MAP 4170 Name:______________________________

Test 4 April 24, 2014

Each problem is worth 10 points. Show sufficient work for full credit.

1. A bond sells for 950 at an annual yield rate of 6% and for 964 at an annual yield rate of 5.9%. Let X denote the modified duration of the bond at 6% and Y denote the modified duration of the bond at 5.9%. Determine the approximate value of $X - Y$ to the nearest tenth.

   (A) −0.2 (B) −0.1 (C) 0.0 (D) 0.1 (E) 0.2

$P(.06) = 950$ $\qquad \Delta P \doteq -P(i) \cdot \text{Mod } D \cdot \Delta i$

$P(.059) = 964$

$i = .06 \to i = .059$: $\Delta P = 14$ $\quad P(i) = 950$ $\quad \Delta i = -.001$ $\quad \text{Mod } D = X$

$\Rightarrow X = \frac{14}{-.001}\left(-\frac{1}{950}\right) = \frac{14000}{950}$

$i = .059 \to i = .06$: $\Delta P = -14$ $\quad P(i) = 964$ $\quad \Delta i = .001$ $\quad \text{Mod } D = Y$

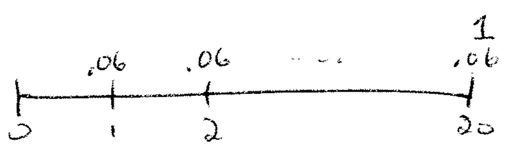
$\Rightarrow Y = \frac{14000}{964}$

$\therefore X - Y = \frac{14000}{950} - \frac{14000}{964} \doteq 0.214$

2. A 20-year bond, redeemable at par, has 6% annual coupons. Using a 5% annual effective interest rate, determine the Macaulay duration of the bond, in years, to the hundredths place.

   (A) 11.47 (B) 12.16 (C) 12.46 (D) 12.62 (E) 13.09

Per dollar of Face amount (F = 1)

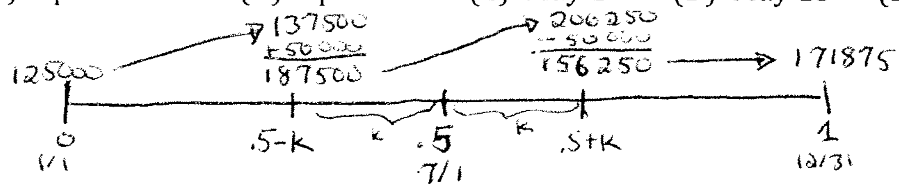


$$\text{Mac } D = \frac{.06\,(Ia)_{\overline{20}|.05} + 20\,v_{.05}^{20}}{P(.05)} \doteq 12.62$$

3. The balance in an investment account on January 1 is 125,000. A deposit of 50,000 is made k months prior to July 1, and a withdrawal of 50,000 is made k months after July 1. The balance on December 31 is 171,875.

The balance in the account immediately before the deposit is 137,500 and the balance in the account immediately before the withdrawal is 206,250. The time weighted return for the year is equal to the dollar weighted return for the year. Determine the date that the deposit was made.

(A) April 1 (B) April 15 (C) May 1 (D) May 15 (E) June 1



$$i_{TW} = \frac{137500}{125000}\cdot\frac{206250}{187500}\cdot\frac{171875}{156250} - 1 = .331 = i_{DW} = i$$

$$125000(1+\overset{.331}{i}) + 50000(1+.331(1-(.5-k))) - 50000(1+.331(1-(.5+k))) = 171875$$

$$\Rightarrow k \doteq .166163 \Rightarrow .5-k = .333837$$

$(.333837)(12) \doteq 4 \Rightarrow$ deposit is 4 months after Jan. 1.

$\Rightarrow$ deposit is on May 1

4. A portfolio consists of three assets, A, B, and C. Using an annual effective interest rate of 10%, you are given:

Asset A has a price of 6000 and modified duration of 9 years.
Asset B has a price of 1000 and modified duration of X years.
Asset C has a price of 3000 and modified duration of X+2 years.

