MAP 4170 Name:______________________________
Test 4 Date: July 25, 2013

Show sufficient work and clearly mark your answers. Each problem is worth 10 points.

1. Zero coupon bonds, redeemable at 1000, with durations of 1, 2, and 3 years are currently selling for 970.87, 938.04, and 901.94, respectively. Determine the annual effective yield on 3-year 4% annual coupon bonds that is consistent with this term structure of interest rates.

(A) 3.487%

(B) 3.587%

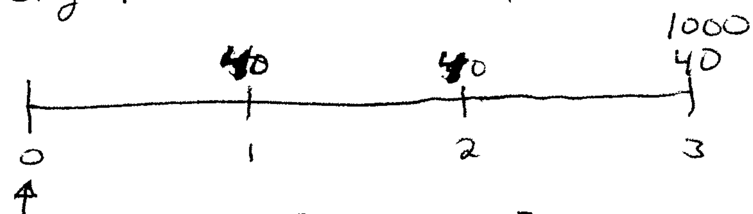
(C) 3.687%

(D) 3.787%

(E) 3.887%

$v_{s_1} = .97087 \quad v_{s_2}^2 = .93804 \quad v_{s_3}^3 = .90194$

Using $F = 1000$ (arbitrary)



$$P = 40v_{s_1} + 40v_{s_2}^2 + 1040v_{s_3}^3 = 40a_{\overline{3}|i} + 1000v_i^3$$

$$\Longrightarrow i \doteq 3.487\%$$

2. Liabilities of 12125 and 13125, due at the end of 1 and 2 years respectively, are to be exactly matched using a 1-year zero-coupon bond and a 2-year 5% annual coupon bond. The 1-year bond can be bought to yield 3% annual effective and the 2-year bond can be bought to yield 4% annual effective. Determine the cost to exactly match the liabilities.

(A) 23900

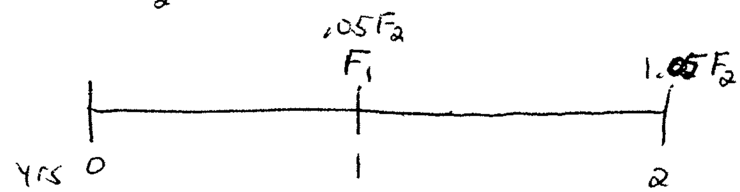
(B) 23910

(C) 24020

(D) 24030

(E) 24040

Let F_1 = face amount of 1-year bond
F_2 = ———————— 2-year bond



$$\therefore \left.\begin{aligned} F_1 + .05F_2 &= 12125 \\ 1.05F_2 &= 13125 \end{aligned}\right\} \Longrightarrow \begin{aligned} F_2 &= 12500 \quad F_2(.05) = 625 \\ F_1 &= 11500 \end{aligned}$$

$$\therefore \text{Cost} = \overbrace{11500v_{.03}}^{=P_1(.03)} + \Big(\overbrace{625a_{\overline{2}|.04} + 12500v_{.04}^2}^{=P_2(.04)}\Big)$$

$$\doteq 23900$$

3. A liability of 1000 due at the end of 2 years is fully immunized at 3% annual effective with 1-year and 3-year zero coupon bonds. Determine the excess of the present value of assets over the present value of liabilities if the interest rate is changed to 6% annual effective.

(A) 0.23

(B) 0.37

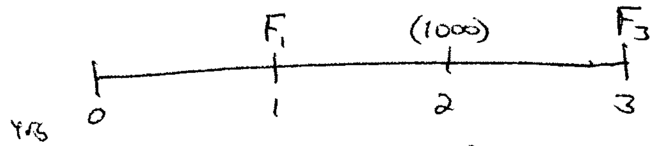
(C) 0.86

(D) 1.11

(E) 1.35

Let F_1 = face amount of 1-year bond

F_3 = ________ 3-year bond



$$\left.\begin{array}{l} F_1 v + F_3 v^3 = 1000 v^2 \\ F_1 v + 3F_3 v^3 = 2000 v^2 \end{array}\right\} \Rightarrow \begin{array}{l} F_3 = 515 \\ F_1 = 485.44 \end{array}$$

$\therefore$ excess at 6% is

$$E = \left(F_3 v_{.06}^3 + F_1 v_{.06}\right) - 1000 v_{.06}^2 \doteq 0.37$$

4. An investment account has a beginning of year balance on January 1 of X and an end of year balance on December 31 of Y. The only account transaction during the year was a deposit of 8000 on July 31. The account balance immediately after the transaction was 40000. Both the dollar weighted and time weighted interest rates for the account for the year are equal to 18%. Determine the end of year balance, Y.

