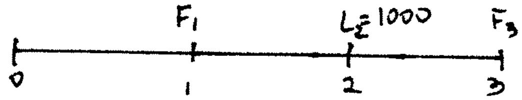
MAP 4170 — Name: ____________________

Test 4 — Date: July 28, 2014

Show sufficient work and clearly mark your answers. Each problem is worth 10 points.

1. A liability of 1,000 due at the end of 2 years is fully immunized at 4% annual effective with 1-year and 3-year zero coupon bonds. Determine the excess of the present value of assets over the present value of liabilities if the interest reate is changed to 5% annual effective.



$$\begin{cases} F_1 \, v_{0.04} + F_3 \, v_{0.04}^3 = 1000 \, v_{0.04}^2 \\ F_1 \, v_{0.04} + 3 F_3 \, v_{0.04}^3 = (2)(1000) \, v_{0.04}^2 \end{cases}$$

(A) 0.088

(B) 0.0415

$$F_3 = 500 \, v_{0.04}^{-1} = 520$$

(C) 3.515

$$F_1 = 500 \, v_{0.04} = 480.769231$$

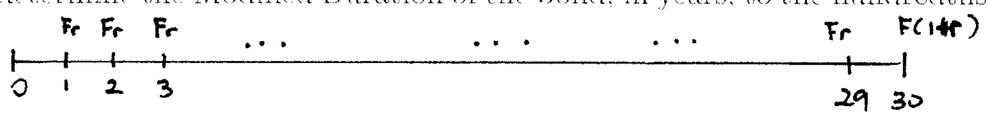
(D) 0.214

$$P^A(0.05) - P^L(0.05) = \left(F_1 v_{0.05} + F_3 v_{0.05}^3\right) - 1000 \, v_{0.05}^2$$

(E) 0.0498

$$= 0.0415$$

2. A 30-year bond redeemable at par, has 5% annual coupons. Using a 4% annual effective interest rate, determine the Modified Duration of the bond, in years, to the hundredths place.



(A) 16.46

(B) 17.19

$$\text{Mod D} = v_{0.04} \, \text{Mac D}$$

and

(C) 16.53

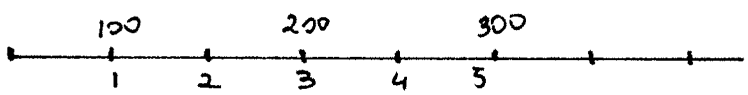
$$\text{Mac D} = \frac{(0.05)(Ia)_{\overline{30}|0.04} + 30 \, v_{0.04}^{30}}{(0.05) \, a_{\overline{30}|0.04} + v_{0.04}^{30}} = 17.1941$$

(D) 15.82

(E) 17.29

$$\text{Mod D} = (17.1941)(1.04)^{-1} = 16.5327$$

3. Payment of 100, 200, and 300 are to be made at the end of years 1, 3, and 5 respectively. Determine the convexity of this sequence of payments using an annual effective interest rate of 7%.



(A) 15.89

$$P(i) = 100(1+i)^{-1} + 200(1+i)^{-3} + 300(1+i)^{-5}$$

(B) 17.27

$$P'(i) = -100(1+i)^{-2} + (-3)(200)(1+i)^{-4} + (-5)(300)(1+i)^{-6}$$

(C) 19.83

$$P''(i) = 200(1+i)^{-3} + 2400(1+i)^{-5} + 9000(1+i)^{-7}$$

(D) 16.01

$$\text{Mod C} = \frac{P''(0.07)}{P(0.07)} = \boxed{15.89}$$

(E) 16.31

or use

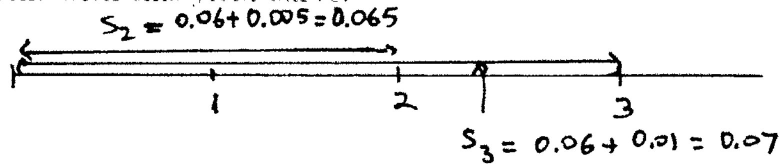
$$\text{Mod C} = v_{0.07}^2 \left[\text{Mac D} + \text{Mac C}\right] = \boxed{15.89}$$

with

$$\text{Mac D} = \frac{100 \, v_{0.07} + (200)(3) \, v_{0.07}^3 + (300)(5) \, v_{0.07}^5}{100 \, v_{0.07} + 200 \, v_{0.07}^3 + 300 \, v_{0.07}^5} = 3.512$$

$$\text{Mac C} = \frac{100 \, v_{0.07} + (200)(9) \, v_{0.07}^3 + 300(25) \, v_{0.07}^5}{100 \, v_{0.07} + 200 \, v_{0.07}^3 + 300 \, v_{0.07}^5} = 14.68$$

4. Consider the yield curve given by the equation $i_k = 0.06 + 0.005(k-1)$, where i_k is the annual effective rate of return on zero coupon bonds with maturity of k years. Determine $f_{[2,3]}$ that is consistent with this yield curve.