The Macaulay duration of the portfolio is 11 years using an annual effective interest rate of 10%. Determine the Macaulay duration, in years, of Asset C.

(A) 9 (B) 10 (C) 11 (D) 12 (E) 13

Total Price $= P_{port.} = 10000$

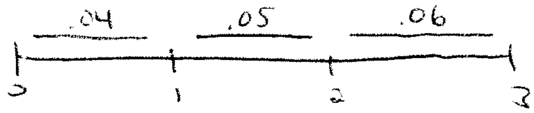
$\therefore$ weights: $\omega_A = \frac{6000}{10000} = .6 \quad \omega_B = .1 \quad \omega_C = .3$

$$\text{Mod } D_{port.} = 11v_{.1} = \frac{11}{1.1} = 10 = .6(9) + .1X + .3(X+2)$$

$$\Rightarrow X = 10 \Rightarrow X+2 = 12$$

$$\therefore \text{Mod } D_C = 12 \Rightarrow \text{Mac } D_C = 12(1.1) = 13.2$$

5. You are given:

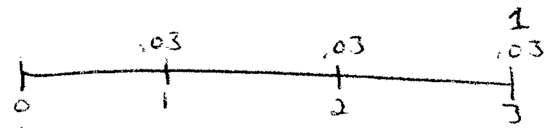


The 1-year spot rate is currently 4%.
The 1-year deferred 1-year forward rate consistent with the current term structure of interest rates is 5%.
The 2-year deferred 1-year forward rate consistent with the current term structure of interest rates is 6%.

Determine the annual yield rate for 3-year, 3% annual coupon bonds that is consistent with these rates.

(A) 4.98% (B) 5.14% (C) 5.37% (D) 5.96% (E) 6.15%

Per dollar of Face Amount (F=1)



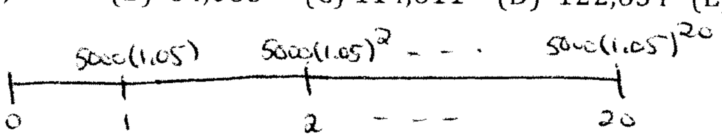
$$P = .03\, a_{\overline{3}|i} + v_i^3 = \frac{.03}{1.04} + \frac{.03}{(1.04)(1.05)} + \frac{1.03}{(1.04)(1.05)(1.06)}$$

$$\Longrightarrow i \doteq .0498$$

6. An insurance company has an obligation to pay the medical costs for a claimant. Average annual claims costs today are 5000, and medical inflation is expected to be 5% per year. The claimant is expected to live an additional 20 years.

Claim payments are made at yearly intervals, with the first claim payment to be made one year from today. Determine the present value of the obligation if the annual interest rate is 7%.

(A) 82,514 (B) 84,085 (C) 114,611 (D) 122,634 (E) Cannot be determined



$$v = v_{.07} = \frac{1}{1.07}$$

$$PV = 5000(1.05)v + 5000(1.05)^2 v^2 + \cdots \text{ (20 terms)}$$

$$= \frac{5000(1.05)}{1.07}\left[1 + \frac{1.05}{1.07} + \cdots \text{ (20 terms)}\right]$$

$$= \frac{5000(1.05)}{1.07} \cdot \ddot{a}_{\overline{20}|\left(\frac{1.07}{1.05}-1\right)} \doteq$$

7. You are given the following annual effective interest rates:

Year	Investment Rates		Portfolio Rates
Y	i_1^Y	i_2^Y	i^{Y+2}
2008	8.00%	7.75%	5.00%
2009	7.25%	6.50%	4.00%
2010	6.00%	6.00%	3.00%
2011	6.25%	5.75%	3.25%
2012	4.50%	4.00%	3.50%
2013	4.00%	4.00%	
2014	4.50%		

A deposit of 1000 is made at the beginning of year 2010. Alex and Veronica are to determine the accumulated value of the deposit on January 1, 2013. Alex used portfolio rates and determined the accumulated value is X. Veronica used investment/portfolio rates and determined the accumulated value is Y. Determine the sum $X + Y$.

