MAP 4170 Name:_____
Test 4 Date: December 5, 2013

Show sufficient work and clearly mark your answers. Each problem is worth 10 points.

1. Rob purchased two bonds to form a portfolio as given below:

(i) Bond A is a 20-year bond with coupons at 5%, a duration of 14 and a purchase price of 1297.

(ii) Bond B is a 20-year bond with coupons at 3%, a duration of 15.32 and a purchase price of P.

For both bonds the duration and price were determined using an annual effective interest rate i. Using the same interest rate, the duration of the portfolio is 14.575. Determine P.

A) 950 B) 975 C) 1000 D) 1025 E) 1050

$$14.575 = \left(\frac{1297}{P+1297}\right)(14) + \left(\frac{P}{P+1297}\right)(15.32)$$

$$\Rightarrow P \doteq 1001.04$$

2. You are given the following table of interest rates:

Calendar Year of Investment	Investment Year Rates			Portfolio Rates
Y	i_1^Y	i_2^Y	i_3^Y	i^{Y+3}
2002	0.05	0.06	0.07	0.04
2003	0.06	0.06	0.06	X%
2004	0.08	0.08	0.10	(X+2) %

1000 is invested on 01/01/2002 and another 1000 is invested on 01/01/2004. The total amount in the account as of 12/31/2006 is 2565. Determine the portfolio rate for 2007.

A) 3.5% B) 4.0% C) 4.5% D) 5.0% E) 5.5%

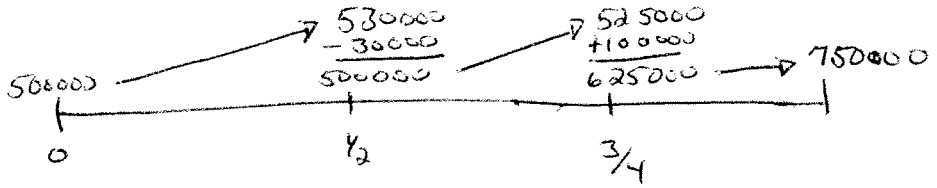
$$2565 = 1000(1.05)(1.06)(1.07)(1.04)(1+.01X) + 1000(1.08)(1.08)(1.10)$$

$$\Rightarrow X \doteq 3.5$$

$$\Rightarrow X+2 = 5.5$$

3. On January 1 a pension fund has \$500,000. The balance on June 30 is \$530,000 and on July 1 benefit payments of \$30,000 are paid from the fund. A contribution of \$100,000 is paid into the fund on date October 1. Immediately before the contribution, the balance in the fund is \$525,000. The balance on December 31 is \$750,000. Kevin calculates the time weighted rate of return on the fund to be K and Linda calculates the dollar weighted rate of return on the fund to be L. Determine the ratio K/L

A) 0.90 B) 0.95 C) 1.00 D) 1.05 E) 1.10



$$1+K = \frac{530000}{500000} \cdot \frac{525000}{500000} \cdot \frac{750000}{625000} \Rightarrow K = .3356$$

$$500000(1+L) - 30000\left(1+\tfrac{1}{2}L\right) + 100000\left(1+\tfrac{1}{4}L\right) = 750000$$

$$\Rightarrow L = \frac{180000}{510000} \doteq .3529 \qquad \therefore \frac{K}{L} \doteq 0.95$$

4. A bond is priced at 1412.50 to yield 6% annual effective, and the Macaulay duration using this interest rate is 13.6. Determine the price of the bond if the interest rate is changed to 6.5%.

A) Less than 1318
B) Greater than or equal to 1318 but less than 1322
C) Greater than or equal to 1322 but less than 1326
D) Greater than or equal to 1326 but less than 1330
E) Greater than or equal to 1330

$\text{MacD} = 13.6$

$\text{ModD} = 13.6/1.06$

$$P(i) \doteq P(i_0) + P'(i_0)(i - i_0)$$

$$P(.06) = 1412.50 \qquad \text{ModD} = \frac{13.6}{1.06} = -\frac{P'(.06)}{P(.06)}$$

$$\Rightarrow P'(.06) = -1412.50\left(\frac{13.6}{1.06}\right)$$

$$\therefore P(.065) \doteq 1412.50 - 1412.50\left(\frac{13.6}{1.06}\right)(.005)$$

$$= 1321.89$$

For 5. and 6.
Consider a yield curve defined by the equation $i_k = .09 - (.06)(.94)^{k-1}$ where i_k is the annual effective rate of return for zero coupon bonds with maturity of k years.