(A) 9.2%

(B) 8.5%

(C) 9.3%

(D) 8.0%

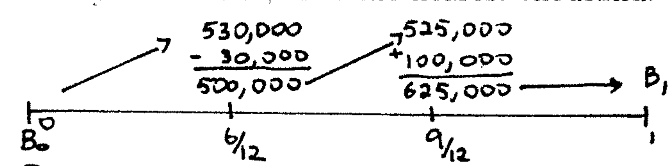
(E) 8.2%

$$(1+S_2)^2(1+f_{[2,3]}) = (1+S_3)^3$$

$$\Rightarrow 1 + f_{[2,3]} = \frac{(1.07)^3}{(1.065)^2} = 1.080071$$

$$f_{[2,3]} = \boxed{0.080071}$$

5. On January 1 a pension fund has $X. The balance on June 30 is $530,000 and on July 1 benefit payments of $30,000 are paid from the fund. A contribution of $100,000 is paid into the fund on date October 1. Immediately before the contribution, the balance in the fund is $525,000. The balance on December 31 is $Y. The time weighted rate of return on the fund is $i_{tw} = 0.3356$ and he dollar weighted rate of return on the fund is $i_{dw} = 0.353$. Find the beginning of the year balance, X to the nearest thousand.



(A) 499,000

(B) 750,000

(C) 500,000

(D) 61,000

(E) 630,000

① $1.03356 = \frac{530{,}000}{B_0} \cdot \frac{525{,}000}{500{,}000} \cdot \frac{B_1}{625{,}000} \Rightarrow B_1 = 1.5\,B_0$

② $B_0(1.353) - 30{,}000\left(1+(1/2)(0.353)\right) + 100{,}000\left(1+(1/4)0.353\right) = B_1$

$\Rightarrow 1.353\,B_0 - B_1 = -73{,}530$

$\Rightarrow B_0\,(1.353 - 1.5) = -73{,}530$

$\boxed{B_0 = 500{,}204.0816}$

6. A portfolio consists of three assets, A, B, and C. Using an annual effective interest rate of 8%, you are given;

Asset A has a price of 4000 and modified duration of 5 years.

Asset B has a price of 3000 and modified duration of $Y+1$ years.

Asset C has a price of 3000 and modified duration of Y years.

The Macaulay duration of the portfolio is 21.6 years using an annual effective interest rate of 8%. Determine the Macaulay duration, in years, of Asset B.

(A) 32.94 years (B) 28.24 years (C) 30.5 years (D) 31.18 years (E) 31.5 years

$\begin{cases} \text{Mod } D^A = 5 \\ P^A = 4{,}000 \end{cases}$

$\begin{cases} \text{Mod } D^B = Y+1 \\ P^B = 3{,}000 \end{cases}$

$\begin{cases} \text{Mod } D^C = Y \\ P^C = 3{,}000 \end{cases}$

$P^{Total} = 4{,}000 + 3{,}000 + 3{,}000 = 10{,}000$

$$\text{Mod } D = v_{.08}\,\text{Mac } D = \frac{21.6}{1.08} = 20 = \frac{5(4{,}000)}{10{,}000} + \frac{(Y+1)(3{,}000)}{10{,}000} + \frac{Y(3{,}000)}{10{,}000}$$

$\Rightarrow 20 = 2 + 0.3(Y+1) + 0.3Y$

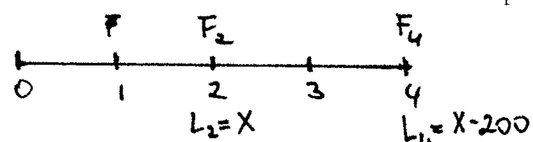
$\Rightarrow Y = \frac{59}{2}$ and $Y+1 = 30.5 = \text{Mod } D^B$

thus $\text{Mac } D^B = (1.08)(30.5) = \boxed{32.94 \text{ years}}$

7. Bond A is a 2-year zero-coupon bond with a redemption value of F_2. It can be bought to yield 10% annual effective.
Bond B is a 4-year zero coupon bond with a redemption value of F_4. It can be bought to yield 5% annual effective.
Celia has the following obligations:

$$\begin{cases} X & \text{due at the end of 2 years} \\ X - 200 & \text{due at the end of 4 years} \end{cases}$$

Celia pays a total of 15,000 to purchase enough of each type of bond in order to exactly match the obligations. Determine X to the nearest hundredths place.



