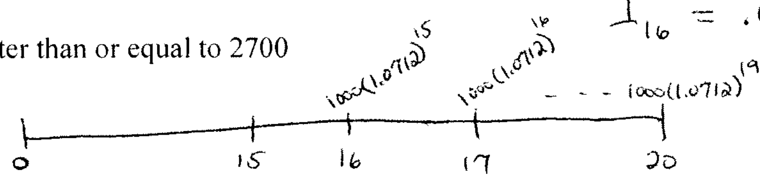
   (B) Greater than or equal to 2400, but less than 2500

   (C) Greater than or equal to 2500, but less than 2600

   (D) Greater than or equal to 2600, but less than 2700

   (E) Greater than or equal to 2700

$$R_1 = 1000$$
$$R_{16} = 1000(1.0712)^{15}$$
$$P_{16} = R_{16} - I_{16}$$
$$I_{16} = .03 \cdot B_{15}$$




$$B_{15} = \frac{1000(1.0712)^{15}}{1.03}\left(1 + \frac{1.0712}{1.03} + \dots (5 \text{ terms})\right) = \frac{1000(1.0712)^{15}}{1.03} \cdot s_{\overline{5}|.04}$$

$$\therefore B_{15} = 14754.54 \Rightarrow I_{16} \doteq 442.64 \Rightarrow P_{16} \doteq 2363.18$$

2. A 20-year bond with annual coupons of 50 was bought to yield 5% annual effective. The amount of write-up for the $10^{th}$ year is 7.31. Determine $\underbrace{\text{the redemption value}}_{C = B_{20}}$.

   (A) 750

   (B) 900

   (C) 1000

   (D) 1200

   (E) 1250

$$P_{10} = -7.31$$

$$Fr = 50 = I_{10} + P_{10} \Rightarrow I_{10} = 57.31$$

$$I_{10} = .05 B_9 \Rightarrow B_9 = 1146.20$$

$$B_9 = 50\, a_{\overline{11}|.05} + C v_{.05}^{11} \Rightarrow C \doteq 1250.05$$

3. Eddy purchases a 1000 face value 20-year bond with 6% semiannual coupons and redemption value of 1200 at a price to yield 8% compounded semiannually. Eddy invests each coupon received in an account that pays 10% compounded semiannually. Immediately after receiving the coupon at the end of the 8th year, Eddy sells the bond to Greg at a price that yields Eddy 10% per annum over the time in which he owns the bond. Determine the nominal interest rate, payable semiannually, that Greg bought the bond to yield.

(A) 3%

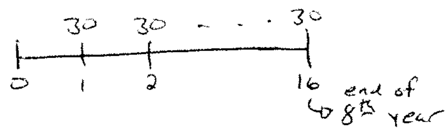
(B) 4%

(C) 5%

(D) 6%

(E) 7%

$$P^{Eddy} = 30\, a_{\overline{40}|.04} + 1200\, v_{.04}^{40} \doteq 843.73$$




$$AV_{16} = 30\, s_{\overline{16}|.05} \doteq 709.72$$

$$P^{Eddy}(1.1)^8 = AV_{16} + P^{Greg} \Rightarrow P^{Greg} \doteq 1098.89$$

$$P^{Greg} = 30\, a_{\overline{24}|j} + 1200\, v_j^{24} \Rightarrow j \doteq .03$$

$$\Rightarrow j^{(2)} = 2j \doteq .06$$

4. A 1000 face value 30-year callable bond with 4% annual coupons may be called at the end of any year beginning with the 10th year. If the bond is called before the 20th year, then the redemption value is 1200. If the bond is called at the end of the 20th year or before the 30th year, then the redemption value is 1100. If not called the bond matures at a redemption value of 1200. The bond is bought at the maximum price that guarantees an annual yield of at least 4%. Determine the annual yield on the bond if the bond was called at the end of the 15th year.

(A) 4.20%

(B) 4.35%

(C) 4.50%

(D) 4.65%

(E) 4.80%

| Call Time | P(.04) |
|---|---|
| 10 | $40\, a_{\overline{10}|.04} + 1200\, v_{.04}^{10} \doteq 1135.11$ |
| 19 | $40\, a_{\overline{19}|.04} + 1200\, v_{.04}^{19} \doteq 1094.92$ |
| 20 | $40\, a_{\overline{20}|.04} + 1100\, v_{.04}^{20} \doteq 1045.63$ |
| 29 | $40\, a_{\overline{29}|.04} + 1100\, v_{.04}^{29} \doteq 1032.06$ |
| 30 | $40\, a_{\overline{30}|.04} + 1200\, v_{.04}^{30} \doteq 1061.66$ |

$\therefore$ bond is bought for $P = 1032.06$

Bond called at time $n = 15 \Rightarrow P = 40\, a_{\overline{15}|i} + 1200\, v_i^{15}$

$$\Rightarrow i \doteq 4.65\%$$

5. A 30-year loan at 4% annual effective is repaid with annual payments. The first payment is 1000 and each subsequent payment is 50 more than its preceding payment. Determine the amount of interest paid on the loan during the middle 10-year period of the term of the loan.