5. Determine the annual yield on 3-year 5% annual coupon bonds that is consistent with this yields curve.

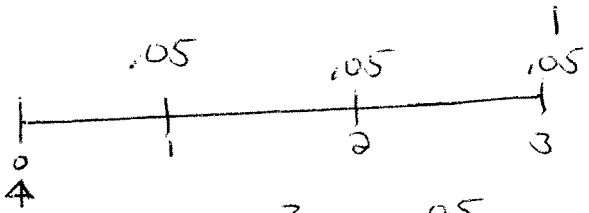
A) 3.62% B) 3.64% C) 3.66% D) 3.68% E) 3.70%

$S_1 = i_1 = .03$

$S_2 = i_2 = .09 - .06(.94) = .0336$

$S_3 = i_3 = .09 - .06(.94)^2 = .036984$

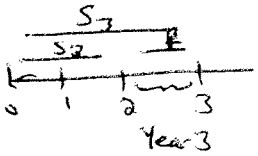
Per dollar of face value



$.05\, a_{\overline{3}|i} + v_i^3 = \frac{.05}{1.03} + \frac{.05}{(1.0336)^2} + \frac{1.05}{(1.036984)^3} \Rightarrow i \doteq .0368$

6. Determine the 1-year forward rate for year 3 implied by this yield curve.

A) 4.38% B) 4.40% C) 4.42% D) 4.44% E) 4.46%



$f = f_{[2,3]}$

$(1+S_2)^2(1+f) = (1+S_3)^3$

$f = \frac{(1.036984)^3}{(1.0336)^2} - 1 \doteq .0438$

7. Bond A is a 1-year zero-coupon bond with a redemption value of F_1. It can be bought to yield 10% annual effective.

Bond B is a 2-year zero-coupon bond with a redemption value of F_2. It can be bought to yield 5% annual effective.

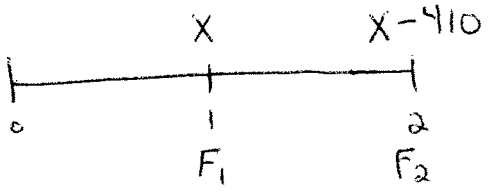
Cecil has the following obligations:

X due at the end of 1 year
$X - 410$ due at the end of 2 years

Cecil pays a total of 13,300 to purchase enough of each type of bond in order to exactly match the obligations. Determine X.

(A) 7,510 (B) 7,530 (C) 7,550 (D) 7,570 (E) 7,590

L



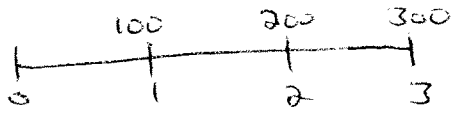
Exact Matching
$\Rightarrow F_1 = X$
$F_2 = X - 410$

$$P = 13300 = F_1 v_{.1} + F_2 v_{.05}^2$$

$$\therefore 13300 = X v_{.1} + (X-410) v_{.05}^2 \Rightarrow X = 7528.07$$

8. Payments of 100, 200, and 300 are to be made at the end of years 1, 2, and 3, respectively. Determine the modified convexity of this sequence of payments using an annual effective interest rate of 6%.

(A) 6.67 (B) 6.87 (C) 7.07 (D) 7.27 (E) 7.47



$$P(i) = 100(1+i)^{-1} + 200(1+i)^{-2} + 300(1+i)^{-3}$$

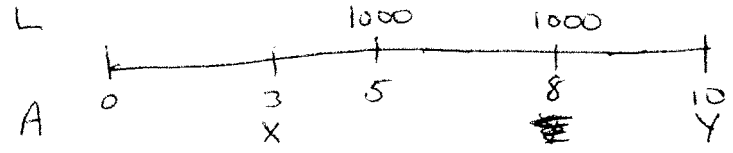
$$P'(i) = -100(1+i)^{-2} - 400(1+i)^{-3} - 900(1+i)^{-4}$$

$$P''(i) = 200(1+i)^{-3} + 1200(1+i)^{-4} + 3600(1+i)^{-5}$$

$$\text{ModC}_{.06} = \frac{P''(.06)}{P(.06)} \doteq \frac{3808.5657}{524.2247} \doteq 7.27$$

9. Liability payments consist of 1,000 due at the end of 5 years and another 1000 due at the end of 8 years. Assets of amount $\$X$ due in 3 year and $\$Y$ due in 10 years have the same present value and duration as the liabilities. Interest is at an annual effective rate of 10%. Determine X.

A) 768 B) 778 C) 788 D) 798 E) 808



$$\text{PV: } \left(1000v^5 + 1000v^8 = Xv^3 + Yv^{10}\right) * 10$$

$$\text{D: } 5000v^5 + 8000v^8 = 3Xv^3 + 10Yv^{10}$$

$$5000v^5 + 2000v^8 = 7Xv^3$$

$$\Rightarrow X = \frac{5000v_{.1}^2 + 2000v_{.1}^5}{7} \doteq 767.72$$

10. You are given the following information about two bonds that will mature at par value in 10 years:

(i) Bond A has a par value of 10000, 5% annual coupons, and is priced at 10963.

(ii) Bond B has a par value of 1000, 4% annual coupons, and is priced at 1013.

Determine the 10-year spot rate consistent with the pricing of these bonds.

(A) 3.7% (B) 3.8% (C) 3.9% (D) 4.0% (E) 4.1%

$$\begin{cases} 10963 = 500a_{\overline{10}|} + 10000v^{10} \\ \left(1013 = 40a_{\overline{10}|} + 1000v^{10}\right)(12.5) \end{cases}$$

$$1699.5 = 2500v^{10}$$

$$\Rightarrow v^{10} = (1+s_{10})^{-10} = \frac{1699.5}{2500}$$

$$\Rightarrow s_{10} \doteq .039$$