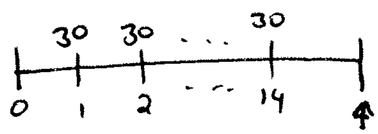
MAP 4170 Name:______________________________

Test 3 Date: July 6, 2016

Show sufficient work and clearly mark your answers. Each problem is worth 10 points.

1. Gus buys a 20-year 1000 face value bond, redeemable at par, with 6% semiannual coupons at a price to yield 8%, compounded semiannually. Gus invests each coupon received in an account that earns 10%, compounded semiannually. Immediately before receiving the 15th coupon, Gus sells the bond to Olivia at a price that yields Olivia 4%, compounded semiannually. Determine the nominal yield, compounded semiannually, that Gus received over the time in which he owns the bond.

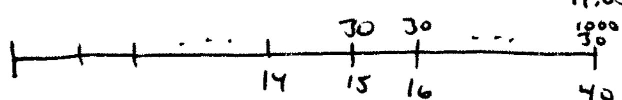
(A) 5.59%



$AV = 30\ddot{s}_{\overline{14}|.05}$

(B) 5.70%

(C) 8.97%



(D) 11.17%

$P^{Olivia} = 30 + 30a_{\overline{25}|.02} + 1000v^{25}_{.02}$

(E) 11.40%

$P^{Gus} = 30a_{\overline{40}|.04} + 1000v^{40}_{.04}$

$$\therefore \left[30a_{\overline{40}|.04} + 1000v^{40}_{.04}\right]\left(1+\frac{i^{(2)}}{2}\right)^{15} = 30\ddot{s}_{\overline{14}|.05} + 30 + 30a_{\overline{25}|.02} + 1000v^{25}_{.02}$$

$$\Rightarrow i^{(2)} \doteq 11.4\%$$

2. For a bond with coupons of 140, you are given:
   i. $P_{16} + P_{18} + \cdots + P_{34} = -43.60$
   ii. $P_{16} = -3.80$
   Determine the price of the bond.

(A) 9500

$$P_{16} + P_{18} + \cdots + P_{34} = P_{16}\left(1 + (1+i)^2 + \cdots (10\text{ terms})\right)$$

(B) 9550

$$= P_{16} \cdot s_{\overline{10}|[(1+i)^2-1]}$$

(C) 9600

(D) 9650

$$\therefore 3.80 \cdot s_{\overline{10}|[(1+i)^2-1]} = 43.6 \Rightarrow i = .01498\cdots$$

(E) 9700

$$I_{16} = R - P_{16} = 143.80 = i \cdot B_{15} \Rightarrow B_{15} = 9599.08$$

$$P = B_0 = 140a_{\overline{15}|} + B_{15} \cdot v^{15} \Rightarrow P = 9548.37$$

3. A 30-year loan is repaid with level monthly payments of 2000. The loan interest rate is 6% annual effective for the first 15 years and 9% annual effective thereafter. Determine the amount of principle repaid in the 150th payment.

(A) Less than or equal to 870

(B) Greater than 870 but less than or equal to 875

(C) Greater than 875 but less than or equal to 880

(D) Greater than 880 but less than or equal to 885

(E) Greater than 885

$$P_{150} = 2000 - I_{150}$$

$$I_{150} = i \cdot B_{149} \qquad (1+i)^{12} = 1.06$$

$$B_{180} = 2000\, a_{\overline{180}|j} \qquad (1+j)^{12} = 1.09$$

$$B_{149} = 2000\, a_{\overline{31}|i} + B_{180} \cdot v_i^{31}$$

$$\therefore \quad P_{150} = 877.55$$

4. A 20-year loan at 7% annual effective is repaid with annual payments. Each payment for the first 10 years is 120% of the amount of interest due at time of payment. Each payment for the last 10 years is 12,365. Determine the amount borrowed.