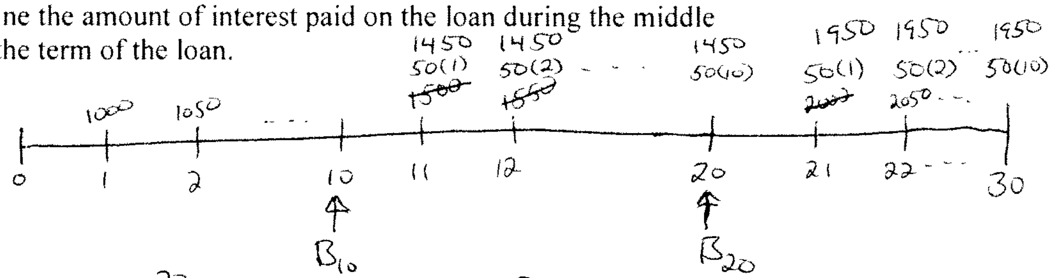
(A) 9000

(B) 9200

(C) 9400

(D) 9600

(E) 9800




$$\sum_{K=11}^{20} I_K = \sum_{K=11}^{20} R_K - \sum_{K=11}^{20} P_K = \sum_{K=11}^{20} R_K - (B_{10} - B_{20})$$

$$B_{10} \overset{Pro}{=} 1450\, a_{\overline{20}|.04} + 50\,(Ia)_{\overline{20}|.04} \doteq 25963.72$$

$$B_{20} \overset{Pro}{=} 1950\, a_{\overline{10}|.04} + 50\,(Ia)_{\overline{10}|.04} \doteq 17915.86$$

$$\sum_{K=11}^{20} R_K = 1500 + 1550 + \cdots + 1950 \quad \frac{\text{Arithmetic}}{10\text{ terms}} \quad \left(\frac{1500+1950}{2}\right)(10) = 17250$$

$$\therefore \sum_{K=11}^{20} I_K = 17250 - (25963.72 - 17915.86) = 9202.14$$

6. A 20-year 1000 par value bond with 8% semiannual coupons is bought to yield 5% annual effective. Determine the book value of the bond immediately before the $12^{th}$ coupon is paid.

(A) 1300

(B) 1307

(C) 1327

(D) 1340

(E) 1347

We must assume $F = C = 1000$ to proceed.

$n = 40$

$Fr = 40$

$i = (1.05)^{1/2} - 1$

$$B_{11} = 40\, a_{\overline{29}|i} + 1000\, v_i^{29} \doteq 1314.28$$

$$B_{12}^{before} = B_{11}(1+i) \doteq 1346.74$$

7. Dale pays back a 100,000 loan with annual payments over 10 years using the sinking fund method. The lender charges an interest rate of 5% annual effective, and the sinking fund interest rate is 6% annual effective. Dale makes a total annual payment of 12,000 each year. Determine the additional amount necessary at the time of the last scheduled payment in order for Dale to completely pay off the loan.

(A) 7735

(B) 9735

(C) 10735

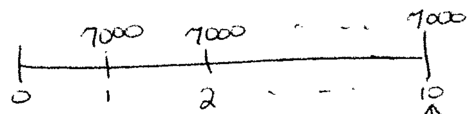
(D) 12735

(E) 14735

$$R^{I} = 100000(.05) = 5000$$

$$\therefore R^{SF} = 12000 - 5000 = 7000$$

SF:




$$AV = 7000\, s_{\overline{10}|.06} \doteq 92265.56$$

Dale must pay an additional $100000 - 92265.56 \doteq 7734.44$

8. Lisa lends Mary 10,000. To repay the loan, Mary makes payments at the end of each year for 10 years. Each payment consists of a principal payment of 1000 plus interest on the unpaid balance at 4%. Lisa invests each payment from Mary into an account that pays 5% annual effective. Determine Lisa's yield rate, as a nominal rate compounded monthly, on the loan.

