MAP 4170 Name:______________________________

Test 4 December 4, 2012

Each problem is worth 10 points. Show sufficient work for full credit.

1. A 3-year bond with 5% annual coupons, redeemable at par, can be bought to yield 6% compounded annually. The current 1-year forward rates for years 2 and 3 are 6% and 8%, respectively. Determine the current 1-year spot rate that is consistent with the above rates.

   (A) 1.2% (B) 2.2% (C) 3.2% (D) 4.2% (E) 5.2%

$s_1 = ?$ $\quad f_{[1,2]} = .06 \quad f_{[2,3]} = .08$

Picture:

Per dollar of face value $(F=1)$, we get

$$P = .05\, a_{\overline{3}|.06} + v^3_{.06} = \frac{.05}{1+s_1} + \frac{.05}{(1+s_1)(1.06)} + \frac{1.05}{(1+s_1)(1.06)(1.08)}$$

$$\Longrightarrow s_1 \doteq .042$$

2. A 20-year bond, redeemable at par, has 6% annual coupons. Using a 6% annual effective interest rate, determine the (Macaulay) duration of the bond.

   (A) 10.82 (B) 11.47 (C) 11.82 (D) 12.16 (E) 12.89

$\underbrace{F = C \qquad r = i}$ $\qquad (Ia)_{\overline{n}|} = \dfrac{\ddot{a}_{\overline{n}|} - nv^n}{r}$

$$\therefore \text{MacD} = \frac{\cancel{F}r\,(Ia)_{\overline{n}|} + n\cdot\cancel{F}\cdot v^n}{\cancel{F}r\,a_{\overline{n}|} + \cancel{F}v^n} = \frac{\ddot{a}_{\overline{n}|} - nv^n + nv^n}{1 - v^n + v^n}$$

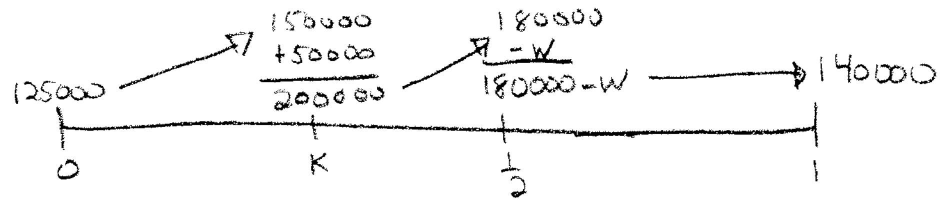
$\hookrightarrow a_{\overline{n}|} = \dfrac{1-v^n}{r}$

$$\therefore \text{MacD} = \ddot{a}_{\overline{n}|}$$

We have $\text{MacD} = \ddot{a}_{\overline{20}|.06} \doteq 12.16$

3. The balance in an investment account on January 1 is 125,000. On some date prior to June 30, a deposit of 50,000 is made into the account. The balance immediately prior to this deposit is 150,000. On June 30 the account balance is 180,000 and on July 1 a withdrawal is taken from the account. The balance on December 31 is 140,000. For the account, the dollar weighted return for the year is 15% and the time weighted return for the year is 20%. Determine the month in which the deposit is made.

(A) February (B) March (C) April (D) May (E) June



$$i_{TW} = 0.2 \Rightarrow 1.2 = \frac{150000}{125000} \cdot \frac{180000}{200000} \cdot \frac{140000}{180000 - W} \Rightarrow \underline{W = 54000}$$

$$i_{DW} = 0.15 \Rightarrow 125000(1.15) + 50000(1 + .15(1-k)) - 54000(1 + .15(\tfrac{1}{2})) = 140000$$

$$\Rightarrow k = .42\bar{6} \text{ years} = 5.12 \text{ months (in June)}$$

4. A portfolio consists of the following two investments:

Investment A is a 10-year annual payment annuity with initial payment *R* and each subsequent payment increasing by 20%. Using a 5% annual effective interest rate, the present value of this annuity is *P* and the Macaulay duration is 6.57 years.

