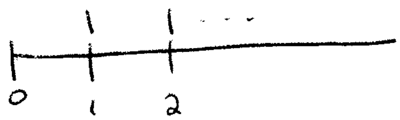
MAP 4170 Name:________________________

Test 4 Date: December 8, 2011

Show sufficient work and clearly mark your answers. Each problem is worth 10 points.

1. The modified duration of a perpetuity immediate with level annual payments of 1 is 20, when calculated using an annual effective interest rate of i. Determine i.

(A) 4.9% (B) 5.0% (C) 5.1% (D) 5.2% (E) 5.3%



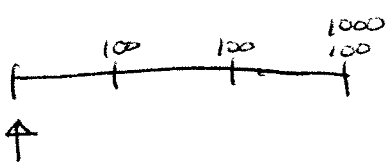
$$MacD = \frac{v+2v^2+\cdots}{v+v^2+\cdots} = \frac{(Ia)_{\overline{\infty|}}}{a_{\overline{\infty|}}} = \frac{\frac{\ddot{a}_{\overline{\infty|}}}{i}}{\frac{1}{i}} = \ddot{a}_{\overline{\infty|}}$$

$$\therefore ModD = \ddot{a}_{\overline{\infty|}} \cdot v = a_{\overline{\infty|}} = \frac{1}{i} = 20$$

$$\Longrightarrow i = .05$$

2. A 3-year bond with annual coupons of 100 and redemption value of 1000 can be bought at an annual effective yield rate of 3%. The current 1-year spot rate is 2%, and the current 2-year spot rate is 3%. Determine the $\underbrace{\text{forward rate during year 3}}_{=f}$ that is consistent with the pricing of the bond.

(A) 3.0% (B) 3.1% (C) 3.2% (D) 4.2% (E) 4.3%



$$P = 100\,a_{\overline{3|}.03} + 1000\,v^3_{.03} = \frac{100}{1.02} + \frac{100}{1.03^2} + \frac{1100}{(1.03)^2(1+f)}$$

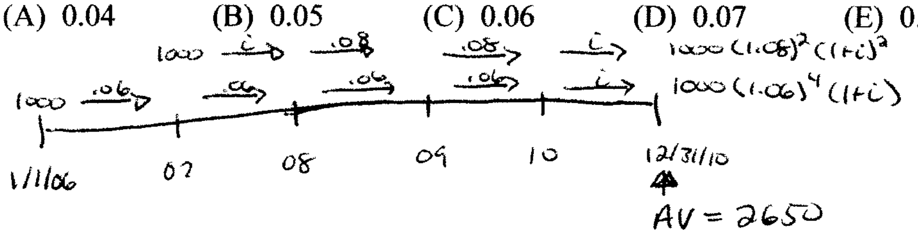
$$\Longrightarrow f \doteq .031$$

3. You are given the following table of interest rates:

Calendar Year of Investment	Investment Year Rates			Portfolio Rates
y	i_1^y	i_2^y	i_3^y	i^{y+2}
2005	0.05	0.06	0.07	0.06
2006	0.06	0.06	0.06	0.06
2007	i	0.08	0.08	i

1000 is invested on 01/01/2006 and another 1000 is invested on 01/01/2007. The total amount as of 12/31/2010 is 2650. Determine i.

(A) 0.04 (B) 0.05 (C) 0.06 (D) 0.07 (E) 0.08

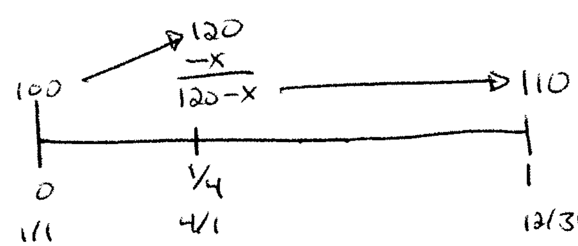


$\therefore \quad 1000(1.08)^2(1+i)^2 + 1000(1.06)^4(1+i) - 2650 = 0$ (quadratic in $1+i$)

$\Longrightarrow 1+i \doteq 1.06 \quad \therefore \; i = .06$

4. A deposit of 100 is made into an account on January 1, 2010. On April 1, 2010, a withdrawal of X is made. The account value before the withdrawal is 120. There are no other transactions during 2010, and the account balance on December 31, 2010, is 110. The time weighted return during 2010 is 32%. Determine the dollar weighted return during 2010.