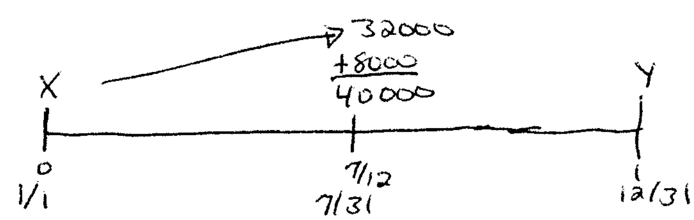
(A) 42000

(B) 43000

(C) 44000

(D) 45000

(E) 46000



$$i_{DW} = .18 \Rightarrow X(1.18) + 8000\left(1 + \tfrac{5}{12}(.18)\right) = Y \quad (1)$$

$$i_{TW} = .18 \Rightarrow 1.18 = \frac{32000}{X} \cdot \frac{Y}{40000} \quad (2)$$

$$\left.\begin{array}{ll} (1) \Rightarrow & 1.18X + 8600 = Y \\ (2) \Rightarrow & 1.18X = 0.8Y \end{array}\right\} \Rightarrow Y = 43000$$

5. George wants to buy a car in 10 years. The car George wants currently costs 20000, and he assumes inflation will increase the price of the car by 4% each year. George will make monthly deposits into an account beginning one month from today in order to have exactly enough to buy the car outright. Determine the amount George needs to deposit each month if he receives 6% compounded monthly on the deposits.

(A) 150

(B) 160

(C) 170

(D) 180

(E) 190

George needs $20000(1.04)^{10} \doteq 29604.89$ in 10 years

$\therefore \quad R\,s_{\overline{120}|.005} = 29604.89$

$\Longrightarrow R \doteq 180$

6. You are given:

(i) The annual effective yield on 1-year zero-coupon bonds is 3%.
(ii) The annual effective yield on 3-year zero-coupon bonds is 3.5%.

If a 3-year 1000 par value bond, redeemable at par, with 5% annual coupons is selling at a premium of 42.53, then determine the annual effective forward rate for year 2.

(A) 3.1%

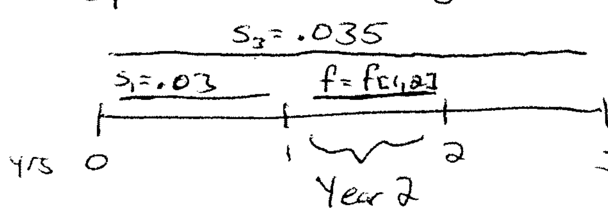
(B) 3.2%

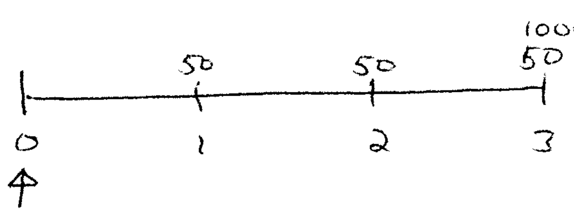
(C) 3.3%

(D) 3.4%

(E) 3.5%

$S_1 = .03 \qquad S_3 = .035$





$$P = 1000 + 42.53 = \frac{50}{1.03} + \frac{50}{(1+f)(1.03)} + \frac{1050}{(1.035)^3}$$

$\Longrightarrow f \doteq .034$

7. A 1000 face value 10-year bond with 5% annual coupons, redeemable at 1200, is selling for 1123.77. A 100 face value 10-year bond with 5% annual coupons, redeemable at par, is selling for P. Determine the price, P, consistent with a 10-year spot rate of 3%.

(A) 92.5

(B) 95.0

(C) 97.5

(D) 100.0

(E) 102.5

$$1123.77 = 50\, a_{\overline{10|}} + 1200\, v^{10}$$
$$-\ (P = 5\, a_{\overline{10|}} + 100\, v^{10}) \times 10$$
$$\therefore 1123.77 - 10P = 200\, v^{10}$$
$$v^{10} = v_{.03}^{10} = \frac{1}{(1.03)^{10}}$$
$$\therefore P \doteq 97.5$$

8. A portfolio consists of the following three bonds:

Bond A is a 5-year zero-coupon bond, redeemable at 1000.

