MAP 4170
Test 3

Name: Key

Date: July 9, 2015

Show sufficient work and clearly mark your answers. Each problem is worth 10 points.

1. A 10-year loan of 100,000 is to be repaid with annual payments using the sinking fund method. The lender charges an interest rate of 8% annual effective, and the total annual payment each year is 15,490. Immediately after the final payment, the borrower has 7500 more than is necessary to repay the loan. Determine the amount of interest earned in the sinking fund during the sixth year.

(A) 3400

(B) 3415

(C) 3430

(D) 3445

(E) 3460

$R^{I} = 100000(.08) = 8000 \Rightarrow R^{SF} = 15490 - 8000 = 7490$

$\therefore 7490 \cdot s_{\overline{10|}i} = 100000 + 7500 = 107,500$

$\Rightarrow i \doteq 7.804\cdots\%$

$I_6 = B_5^{SF} \cdot i = (7490\, s_{\overline{5|}i}) \cdot i \doteq 3415.80$

2. A 10-year bond with semiannual coupons of 50 is bought to yield 6% annual effective. The accumulation of discount in the 8th coupon is 5. Determine the price of the bond.

(A) 890

(B) 900

(C) 1550

(D) 1800

(E) 1830

$i = seir = (1.06)^{1/2} - 1 = .02956\cdots \boxed{1}$

$P_8 = -5$

$Fr = 50 = I_K + P_K \Rightarrow I_8 = 55$

$\therefore B_7 = \frac{I_8}{i} = \frac{55}{\boxed{1}} = 1860.43\cdots \boxed{2}$

$P = B_0 = 50\, a_{\overline{7|}i} + B_7 \cdot v^7 = 50\, a_{\overline{7|}i} + \boxed{2} \cdot v^7$

$\Rightarrow P \doteq 1829.23$

3. A special 3-year bond with annual coupons has a coupon of 85 at then end of the first year, 90 at the end of the second year, and 100 at the end of the third year. The redemption value is 1300. The book value immediately after the first coupon is paid is 1215. Determine the price of the bond.

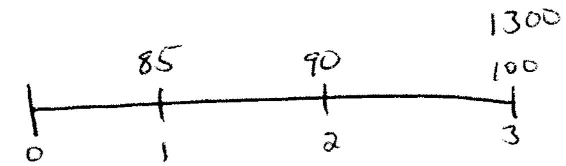
(A) 1150

(B) 1160

(C) 1170

(D) 1180

(E) 1190



$B_1 \overset{Pro}{=} 90v + 1400v^2 \qquad v = adf$

$\therefore 1215 = 90v + 1400v^2$ (quadratic in v)

$\Longrightarrow v = 0.9$

$\therefore P = 85v + 90v^2 + 1400v^3 = 1170$

4. A loan of 6595 at a 5% annual effective interest rate is repaid with annual payments. The first payment is 100 and subsequent payments are X more than their preceding payment. The balance immediately after the fifth payment is 7550. Determine the balance immediately after the twentieth payment. $i = .05 = aeir$

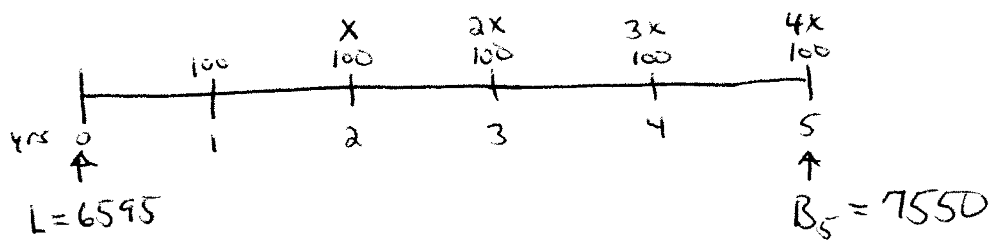
(A) 3725

(B) 4150

(C) 4825

(D) 5275

(E) 6375



$\therefore 7550 \overset{Retro}{=} 6595(1.05)^5 - 100s_{\overline{5}|.05} - X(Is)_{\overline{4}|.05}$