(A) 26% (B) 29% (C) 32% (D) 35% (E) 38%



$i_{TW} = .32 \Longrightarrow \frac{120}{100} \cdot \frac{110}{120-X} = 1.32 \Longrightarrow X = 20$

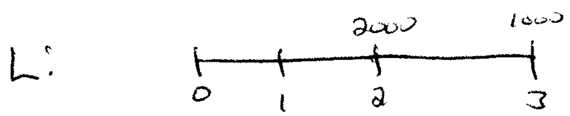
$i = i_{DW}: \quad 100(1+i) - 20(1+\frac{3}{4}i) = 110$

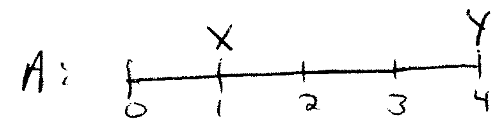
$\Longrightarrow i_{DW} = \frac{30}{85} \doteq 0.35$

5. Leo has liabilities of 2000 due in 2 years from now and another 1000 due in 3 years from now. He is to receive assets 1 year from now and 4 years from now in such a way as to have the assets have the same present value and duration as the liabilities when using an annual discount factor of 0.9. Determine the amount of assets Leo is to receive 1 year from now.

$v = .9$

(A) 1410 (B) 1430 (C) 1450 (D) 1470 (E) 1490





$PV_L = 2000v^2 + 1000v^3$ $\qquad PV_A = Xv + Yv^4$

Numerator of $MacD_L = 4000v^2 + 3000v^3$ $\qquad$ Numerator of $MacD_A = Xv + 4Yv^4$

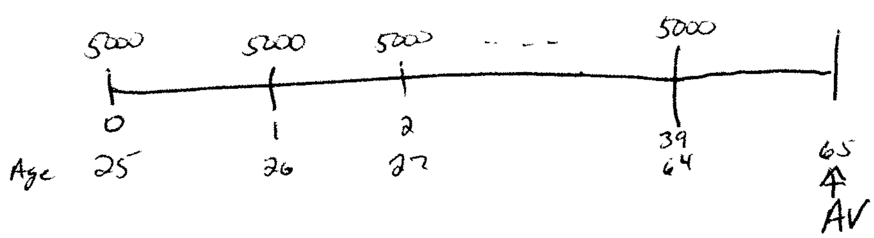
$$\therefore \begin{cases} (Xv + Yv^4 = 2000v^2 + 1000v^3)(4) \\ -(Xv + 4Yv^4 = 4000v^2 + 3000v^3) \end{cases}$$

$$3Xv = 4000v^2 + 1000v^3$$

$$X = \frac{4000(.81) + 1000(.729)}{3(.9)} \doteq 1470$$

6. Corey and Chumlee are 24 year old friends who have decided to start saving for their respective retirements. They each will open retirement accounts and deposit 5000 into their own accounts on every birthday starting on their 25th birthday and ending on their 64th birthday. In estimating how much money they will have on their 65th birthday, Corey uses an annual effective interest rate of 8% and determines they will each have *X*. Chumlee also uses a nominal annual effective interest rate of 8% but he factors in an inflation rate of 3% and uses the real rate of return to estimate the amount they will each have at age 65. Chumlee's estimate is *Y*. Determine *X* – *Y*.

(A) 690,000 (B) 710,000 (C) 735,000 (D) 765,000 (E) 790,000



Corey: $AV = 5000\,\ddot{s}_{\overline{40}|.08} \doteq 1398900 = X$

Chumlee: $1 + i' = \frac{1.08}{1.03} \Rightarrow i' = 0.048543689$

$\therefore AV = 5000\,\ddot{s}_{\overline{40}|i'} \doteq 611250 = Y$

$\therefore X - Y \doteq 787650$

7. Consider a yield curve defined by the following equation:

$$i_k = 0.09 + 0.002k - 0.001k^2$$

where i_k is the annual effective rate of return for zero coupon bonds with maturity of k years. Determine the 2-year spot rate defined by this yield curve.