(A) 2250 (B) 2280 (C) 2300 (D) 2330 (E) 2360

r	2010	2011	2012
A	.05	.04	.03
V	.06	.06	.03

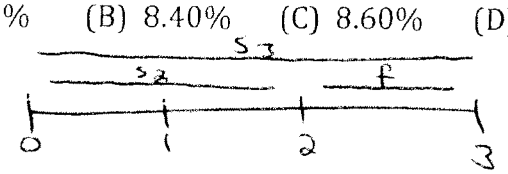
$X = 1000(1.05)(1.04)(1.03)$

$Y = 1000(1.06)(1.06)(1.03)$

$\therefore X + Y \doteq 2282$

8. Consider the yield curve given by the equation $i_k = 0.08 + 0.002(k-1)$, where i_k is the annual effective rate of return on zero coupon bonds with maturity of k years. Determine the 1-year forward rate for year 3 that is consistent with this yield curve.

(A) 8.20% (B) 8.40% (C) 8.60% (D) 8.80% (E) 9.00%



$f = f_{[2,3]}$

$s_3 = .084$

$s_2 = .082$

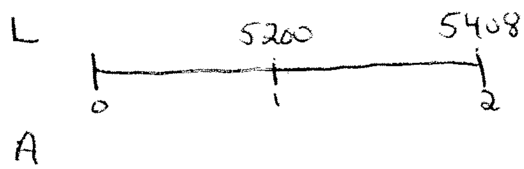
$$\therefore f = \frac{(1.084)^3}{(1.082)^2} - 1 \doteq .088$$

9. Liabilities are 5200 due in one year and 5408 due in two years. The following two bonds are to be used to exactly match the liabilities:

Bond A: a 1-year 1000 face-value bond, redeemable at par, with r % annual coupons
Bond B: a 2-year 1000 face-value bond, redeemable at par, with r % annual coupons

Both bonds can be bought to yield 4% annual effective. Determine the total cost in order to exactly match his liabilities.

(A) 9977 (B) 10000 (C) 10032 (D) 10088 (E) 10137



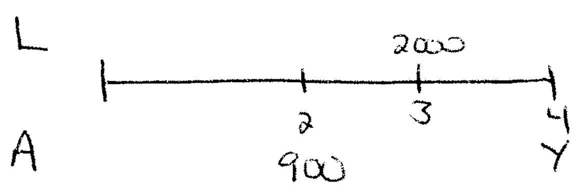
Since both bonds yield 4% annual effective,

Total Cost = PV(A) @ 4% = PV(L) @ 4%

$$= 5200\,v_{.04} + 5408\,v_{.04}^2 = 10000$$

10. A liability of 2000 at the end of 3 years is to be fully immunized, at a nominal interest rate of i compounded monthly, using an asset of 900 at the end of 2 years and another asset of Y at the end of 4 years. Determine Y.

(A) 900 (B) 1010 (C) 1111 (D) 1212 (E) 1313



Let $v = adf$

$$\begin{cases} (900\,v^2 + Y \cdot v^4 = 2000\,v^3)\ (-2) \\ 1800\,v^2 + 4Y \cdot v^4 = 6000\,v^3 \end{cases}$$

$$2Y\,v^4 = 2000\,v^3 \Rightarrow Y = 1000\,v^{-1}$$

$$\therefore\ 900\,v^2 + (1000\,v^{-1}) \cdot v^4 = 2000\,v^3$$

$$\Rightarrow v = .9$$

$$\therefore\ Y = \frac{1000}{.9} \doteq 1111$$