Investment B is a 10-year zero coupon bond redeemable at 10000. $P_B = 10000 v^{10}_{.05}$

The modified duration of the portfolio is 7.55 years using an annual effective interest rate of 5%. Determine *P*.

(A) 9187 (B) 9373 (C) 9589 (D) 9737 (E) 9905

$$\text{ModD} = v \cdot \text{MacD} \Rightarrow \text{MacD}_{\text{Portfolio}} = 7.55(1.05) = 7.9275$$

$P_A = P$ $\quad\quad$ $P_B = 10000v^{10}$

$\text{MacD}_A = 6.57$ $\quad\quad$ $\text{MacD}_B = 10$

$$\therefore 7.9275 = \left(\frac{P}{P + 10000v^{10}}\right)6.57 + \left(\frac{10000v^{10}}{P + 10000v^{10}}\right)10$$

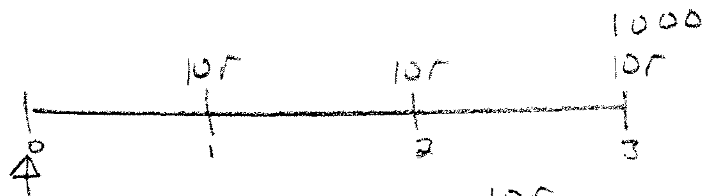
$$\Rightarrow P \doteq 9373$$

5. You are given:

The annual effective yield on 1-year zero-coupon bonds is 4%. $= s_1$
The annual effective yield on 2-year zero-coupon bonds is 5%. $= s_2$
The annual effective yield on 3-year zero-coupon bonds is 6%. $= s_3$

Consistent with these rates, a 3-year annual coupon 1000 face-value bond, redeemable at par, with annual coupon rate *r* %, can be bought for 920.86. Determine *r*.

(A) 1.0 (B) 1.5 (C) 2.0 (D) 2.5 (E) 3.0



$$920.86 = \frac{10r}{1.04} + \frac{10r}{(1.05)^2} + \frac{10r}{(1.06)^2} + \frac{1000}{(1.06)^3}$$

$$\Longrightarrow r \doteq 3$$

6. Jason determines that he can afford to deposit 500 at the beginning of each month beginning on his 25th birthday in order to save for retirement. His last deposit will be one month before his 65th birthday. Jason believes he can realize a nominal annual return of 9% on his investments in the account, and he assumes there will be an annual inflation rate of *r* %. Using the real rate of return, he determines the amount in his account on his 65th birthday will be 1,200,000. Determine *r*.

(A) 1.0 (B) 1.5 (C) 2.0 (D) 2.5 (E) 3.0

$$500\, \ddot{s}_{\overline{480}|j} = 1200000 \Longrightarrow j \doteq .00555$$ monthly effective

i' = real rate of return (annual effective)

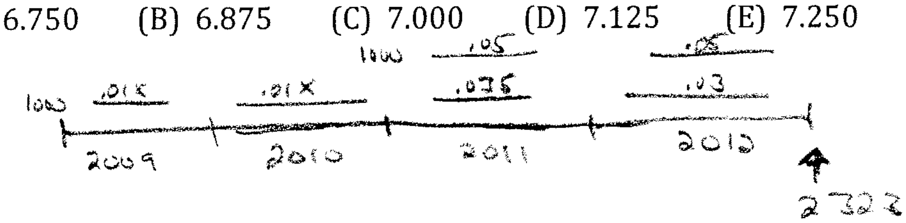
$$1+i' = (1+j)^{12} = \frac{1.09}{1+.01r} \Longrightarrow r \doteq .02$$

7. You are given the following annual interest rates:

Year	Investment Rates		Portfolio Rates
Y	i_1^Y	i_2^Y	i^{Y+2}
2008	8%	8%	4%
2009	X%	X%	3.5%
2010	6%	6%	3%
2011	5%	5%	
2012	4%		

A deposit of 1000 is made at the beginning of year 2009 and another 1000 is deposited at the beginning of year 2011. Using the rates above, the accumulated value at the end of year 2012 is 2,323. Determine X.