$\Longrightarrow X = 29.917\cdots \boxed{1}$

Similarly, $B_{20} \overset{Retro}{=} 6595(1.05)^{20} - 100s_{\overline{20}|.05} - X(Is)_{\overline{19}|.05}$

$\Longrightarrow B_{20} \doteq 6373.83$

5. On $\underbrace{\text{January 1, 2015}}^{t=0}$, Cathy borrows 200,000 and will repay the loan using the amortization method. The lender charges Cathy an interest rate 5% annual effective during the first 10 years and 4% annual effective thereafter. Cathy makes annual payments of 12,000 at the end of each year. Determine the amount of interest Cathy pays in the 10 year period extending from $\underbrace{\text{January 1, 2035}}_{t=20}$, to $\underbrace{\text{January 1, 2045}}_{t=30}$.

   (A) Less than 32000

   (B) Greater than or equal to 32000, but less than 35000

   (C) Greater than or equal to 35000, but less than 38000

   (D) Greater than or equal to 38000, but less than 41000

   (E) Greater than or equal to 41000

$$B_0 = 200000 = 12000\, a_{\overline{10}|.05} + B_{10} \cdot v_{.05}^{10} \Longrightarrow B_{10} \doteq 174844.21 \cdots \boxed{1}$$

$$B_{10} = \boxed{1} = 12000\, a_{\overline{10}|.04} + B_{20} \cdot v_{.04}^{10} \Longrightarrow B_{20} \doteq 114738.86 \cdots \boxed{2}$$

$$B_{20} = \boxed{2} = 12000\, a_{\overline{10}|.04} + B_{30} \cdot v_{.04}^{10} \Longrightarrow B_{30} \doteq 25768.26 \cdots \boxed{3}$$

$$\sum_{K=21}^{30} R_K = 12000(10) = 120{,}000 \qquad \sum_{K=21}^{30} P_K = B_{20} - B_{30} = \boxed{2} - \boxed{3} \doteq 88970.60$$

$$\Longrightarrow \sum_{K=21}^{30} I_K \doteq 120000 - 88970.60 = 31029.40$$

6. An n-year loan is repaid with monthly payments of 300, with the first payment due one month after the money is borrowed. The amount of principle repaid in the 44th payment is 2.5 times the amount of principle repaid in the 27th payment. The amount of interest paid in the 32nd payment is 177.12. Determine the loan amount.

   (A) 4500

   (B) 5000

   (C) 5500

   (D) 6000

   (E) 6500

$$P_{44} = (2.5) P_{27} \qquad P_{44} = P_{27} (1+i)^{17} \quad i = \text{meir}$$

$$\therefore (1+i)^{17} = 2.5 \Longrightarrow i = .0553 \cdots \boxed{1}$$

$$I_{32} = 177.12 = B_{31} \cdot i \Longrightarrow B_{31} = \frac{177.12}{\boxed{1}} = 3198.35 \cdots \boxed{2}$$

$$L = B_0 = 300\, a_{\overline{31}|i} + B_{31} \cdot v_i^{31}$$

$$\Longrightarrow B_0 \doteq 4999.93$$

7. Betty and Lou each buy an n-year bond with annual coupons at a price to yield i, annual effective. Both bonds have the same face value, and both bonds have the same redemption value. Betty's annual coupon rate is r, whereas Lou's annual coupon rate is $r + i$. The present value of the redemption value is 230. Betty pays 1790 for her bond. Given $\frac{r}{i} = 1.5$, determine the price Lou pays for his bond.