(A) Less than or equal to 95,000

(B) Greater than 95,000 but less than or equal to 105,000

(C) Greater than 105,000 but less than or equal to 115,000

(D) Greater than 115,000 but less than or equal to 125,000

(E) Greater than 125,000

0	1	2	…	10	11	12	…	20
	$1.2I_1$	$1.2I_2$	…	$1.2I_{10}$	12365	12365	…	12365

$$B_{10} = 12365 \cdot a_{\overline{10}|.07}$$

$$R_1 = 1.2I_1 = I_1 + P_1 \Rightarrow P_1 = .2I_1 = .2(.07)\cdot B_0 = .014B_0$$

$$\therefore B_1 = B_0 - P_1 = B_0 - .014B_0 = .986B_0$$

$$R_2 = 1.2I_2 = I_2 + P_2 \Rightarrow P_2 = .2I_2 = .2(.07)B_1 = .014B_1$$

$$\therefore B_2 = B_1 - P_2 = B_1 - .014B_1 = .986B_1 = (.986)^2 \cdot B_0$$

$$\vdots$$

$$B_{10} = (.986)^{10} B_0 = 12365\, a_{\overline{10}|.07}$$

$$\Longrightarrow B_0 = L = 99{,}996.22$$

5. A 20-year loan of 100,000 is to be repaid with annual payments using the sinking fund method. The lender charges an interest rate of 6% annual effective. The balance in the sinking fund immediately after the deposit at the end of the fourth year is 12,170. The net amount of interest paid on the loan during the fifth year is 5300. Determine the balance in the sinking fund immediately after the deposit at the end of the tenth year.

(A) 36,375

(B) 36,575

(C) 36,775

(D) 36,975

(E) 37,175

$R^I = 6000 \qquad B_4^{SF} = R^{SF} \cdot s_{\overline{4}|j} = 12170$

$5300 = 6000 - j \cdot B_4 = 6000 - 12170j$

$\Rightarrow j = \frac{700}{12170}$

$\therefore R^{SF} \doteq 2792.22$

$B_{10}^{SF} = R^{SF} s_{\overline{10}|j} = 36{,}377.62$

6. A 40-year 1000 face value callable bond with 4% annual coupons is bought at the maximum price to yield 3% annual effective. The bond can be called as follows:

at the end of years 16 through 20 with a redemption value of 1200
at the end of years 26 through 30 with a redemption value of 1300
at the end of years 36 through 40 with a redemption value of 1400

Determine the condition in which the buyer of the bond will realize the greatest yield.

(A) the bond is redeemed at the end of year 20

(B) the bond is redeemed at the end of year 26

(C) the bond is redeemed at the end of year 30

(D) the bond is redeemed at the end of year 36

(E) the correct answer is not given above

n	$P(.03)$	$i : P(i) = 1250.24$
16	1250.24 *	3%
20	1259.50	3.05%
26	1317.87	3.298%
30	1319.60	3.279%
36	1356.33	3.375% ←
40	1353.77	3.346%

$P = 1250.24$

The greatest yield is 3.375% annual effective and occurs if the bond is called at the end of year 36.

7. A 10-year bond with semiannual coupons and a redemption value of 1000 is bought to yield 6.09% annual effective. The first coupon is 20 and each subsequent coupon is 3% more than its preceding coupon. Calculate the amount of the principle adjustment in the coupon paid at the end of the second half of the third year. $P_6 = ?$

(A) – 6.2

(B) – 6.4

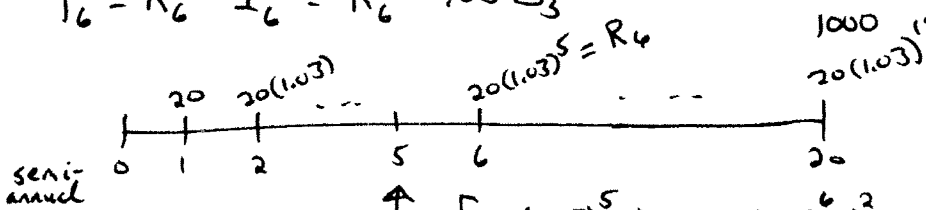
(C) – 6.6

(D) – 6.8

(E) – 7.0

aeir = 6.09% $\Rightarrow$ seir = 3%

$P_6 = R_6 - I_6 = R_6 - .03 B_5$



$B_5 = \left[20(1.03)^5 v_{.03} + 20(1.03)^6 v_{.03}^2 + \cdots (15 \text{ terms})\right] + 1000 v_{.03}^{15}$

$= \frac{20(1.03)^5}{1.03}\left[1 + \frac{1.03}{1.03} + \cdots (15 \text{ terms})\right] + 1000 v_{.03}^{15}$

$\therefore B_5 = 20(1.03)^4(15) + 1000 v_{.03}^{15}$

$R_6 = 20(1.03)^5$ $\therefore P_6 = -6.2$

8. A 10-year loan of 50,000 at an interest rate of 8% annual effective is repaid with quarterly payments. The first payment is 1000 and each subsequent payment during the first five years is 100 more than its preceding payment. The outstanding balance, after the payment at the end of year 5, is amortized with level payments of X at the end of each year for the final five years. Determine X.