(A) 0.04 (B) 0.06 (C) 0,07 (D) 0.08 (E) 0.09

$$S_2 = i_2 = .09 + .002(2) - .001(4) = .09$$

8. Joe must pay liabilities of 1000 due 6 months from now and another 1000 due one year from now. The following are two available investments.

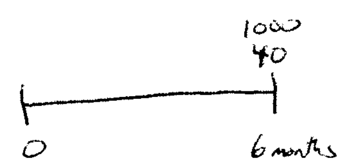
Bond A (i) a 6-month bond with face amount of 1000, an 8% nominal annual coupon rate convertible semiannually, and a 6% nominal annual yield rate convertible semiannually;

Bond B (ii) a one year bond with face amount of 1000, a 5% nominal annual coupon rate convertible semiannually, and a 7% nominal annual yield rate convertible semiannually

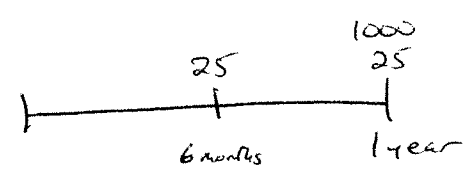
Determine the total cost for Joe to exactly match his liabilities using these two investments.

(A) 1905 (B) 1910 (C) 1915 (D) 1920 (E) 1925

A:



B:



$$P_A = \frac{1040}{1.03} \doteq 1009.71$$

$$P_B = 25a_{\overline{2}|.035} + 1000v^2_{.035} \doteq 981.00$$

Let X = # of Bond A bought
Y = ——— B ———

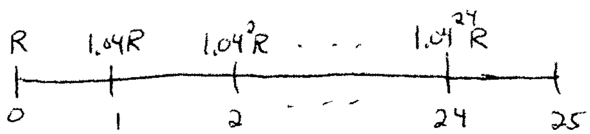
$$\therefore \begin{cases} 1000 = 1040X + 25Y \\ 1000 = 1025Y \end{cases}$$

$$Y = \frac{1000}{1025} \doteq 0.9756 \qquad X = 0.938$$

$$\therefore \text{Total Cost} = 1009.71X + 981Y \doteq 1905$$

9. A 25-year annuity due has annual payments that increase by 4% each year. Determine the duration of this annuity using an annual effective interest rate of 4%.

(A) 11.5 (B) 12 (C) 12.5 (D) 13 (E) 13.5



$$v = \frac{1}{1.04}$$

$$\text{Mac}\,D = \frac{R(0) + \cancel{1.04}R\cancel{v} + 2\cdot\cancel{1.04^2}R\cancel{v^2} + \cdots + 24\cancel{(1.04)^{24}}R\cancel{v^{24}}}{R + \cancel{1.04}R\cancel{v} + \cancel{1.04^2}R\cancel{v^2} + \cdots + \cancel{1.04^{24}}R\cancel{v^{24}}}$$

$$= \frac{R(0+1+2+\cdots+24)}{R(1+1+1+\cdots+1)} = \frac{\frac{24\cdot 25}{2}}{25} = 12$$

10. Data for a two bond portfolio is:

Bond A is a 1000 face value 10 year bond, redeemable at par, with annual coupons of 90. It has Macaulay duration of 7 years using an annual effective interest rate of 9%.

Bond B is a 20 year zero-coupon bond that can be purchased for 1000 to yield 9%.

In what range is the Modified duration, D, of the two bond portfolio using an annual effective interest rate of 9%?

(A) $D < 11.2$ (B) $11.2 < D < 12.4$ (C) $12.4 < D < 13.6$

(D) $13.6 < D < 14.8$ (E) $D > 14.8$

$$P_A = 90\,a_{\overline{10}|.09} + 1000\,v^{10}_{.09} = 1000 = P_B$$

$$\text{Mac}\,D_A = 7 \qquad \text{Mac}\,D_B = 20$$

$$\text{Mac}\,D_{\text{Portfolio}} = \tfrac{1}{2}(7) + \tfrac{1}{2}(20) = 13.5$$

$$\therefore\ \text{Mod}\,D_{\text{Portfolio}} = 13.5v = \frac{13.5}{1.09} \doteq 12.385$$