Show sufficient work and clearly mark your answers. Each problem is worth 10 points.

1. A 20-year bond with annual coupons of 50, redeemable at 1000, can be redeemed at the end of year 16 through 20. The bond is bought at the maximum price that guarantees an annual effective yield of 6%. Determine the annual effective yield if the bond is called at the end of year 18.

   *(A) 6.06%  
   (B) 6.08%  
   (C) 6.10%  
   (D) 6.12%  
   (E) 6.14%  

   

   Time | P(0.06)  
   --- | ---  
   16 | 50 A(16|0.06) + 1000 2.16 \[= 898.94\]  
   20 | 50 A(20|0.06) + 1000 2.20 \[= 885.30\]  

   Buyer pays $885.30  

   \[885.30 = 50 A(18|i) + 1000 2^18 \Rightarrow \hat{i} = 6.06\%\]  

2. A loan of 12,000 is being repaid in 10 years with monthly payments of 100, plus interest on the unpaid balance at 6% per annum compounded monthly. Determine the amount of the 84th payment.

   *(A) 117.0  
   (B) 117.5  
   (C) 118.0  
   (D) 118.5  
   (E) 119.0  

   \[R_{84} = 100 + m \cdot B_{83}\]  

   \[m = m \cdot i \tau = \frac{0.06}{12} = 0.005\]  

   \[B_{83} = 12000 - 83(100) = 3700\]  

   \[\therefore R_{84} = 118.5\]
3. Bob takes out a loan of $L$ at an annual effective interest rate of $i$. You are given:

(i) The first payment is made at the end of year 3.
(ii) Four equal annual payments are made to repay the loan in full at the end of 6 years.
(iii) The balance immediately after the payment made at the end of year 4 is $0.6L$.

Determine $i$.

(A) 6.00%

\[ L = R \frac{a_{n1}}{.01} \cdot v^2 \implies \frac{1}{.6} = \frac{a_{n1} \cdot v^2}{a_{21}} \]

(B) 6.17%

\[ .6L = R a_{31} \]

(C) 6.33%

\[ a_{41} = a_{21} \cdot (1 + v^2) \implies \frac{1}{.6} = \frac{(1 + v^2) \cdot v^2}{1} \]

(D) 6.50%

\[ \implies .6 v^4 + .6 v^2 - 1 = 0 \]

\[ \implies v^2 = \frac{-6 \pm \sqrt{(6)^2 - 4(6)(-1)}}{2(6)} = .8844... = (1 + i^2) \]

\[ \implies i = .0633... \]

4. Joe repays a loan of 10,000 by establishing a sinking fund and making 20 equal payments at the end of each year. The sinking fund earns 7\% effective annually.

Immediately after the fifth payment, the yield on the sinking fund increases to 8\% effective annually. At that time, Joe adjusts his sinking fund payment to $X$ so that the sinking fund will accumulate to 10,000 exactly 20 years after the original loan date.

Determine $X$.

(A) 195

\[ R^{SF} \cdot S_{\overline{20}|.07} = 10000 \]

(B) 200

\[ \implies R^{SF} = 243.93 \text{ (original SF payment)} \]

(C) 205

\[ C \]

(D) 210

\[ B^{SF}_5 = R^{SF} \cdot S_{\overline{15}|.07} = 1402.773... \]

(E) 215

\[ B^{SF}_5 \cdot (1.08)^{15} + X \cdot S_{\overline{15}|.08} = 10000 \]

\[ \implies X = 204.41 \]
5. A loan of 1000 is repaid with equal payments at the end of each year for 20 years.

The principal portion of the 13\textsuperscript{th} payment is 1.5 times the principal portion of the 5\textsuperscript{th} payment.

Calculate the total amount of interest paid on the loan.

\textbf{(A)} 632

\[ P_{13} = (1.5) \cdot P_5 \quad \Rightarrow \quad (1+i)^8 = 1.5 \]

\textbf{(B)} 642

\[ P_{13} = P_5 \cdot (1+i)^8 \quad \Rightarrow \quad i = .0519 \ldots \]

\textbf{(C)} 652

\[ 1000 = R \cdot a_{\overline{20}|i} \quad \Rightarrow \quad R = 81.60 \]

\[ \sum_{k=1}^{20} I_k = 20R - L = 20(81.60) - 1000 = 632 \]

6. A loan is amortized over five years with monthly payments at a nominal interest rate of 6\% compounded monthly. The first payment is 1000 and is to be paid one month from the date of the loan. Each succeeding monthly payment will be 5\% lower than the prior payment.

Calculate the outstanding loan balance immediately after the 40\textsuperscript{th} payment is made.