(A) 2770

(B) 2790

(C) 2810

(D) 2830

(E) 2850

Betty: $P^B = Fr\,a_{\overline{n}|} + Cv^n \qquad Cv^n = 230$

$\therefore \quad 1790 = Fr\,a_{\overline{n}|} + 230 \implies Fr\,a_{\overline{n}|} = 1560$

Lou: $P^L = F(r+i)a_{\overline{n}|} + Cv^n = F(r+i)a_{\overline{n}|} + 230$

$\frac{r}{i} = 1.5 \implies i = \frac{r}{1.5} = \frac{r}{(3/2)} = \frac{2r}{3}$

$\therefore \quad r + i = r + \frac{2r}{3} = \frac{5}{3}\cdot r$

$\implies P^L = F\cdot\frac{5}{3}r\cdot a_{\overline{n}|} + 230 = \frac{5}{3}(Fr\,a_{\overline{n}|}) + 230 = \frac{5}{3}(1560) + 230 = 2830$

8. Lee lends Anna 20.000. Anna repays Lee by making payments at the end of each year for ten years. Each payment consists of 2000 to repay the principle, plus interest on the unpaid balance at 3% annual effective. Each time Lee receives a payment, he invests the payment in an account that earns an interest rate of 4% annual effective. Determine the amount Lee has in the account immediately after Anna's last payment.

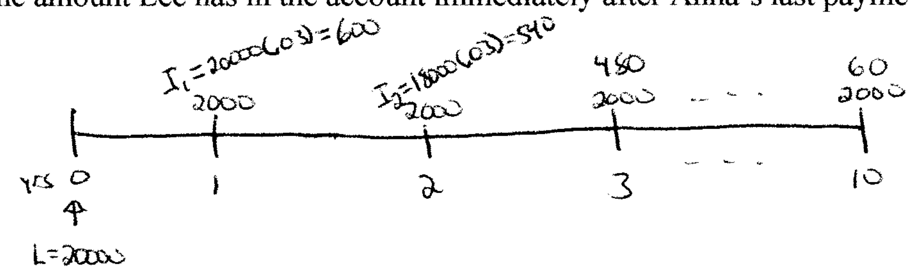
(A) 26850

(B) 27475

(C) 28200

(D) 28850

(E) 29225



$AV_{10} = 2000\,s_{\overline{10}|.04} + 60\,(Ds)_{\overline{10}|.04}$

$\doteq 28206.72$

9. A 20-year callable bond with face amount 1000 and 8% semiannual coupons has a redemption value of 1200. The bond can be redeemed at the end of year 16, 17, 18, 19, or 20. The bond is bought at the maximum price to guarantee a semiannual yield of at least 5%. Determine the buyer's annual effective yield on the bond if the bond is called at the end of year 18.

$r = .04$

(A) 5.1%

(B) 5.2%

(C) 10.2%

(D) 10.4%

(E) 10.6%

n	$P(.05)$
32	$P = 40a_{\overline{32}\rvert .05} + 1200v_{.05}^{32} = 883.94$
40	$P = 40a_{\overline{40}\rvert .05} + 1200v_{.05}^{40} = 856.81$

The bond is bought for 856.81

If redeemed at $n = 36$, then

$$856.81 = 40a_{\overline{36}\rvert i} + 1200v_i^{36}$$

$$\Rightarrow i = .05079\cdots = \text{seir}$$

$$\Rightarrow j = \text{aeir} = (1+i)^2 - 1 \doteq 10.4\%$$

10. Bob buys a 1000 face value 20-year bond, redeemable at par, with 10% quarterly coupons at a price to yield a nominal interest rate of i, compounded quarterly. Immediately after receiving the 10^{th} coupon, Bob sells the bond to Amy at a price that yields Amy a nominal interest rate of 8%, compounded quarterly. Bob's quarterly effective yield over the time he owns the bond is 3%. Determine i.

(A) 8.95%

(B) 9.00%

(C) 9.05%

(D) 9.10%

(E) 9.15%

$r = \frac{.1}{4} = .025 \Rightarrow Fr = 25$ $\quad n = 80$

$$\text{Amy: } P^A = 25a_{\overline{70}\rvert .02} + 1000v_{.02}^{70} \doteq 1187.49$$

$$\therefore P^B = 25a_{\overline{10}\rvert .03} + P^A v_{.03}^{10} \Rightarrow P^B \doteq 1096.86$$

$$\Rightarrow 1096.86 = 25a_{\overline{80}\rvert j} + 1000v_j^{80} \quad j = \text{qeir}$$

$$\Rightarrow j = .02238\cdots$$

$$\Rightarrow i = i^{(4)} = 4j \doteq 8.95\%$$