(A) 6.750 (B) 6.875 (C) 7.000 (D) 7.125 (E) 7.250



$\therefore 2323 = 1000(1+.01X)^2(1.035)(1.03) + 1000(1.05)^2 \implies X \doteq 7$

8. Liabilities of 1000 at time 2 and another 1000 at time 4 are to be offset with asset payments of 500 at time 1, X at time 3, and Y at time 5, where X and Y are such that, when using a periodic discount factor $v = 0.9$, the present value of the liabilities equals the present value of the assets, and the duration of the liabilities equals the duration of the assets. Determine X, to the nearest dollar.

(A) 802 (B) 882 (C) 962 (D) 1042 (E) 1122

Set PV's equal: $\left(1000v^2 + 1000v^4 = 500v + Xv^3 + Yv^5\right)\cdot 5$

Set (numerator) of durations equal: $2000v^2 + 4000v^4 = 500v + 3Xv^3 + 5Yv^5$

$$3000v^2 + 1000v^4 = 2000v + 2Xv^3$$

$$\implies X \doteq 882$$

9. Ed has liabilities of 9100 due in one year and 13,780 due in two years. He can invest in the following two bonds:

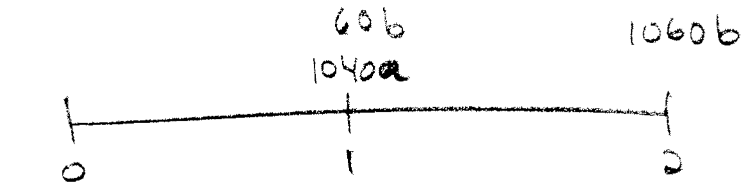
Bond A - a 1-year 1000 face-value bond, redeemable at par, with 4% annual coupons
Bond B - a 2-year 1000 face-value bond, redeemable at par, with 6% annual coupons

Bond A can be bought to yield 4% annual effective and Bond B can be bought to yield 6% annual effective. Determine the total amount Ed will pay in order to exactly match his liabilities.

(A) 21000 (B) 21250 (C) 21500 (D) 21750 (E) 22000

Let a = # of Bond A bought

b = ——— B bought



$\therefore \begin{cases} 60b + 1040a = 9100 \\ 1060b = 13780 \end{cases}$

$b = 13 \qquad a = 8$

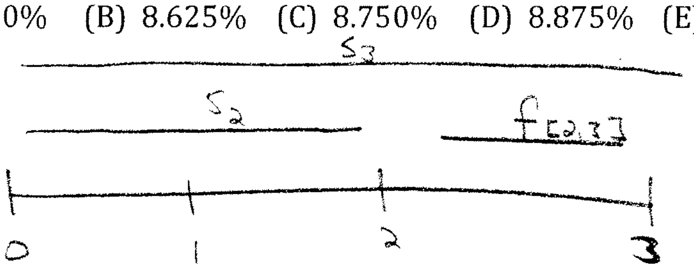
$P_A = 40\, a_{\overline{1}|.04} + 1000 v^1_{.04} = 1000$

$P_B = 1000$ also.

$\therefore$ total paid is $\overset{P_A}{8(1000)} + 13\overset{P_B}{(1000)} = 21000$

10. Consider the yield curve given by the equation $i_k = 0.08 + 0.002k$, where i_k is the annual effective rate of return on zero coupon bonds with maturity of k years. Determine the 1-year forward rate for year 3 that is consistent with this yield curve.

(A) 8.500% (B) 8.625% (C) 8.750% (D) 8.875% (E) 9.000%



$s_2 = i_2 = .084 \qquad s_3 = i_3 = .086$

$\therefore (1.084)^2(1 + f_{[2,3]}) = (1.086)^3 \Rightarrow f_{[2,3]} \doteq .09$