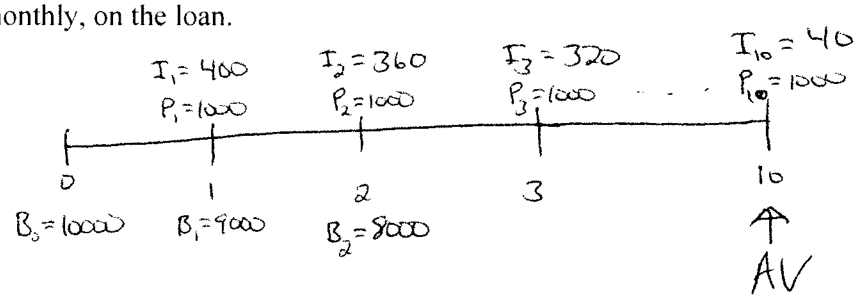
(A) 4.32%

(B) 4.42%

(C) 4.42%

(D) 4.52%

(E) 4.62%




$$AV = 1000\, s_{\overline{10}|.05} + 40\,(Ds)_{\overline{10}|.05} \doteq 15546.74$$

$$\therefore\ 10000\left(1 + \frac{i^{(12)}}{12}\right)^{120} = 15546.74$$

$$\Longrightarrow i^{(12)} \doteq .0442$$

Answer choice (B) or (C) is correct.

9. Sara borrows 60,000 at 6% compounded monthly for 5 years. She originally agrees to make level monthly payments of $X$. Unfortunately, Sara misses the payments at the end of the 11th month up through and including the payment at the end of the 15th month. Sara then continues making the scheduled payments of $X$. Due to the missed payments, there is a balance on the loan at end of the original 5-year period. Sara agrees to repay this balance with 5 level monthly payments of $Y$, with the first payment of $Y$ being made one month later, and $Y$ being calculated using an interest rate of 9% compounded monthly. Determine $Y$. $60000 = X a_{\overline{60}|.005} \Rightarrow X \doteq 1159.97$

(A) 1160

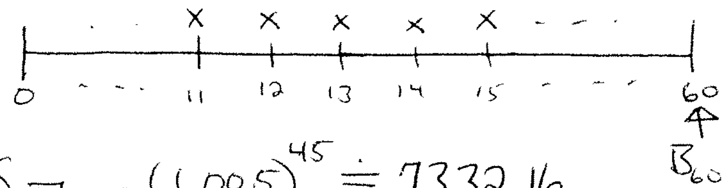
(B) 1190

(C) 1320

(D) 1490

(E) 1500

Missed payments




$$B_{60} = X s_{\overline{51}|.005} (1.005)^{45} \doteq 7332.16$$

New agreement:

$$B_{60} = Y a_{\overline{5}|.0075}$$

$$\Rightarrow Y \doteq 1499.59$$

10. A 10-year 100 face value bond with 8% quarterly coupons is bought to yield 4% compounded quarterly for the first 5 years, and 6% compounded quarterly thereafter. The book value immediately after the coupon payment at the end of year 8 is 112.62. Determine the amount of principal adjustment during the 3rd year.

$Fr = 2$

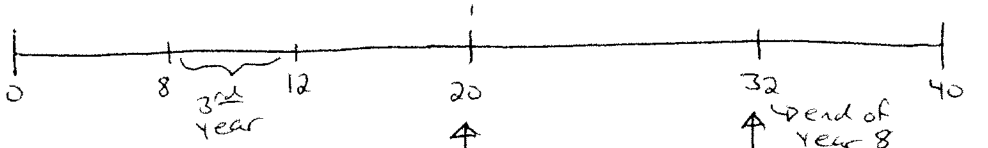
(A) 2.83

(B) 2.93

(C) 3.03

(D) 3.13

(E) 3.23




$B_{32} = 112.62$

$$B_{20} = 2 a_{\overline{12}|.015} + B_{32} \cdot v^{12}_{.015} \doteq 116.01$$

Amount of principal adjustment during year 3 $= \sum_{K=9}^{12} P_K$

$$= P_9 + P_{10} + P_{11} + P_{12} = B_8 - B_{12}$$

$$B_8 = 2 a_{\overline{12}|.01} + B_{20} v^{12}_{.01} \doteq 125.46$$

$$B_{12} = 2 a_{\overline{8}|.01} + B_{20} v^{8}_{.01} \doteq 122.44$$

$$\therefore \sum_{K=9}^{12} P_K \doteq 3.03$$