\textbf{(A)} 1492

\textbf{(B)} 1578

\[ a = \frac{1000 \cdot .95^{40}}{.005} \]

\textbf{(C)} 1701

\[ B_{40} = 1000(.95)^{40} \cdot u + 1000(.95)^{41} \cdot u^2 + \cdots \text{ (20 terms) } \]

\[ = \frac{1000 (.95)^{40}}{.005} \left(1 + \frac{.95}{.005} + \cdots \text{ (20 terms) } \right) \]

\[ \therefore B_{40} = \frac{1000 (.95)^{40}}{.005} \cdot a_{\overline{20}|(.95^{40} - 1)} = 1578.47 \]
7. Bryan buys a 2n-year 1000 par value bond with 7.2% annual coupons at a price $P$. The price assumes an annual effective yield of 12%. At the end of $n$ years, the book value of the bond, $X$, is 45.24 greater than the purchase price, $P$. Assume $v_{0.12}^n < 0.5$. Calculate $X$.

(A) 607
\[ P = 72 \sum_{i=0}^{2n} + 1000 v_{12}^{2n} = 72 \left( \frac{1 - v_{12}^{2n}}{1 - v_{12}} \right) + 1000 v_{12}^{2n} = 600 (1 - v_{12}^{2n}) + 1000 v_{12}^{2n} \]
\[ \Rightarrow P = 600 + 400 v_{12}^{2n} \]

(B) 652
\[ ' ' ' \quad P = 600 + 400 v_{12}^{2n} \]

(C) 886
\[ \text{Likewise} \quad X = P + 45.24 = 600 + 400 v_{12}^{n} \]

(D) 903
\[ \quad \Rightarrow -45.24 = 400 v_{12}^{n} - 400 v_{12}^{n} \Rightarrow 400 v_{12}^{n} - 400 v_{12}^{n} + 45.24 = 0 \]
\[ \Rightarrow v_{12}^{n} = 400 \pm \frac{\sqrt{-400^2 - 4(400)(45.24)}}{2(400)} = \frac{400 \pm 296}{800} = \frac{400 - 296}{800} = .13 \]
\[ \Rightarrow X = 600 + 400 (1.13) = 652 \]

8. A 10-year bond with par value 1000 and annual coupons rate $r$ is redeemable at 1100. You are given:

(i) The price to yield an effective annual interest rate of 4% is $P$.
(ii) The price to yield an effective annual interest rate of 5% is $P - 81.49$.
(iii) The price to yield an effective annual interest rate of $r$ is $X$.

Calculate $X$.

(A) 1040
\[ X = 1000 r a_{\overline{r}^{10}} + 1100 v_r^{10} = 1000 / \left( \frac{1 - \frac{r}{100}}{r} \right) + 1100 v_r^{10} \]
\[ \Rightarrow X = 1000 + 100 v_r^{10} \]

(B) 1050
\[ \boxed{[1]} \]

(C) 1060
\[ P = 1000 r a_{\overline{r}^{10}} + 1100 v_r^{10} = r \cdot (8110, 895... + 743, 12...) \]
\[ \boxed{[2]} \]

(D) 1070
\[ P - 81.49 = 1000 r a_{\overline{r}^{10}} + 1100 v_r^{10} = r \cdot (7721, 734... + 675.30...) \]
\[ \boxed{[3]} \]
\[ \Rightarrow 81.49 = r \cdot (1050 - 1070) + (1050 - 1070) \]
\[ \Rightarrow r = .035... \]
\[ \Rightarrow X = 1070.80 \]
9. Betty buys an n-year 1000 par value bond with 6.5% annual coupons at a price of 798.48. The price assumes an annual effective yield rate of \( i \). The total write-up in book value of the bond during the first 3 years after purchase is 22.50.

Calculate \( i \).

(A) 8.00% \[ B_0 = 798.48 \quad B_3 = 798.48 + 22.50 = 820.98 \]

(B) 8.25% \[ F_F = 65 \]

(C) 8.50% \[ \therefore B_0 = 65 \cdot \frac{1}{3i} + B_3 \cdot \frac{3}{1+i} \]

(D) 8.75%

(E) 9.00% \[ \Rightarrow i = .09 \]

10. A 1000 par value n-year bond maturing at par with annual coupons of 100 is purchased for 1125. The present value of the redemption value is 500.

Find \( n \).

(A) 5 \[ 1125 = 100 \cdot a_{n1} + 1000 \cdot v^n \quad 1000 \cdot v^n = 500 \Rightarrow v^n = .5 \]

(B) 6

(C) 7

(D) 8

(E) 9 \[ \Rightarrow 1125 = 100 \left( \frac{1-.5}{i} \right) + 500 \]

\[ \Rightarrow \quad i = .08 \]

\[ \therefore 1125 = 100 \cdot a_{9,.08} + 1000 \cdot v_{.08} \quad \Rightarrow n = 9 \]