MAP 4170
Test 3

Name:______________________________

Date: July 13, 2017

Show sufficient work and clearly mark your answers. Each problem is worth 10 points.

1. A 20-year bond with annual coupons of 50, redeemable at 1000, can be redeemed at the end of year 16 through 20. The bond is bought at the maximum price that guarantees an annual effective yield of 6%. Determine the annual effective yield if the bond is called at the end of year 18.

(A) 6.06%

(B) 6.08%

(C) 6.10%

(D) 6.12%

(E) 6.14%

Time	P(.06)
16	$50\,a_{\overline{16}\rvert .06} + 1000\,v_{.06}^{16} = 898.94$
⋮	⋮
20	$50\,a_{\overline{20}\rvert .06} + 1000\,v_{.06}^{20} = 885.30$

Buyer pays 885.30

$\therefore\ 885.30 = 50\,a_{\overline{18}\rvert i} + 1000\,v_i^{18} \Rightarrow i = 6.06\%$

2. A loan of 12,000 is being repaid in 10 years with monthly payments of 100, plus interest on the unpaid balance at 6% per annum compounded monthly. Determine the amount of the 84th payment.

(A) 117.0

(B) 117.5

(C) 118.0

(D) 118.5

(E) 119.0

$R_{84} = 100 + m \cdot B_{83}$

$m = \text{meir} = \frac{.06}{12} = .005$

$B_{83} = 12000 - 83(100) = 3700$

$\therefore\ R_{84} = 118.5$

3. Bob takes out a loan of L at an annual effective interest rate of i. You are given:

(i) The first payment is made at the end of year 3.
(ii) Four equal annual payments are made to repay the loan in full at the end of 6 years.
(iii) The balance immediately after the payment made at the end of year 4 is $0.6L$.

Determine i.

(A) 6.00%

(B) 6.17%

(C) 6.33%

(D) 6.50%

(E) 6.67%

$$\left.\begin{array}{l} L = R a_{\overline{4|}} \cdot v^2 \\ .6L = R a_{\overline{2|}} \end{array}\right\} \Rightarrow \frac{1}{.6} = \frac{a_{\overline{4|}} \cdot v^2}{a_{\overline{2|}}}$$

$$a_{\overline{4|}} = a_{\overline{2|}} \cdot (1 + v^2) \Rightarrow \frac{1}{.6} = \frac{(1+v^2)\cdot v^2}{1}$$

$$\Rightarrow .6v^4 + .6v^2 - 1 = 0$$

$$\Rightarrow v^2 = \frac{-.6 \pm \sqrt{(.6)^2 - 4(.6)(-1)}}{2(.6)} = .8844\ldots = (1+i)^{-2}$$

$$\Rightarrow i = .0633\ldots$$

4. Joe repays a loan of 10,000 by establishing a sinking fund and making 20 equal payments at the end of each year. The sinking fund earns 7% effective annually.

Immediately after the fifth payment, the yield on the sinking fund increases to 8% effective annually. At that time, Joe adjusts his sinking fund payment to X so that the sinking fund will accumulate to 10,000 exactly 20 years after the original loan date.

Determine X.

(A) 195

(B) 200

(C) 205

(D) 210

(E) 215

$$R^{SF} \cdot s_{\overline{20|}.07} = 10000$$

$$\Rightarrow R^{SF} = 243.93 \text{ (original SF payment)}$$

$$B_5^{SF} = R^{SF} \cdot s_{\overline{5|}.07} = 1402.773\ldots$$

$$B_5^{SF} \cdot (1.08)^{15} + X \cdot s_{\overline{15|}.08} = 10000$$

$$\Rightarrow X = 204.41$$

5. A loan of 1000 is repaid with equal payments at the end of each year for 20 years.

   The principal portion of the 13th payment is 1.5 times the principal portion of the 5th payment.

   Calculate the total amount of interest paid on the loan.

   (A) 632

   (B) 642

   (C) 652

   (D) 662

   (E) 672

$$\left.\begin{array}{l} P_{13} = (1.5)\cdot P_5 \\ P_{13} = P_5 \cdot (1+i)^8 \end{array}\right\} \Rightarrow (1+i)^8 = 1.5 \Rightarrow i = .0519\ldots$$

$$1000 = R \cdot a_{\overline{20|}i} \Rightarrow R = 81.60$$

$$\therefore \sum_{k=1}^{20} I_k = 20R - L = 20(81.60) - 1000 = 632$$

6. A loan is amortized over five years with monthly payments at a nominal interest rate of 6% compounded monthly. The first payment is 1000 and is to be paid one month from the date of the loan. Each succeeding monthly payment will be 5% lower than the prior payment.

   Calculate the outstanding loan balance immediately after the 40th payment is made.