Bond B is a 30-year bond, redeemable at C, with annual coupons. The first coupon is 50 and each subsequent coupon is 5 more than its preceding coupon. Using an annual effective interest rate of 6%, the bond costs 1500 and its Macaulay duration of 23.

Bond C is a 10 year zero-coupon bond, redeemable at 1000.

Determine the modified duration of the portfolio at a 6% annual effective interest rate.

(A) 12.8

(B) 13.7

(C) 14.7

(D) 15.6

(E) 16.6

$$P^A = 1000\, v_{.06}^5 \qquad P^B = 1500 \qquad P^C = 1000\, v_{.06}^{10}$$
$$MacD^A = 5 \qquad MacD^B = 23 \qquad MacD^C = 10$$
$$P^{Total} = P^A + P^B + P^C$$
$$\therefore MacD^{Portfolio} = \left(\frac{P^A}{P^{Total}}\right)(5) + \left(\frac{P^B}{P^{Total}}\right)(23) + \left(\frac{P^C}{P^{Total}}\right)(10)$$
$$\Longrightarrow MacD^{Portfolio} \doteq 15.61855$$
$$\therefore ModD^{Portfolio} = v_{.06} \cdot MacD^{Portfolio} \doteq 14.7$$

9. Using an annual effective interest rate of 10%, determine the Macaulay duration of a 20-year annuity immediate with level annual payments of K.

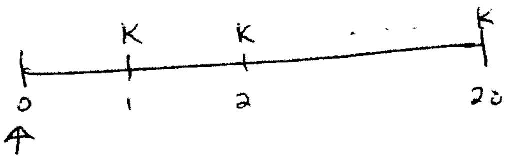
(A) 5.9

(B) 6.5

(C) 6.8

(D) 7.5

(E) 8.2



$$MacD = \frac{Kv + 2Kv^2 + \cdots + 20Kv^{20}}{Kv + Kv^2 + \cdots + Kv^{20}} \quad \left(\text{factor out } K \text{ and cancel}\right)$$

$$\therefore MacD = \frac{(Ia)_{\overline{20|}.1}}{a_{\overline{20|}.1}} \doteq 7.5$$

10. You are given the following table of interest rates:

Calendar Year of Investment	Investment Year Rates			Portfolio Rates
Y	i_1^Y	i_2^Y	i_3^Y	i^{Y+3}
2008	0.05	0.06	0.07	0.04
2009	0.06	0.06	0.06	0.06
2010	0.08	0.08	0.10	0.04

1000 is invested at the beginning of each of years 2008, 2009, and 2010. Determine the amount of interest paid for year 2011.

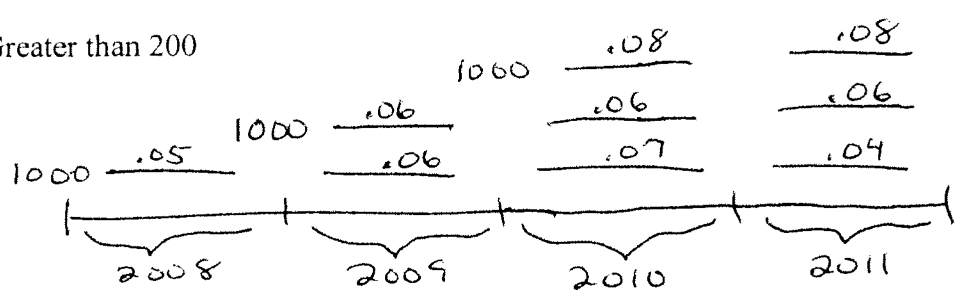
(A) Less than or equal to 185

(B) Greater than 185 but less than or equal to 190

(C) Greater than 190 but less than or equal to 195

(D) Greater than 195 but less than or equal to 200

(E) Greater than 200



∴ the amount of interest paid for year 2011 is

$$I = 1000(1.05)(1.06)(1.07)(.04) + 1000(1.06)(1.06)(.06) + 1000(1.08)(.08) \doteq 201.45$$