(A) 7,274.10

(B) 8,766.99

(C) 6,479.97

(D) 9,195.37

(E) None of the above

$$\Rightarrow \begin{cases} F_2 = X \\ F_4 = X-200 \end{cases} \quad \text{and}$$

$$15{,}000 = X\, v_{0.1}^{2} + (X-200)\, v_{0.05}^{4}$$

$$\Rightarrow X = \frac{15{,}000 + 200\,(1.05)^{-4}}{\left((1.1)^{2} + (1.05)^{-4}\right)} = 9{,}195.3745$$

8. Paul Sr. wants to buy a vintage Limited edition Harley Davidson motorcycle in 6 years. The motorcycle currently costs 16,000, and he assumes inflation will increase the price of the Harley by 3% each year. Paul Sr. will make quaterly deposits into an account beginning three month from today in order to have exactly enough to buy the motorcycle outright. Determine the amount Paul Sr. needs to deposit each quater if he receives 8% compounded quaterly on the deposits.

$$R\, s_{\overline{24}|0.02} = 16{,}000\,(1.03)^{6}$$

(A) 264.96

(B) 628

$$\Rightarrow R = 628$$

(C) 615.68

(D) 286.15

(E) 720

$i^{(4)} = 8\% \Rightarrow i = 0.02$

9. A 1,000 face value 8-year bond with 4% annual coupons, redeemable at 1,200 is selling for 1123.77. A 100 face value 8-year bond with 4% annual coupons, redeemable at par, is selling for P. Determine the price, P, consistent with an 8-year spot rate of 3.5%.

(A) 97.2

(B) 97.5

(C) 92.5

(D) 102.5

(E) 95.0

$$\begin{cases} (1) & 1123.77 = 40\, a_{\overline{8}|} + 1200\, v_{0.035}^{8} \\ (2) & \left(P = 4\, a_{\overline{8}|} + 100\, v_{0.035}^{8}\right)(10) \end{cases}$$

$$\rightarrow \quad 10P = 1123.77 - 200\, v_{0.035}^{8}$$

$$P = \boxed{97.2}$$

10. You are given the following table of interest rates:

Calendar Year or Investment	Intesment Year Rate			Portfolio Rates
Y	i_1^Y	i_2^Y	i_3^Y	i^{Y+3}
2009	0.06	0.07	0.08	0.05
2010	0.07	0.07	0.07	X%
2011	0.08	0.08	0.10	$(X+1)$%

1000 is invested at the beginning of years 2009 and 2010. The total amount of interest paid for year 2013 is 163.23. Find the portfolio rate for 2014.

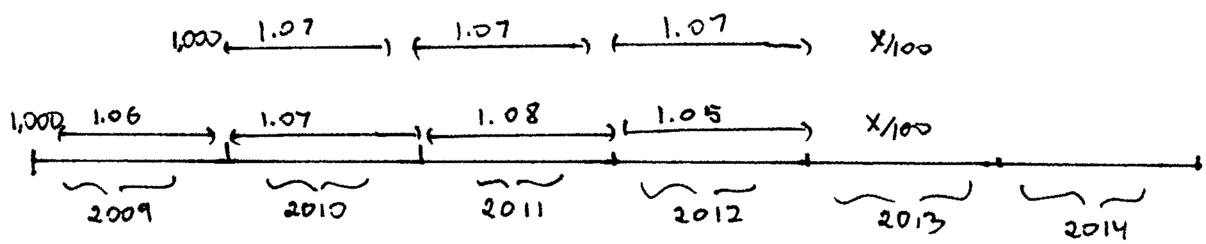
(A) 3.5

(B) 5.5

(C) 7.5

(D) 6.5

(E) 7.75



$$1{,}000\,(1.06)(1.07)(1.08)(1.05)\left(\tfrac{X}{100}\right) + 1000\,(1.07)^3\left(\tfrac{X}{100}\right) = 163.23$$

$$\tfrac{X}{100}\left[1000\,(1.06)(1.07)(1.08)(1.05) + 1000\,(1.07)^3\right] = 163.23$$

$$X = \frac{163.23}{10}\Big/\left((1.06)(1.07)(1.08)(1.05) + (1.07)^3\right)$$

$X = 6.5$ thus $X+1\% = 7.5\%$