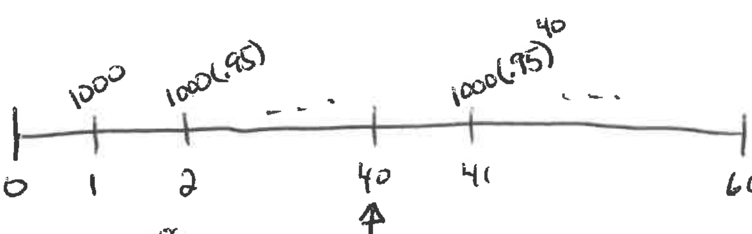
   (A) 1492

   (B) 1578

   (C) 1701

   (D) 1830

   (E) 1966



$m = \text{near} = \frac{.06}{12} = .005$

$v = \frac{1}{1.005}$

B_{40}

$$B_{40} = 1000(.95)^{40}\cdot v + 1000(.95)^{41}\cdot v^2 + \ldots \text{ (20 terms)}$$

$$= \frac{1000(.95)^{40}}{1.005}\left(1 + \frac{.95}{1.005} + \ldots \text{ (20 terms)}\right)$$

$$\therefore B_{40} = \frac{1000(.95)^{40}}{1.005}\cdot \ddot{a}_{\overline{20|}\left(\frac{1.005}{.95}-1\right)} = 1578.47$$

7. Bryan buys a $2n$-year 1000 par value bond with 7.2% annual coupons at a price P. The price assumes an annual effective yield of 12%. At the end of n years, the book value of the bond, X, is 45.24 greater than the purchase price, P. Assume $v_{0.12}^{n} < 0.5$. Calculate X.

(A) 607

(B) 652

(C) 886

(D) 903

(E) 948

$$P = 72a_{\overline{2n}|.12} + 1000v_{.12}^{2n} = 72\left(\frac{1-v^{2n}}{.12}\right) + 1000v^{2n} = 600(1-v^{2n}) + 1000v^{2n}$$

$$\therefore P = 600 + 400v^{2n}$$

Likewise $X = P + 45.24 = 600 + 400v^{n}$

$$\therefore -45.24 = 400v^{2n} - 400v^{n} \Rightarrow 400v^{2n} - 400v^{n} + 45.24 = 0$$

$$\Rightarrow v^{n} = \frac{400 \pm \sqrt{(-400)^2 - 4(400)(45.24)}}{2(400)} = \frac{400 \pm 296}{800}$$

Since $v^{n} < .5$, then $v^{n} = \frac{400-296}{800} = .13$

$$\therefore X = 600 + 400(.13) = 652$$

8. A 10-year bond with par value 1000 and annual coupons rate r is redeemable at 1100. You are given:

(i) The price to yield an effective annual interest rate of 4% is P.
(ii) The price to yield an effective annual interest rate of 5% is $P - 81.49$.
(iii) The price to yield an effective annual interest rate of r is X.

Calculate X.

(A) 1040

(B) 1050

(C) 1060

(D) 1070

(E) 1080

$$X = 1000r\,a_{\overline{10}|r} + 1100v_r^{10} = 1000\cancel{r}\left(\frac{1-v^{10}}{\cancel{r}}\right) + 1100v^{10}$$

$$\therefore X = 1000 + 100v_r^{10}$$

$$P = 1000r\,a_{\overline{10}|.04} + 1100v_{.04}^{10} = r\cdot(\underbrace{8110.895\ldots}_{\boxed{1}}) + \underbrace{743.12\ldots}_{\boxed{2}}$$

$$P - 81.49 = 1000r\,a_{\overline{10}|.05} + 1100v_{.05}^{10} = r\cdot(\underbrace{7721.734\ldots}_{\boxed{3}}) + \underbrace{675.30\ldots}_{\boxed{4}}$$

$$\therefore 81.49 = r\cdot(\boxed{1} - \boxed{3}) + (\boxed{2} - \boxed{4})$$

$$\Rightarrow r = .035\ldots$$

$$\Rightarrow X = 1070.80$$

9. Betty buys an n-year 1000 par value bond with 6.5% annual coupons at a price of 798.48. The price assumes an annual effective yield rate of i. The total write-up in book value of the bond during the first 3 years after purchase is 22.50.

Calculate i.

(A) 8.00%

(B) 8.25%

(C) 8.50%

(D) 8.75%

(E) 9.00%

$B_0 = 798.48 \qquad B_3 = 798.48 + 22.50 = 820.98$

$Fr = 65$

$\therefore \; B_0 = 65\, a_{\overline{3}|i} + B_3 \cdot v_i^3$

$\Rightarrow i = .09$

10. A 1000 par value n-year bond maturing at par with annual coupons of 100 is purchased for 1125. The present value of the redemption value is 500.

Find n.

(A) 5

(B) 6

(C) 7

(D) 8

(E) 9

$1125 = 100\, a_{\overline{n}|} + 1000 v^n \qquad 1000 v^n = 500 \Rightarrow v^n = .5$

$\therefore \; 1125 = 100 \left(\frac{1 - v^n}{i}\right) + 1000 v^n$

$\Rightarrow 1125 = 100 \left(\frac{1 - .5}{i}\right) + 500$

$\Rightarrow i = .08$

$\therefore \; 1125 = 100\, a_{\overline{n}|.08} + 1000 v_{.08}^n \Rightarrow n = 9$