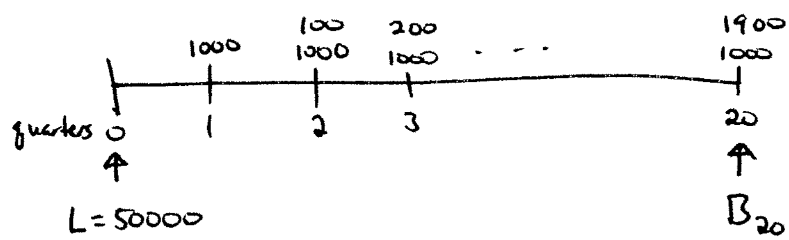
(A) 6470

(B) 6930

(C) 6990

(D) 7510

(E) 7560



$B_{20} = X a_{\overline{5}|.08}$

$B_{20} \overset{\text{Retro}}{=} 50000(1.08)^5 - 1000 s_{\overline{20}|q} - 100 (Is)_{\overline{19}|q}$

$q = \text{qeir} \Rightarrow (1+q)^4 = 1.08$

$\therefore 50000(1.08)^5 - 1000 s_{\overline{20}|q} - 100 (Is)_{\overline{19}|q} = X \cdot a_{\overline{5}|.08}$

$\Rightarrow X = 6987.20$

9. An n-year 1000 par value bond, redeemable at par, with r% semiannual coupons is selling for 885. With $(r-2)$% semiannual coupons instead of the r% semiannual coupons, the bond would sell for 705. Determine the price if the bond had $(r+4)$% semiannual coupons. Assume the same yield rate for all calculations.

(A) 1110

(B) 1155

(C) 1200

(D) 1245

(E) 1290

$$885 = 1000 \cdot \frac{.01r}{2} \cdot a_{\overline{2n}|} + 1000v^{2n} = 5r\,a_{\overline{2n}|} + 1000v^{2n}$$

$$705 = 1000 \cdot \frac{.01r - .02}{2} \cdot a_{\overline{2n}|} + 1000v^{2n} = 5r\,a_{\overline{2n}|} - 10a_{\overline{2n}|} + 1000v^{2n}$$

$$\therefore\ 10a_{\overline{2n}|} = 885 - 705 = 180$$

$$P = 1000 \cdot \frac{.01r + .04}{2} \cdot a_{\overline{2n}|} + 1000v^{2n} = 5r\,a_{\overline{2n}|} + 20a_{\overline{2n}|} + 1000v^{2n}$$

$$= \underbrace{5r\,a_{\overline{2n}|} + 1000v^{2n}}_{885} + 20a_{\overline{2n}|}$$

$$= 885 + 2(180) = 1245$$

10. A 30-year loan of 200,000 at 6% annual effective has annual payments. The payments are 10000 for first 10 years, 20000 for next 10 years, and X for the last 10 years. Calculate the total amount of interest paid during the five year period extending from the beginning of the ninth year through the end of the thirteenth year.

(A) 13,870

(B) 15730

(C) 35,550

(D) 64,270

(E) 66,130

$$\sum_{K=9}^{13} I_K = \sum_{K=9}^{13} R_K - \sum_{K=9}^{13} P_K$$

$$\sum_{K=9}^{13} R_K = R_9 + R_{10} + R_{11} + R_{12} + R_{13}$$

$$= 10000 + 10000 + 20000 + 20000 + 20000$$

$$\Rightarrow \sum_{K=9}^{13} R_K = 80000$$

$$\sum_{K=9}^{13} P_K = B_8 - B_{13}$$

$$B_8 \overset{\text{Retro}}{=} 200000(1.06)^8 - 10000\,s_{\overline{8}|.06}$$

$$B_{13} = B_{10}(1.06)^3 - 20000\,s_{\overline{3}|.06}$$

where $B_{10} = 200000(1.06)^{10} - 10000\,s_{\overline{10}|.06}$

$$\therefore \sum_{K=9}^{13} I_K = 66133.34$$