MAP 4170
Test 4

Name:____________________________

Date: July 30, 2015

Show sufficient work and clearly mark your answers. Each problem is worth 10 points.

1. Ryan turns 21 years old today. She plans to deposit 500 at the beginning of each month starting, starting today, with the last deposit being one month before her 65th birthday. She assumes that she will earn 9% compounded monthly on her deposits, and she assume inflation over this time period to be 3% compounded monthly. Determine the accumulated value on Ryan's 65th birthday of this annuity using the real rate of return.

   (A) less than 1.26 million

   (B) equal to or more than 1.26 million, but less than 1.27 million

   (C) equal to or more than 1.27 million, but less than 1.28 million

   (D) equal to or more than 1.28 million, but less than 1.29 million

   (E) equal to or more than 1.29 million

$n = 44(12) = 528 \qquad 1+i' = \frac{1.0075}{1.0025}$

$$AV = 500\,\ddot{s}_{\overline{528}|i'} = 1{,}292{,}665.62$$

2. Using an annual effective interest rate of 5%, determine the Modified duration of a 20-year 5% annual coupon bond.

   (A) 12.12

   (B) 12.33

   (C) 12.46

   (D) 12.95

   (E) 13.09

Since $F=C$ and $r=i=.05$, the short-cut is

$$\text{Mac } D = \ddot{a}_{\overline{n}|} = \ddot{a}_{\overline{20}|.05}$$

$$\text{Mod } D = \text{Mac } D \cdot v = a_{\overline{20}|.05} \doteq 12.46$$

3. The price of a 10-year 1000 face value bond with 6% annual coupons, redeemable at par, is 1080. The price of a 10-year 100 face value bond with 4% annual coupons, redeemable at par, is 90. Determine the 10-year spot rate that is consistent with the pricing of these bonds.

(A) 6.25%

(B) 6.35%

(C) 6.45%

(D) 6.55%

(E) 6.65%

$$1080 = 60v_{s_1} + 60v_{s_2}^2 + \cdots + 1060v_{s_{10}}^{10}$$

$$+\left[90 = 4v_{s_1} + 4v_{s_2}^2 + \cdots + 104v_{s_{10}}^{10}\right](-15)$$

$$-270 = -500v_{s_{10}}^{10}$$

$$\Rightarrow s_{10} = 6.36\%$$

4. An investment account has a beginning of year balance on January 1 of 100,000. There is one withdrawal of 3000 during the year. Immediately before the withdrawal the balance in the account is 102,000. There is one deposit of 5000 during the year. The deposit is made on July 1, and the balance in the account immediately before the deposit is 103,000. Both the dollar weighted and time weighted interest rates for the account for the year are equal to 16%. Determine the month of the withdrawal.

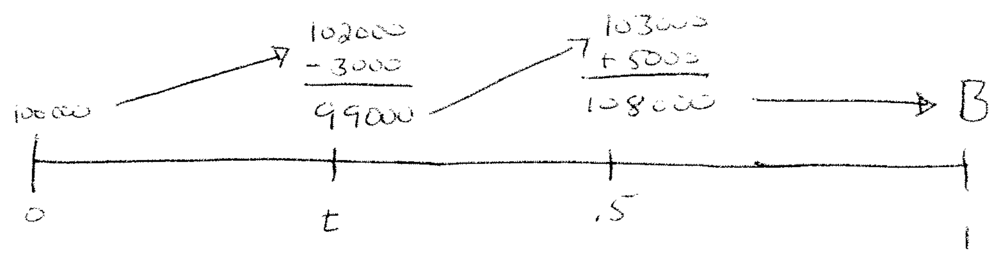
(A) January

(B) February

(C) March

(D) April

(E) May



$$i_{TW} = .16 \Rightarrow 1.16 = \frac{102000}{100000}\cdot\frac{103000}{99000}\cdot\frac{B}{108000}$$

$$\Rightarrow B \doteq 118053.68$$

$$i_{DW} = .16$$

$$\Rightarrow 100000(1.16) - 3000(1+.16(1-t)) + 5000(1+.16(1-.5)) = B$$

$$\Rightarrow t \doteq .2785$$

$\therefore$ withdrawal is made $(.2785)(12) = 3.34$ months after January 1; i.e. in April

5. A 3-year 1000 face value bond, redeemable at 1200, has 5% annual coupons. The annual effective yield rate consistent with the current term structure of interest rates is 3.13%. Given that the 1-year spot rate is 6% and the 3-year spot rate is 3%, determine the price of a 1000 face value 2-year zero-coupon bond. Assume the zero-coupon bond is redeemable at par.

(A) 880

(B) 900

(C) 920

(D) 940

(E) 960

$$P = 50 a_{\overline{3}|.0313} + 1200 v^3_{.0313} = 1235.10$$

$$\therefore 1235.10 = \frac{50}{1.06} + \frac{50}{(1+s_2)^2} + \frac{1250}{(1.03)^3}$$

$$\Rightarrow \frac{50}{(1+s_2)^2} \doteq 44$$

Price of the 2-year bond $= \frac{1000}{(1+s_2)^2} \doteq 880$

6. You are given:

(i) The annual effective yield on 1-year zero-coupon bonds is 3.5%.
(ii) The annual effective yield on 2-year zero-coupon bonds is 3.5%.

If a 3-year 1000 par value bond, redeemable at par, with 3% annual coupons is selling at a discount of 25, then determine the annual effective forward rate for year 3.

(A) 3.9%

(B) 4.1%

(C) 4.3%

(D) 4.5%

(E) 4.7%

$$C = 1000 \Rightarrow P = 975$$

$$\therefore 975 = \frac{30}{1.035} + \frac{30}{(1.035)^2} + \frac{1030}{(1.035)^2(1+f)}$$

$$\Rightarrow f \doteq .0474$$

7. Determine the Macaulay convexity of a 3-year 4% annual coupon bond, redeemable at par, using a 5% annual effective interest rate.

(A) 8.2

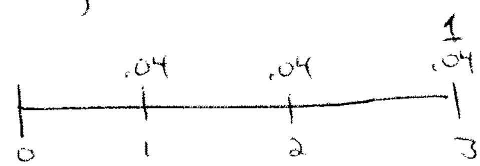
(B) 8.3

(C) 8.4

(D) 8.5

(E) 8.6

With $F = 1$,



$v = \frac{1}{1.05}$

$$\text{Mac } C = \frac{.04(1)^2 v + .04(2)^2 \cdot v^2 + 1.04(3)^2 \cdot v^3}{.04 v + .04 v^2 + 1.04 v^3} \doteq 8.5$$

8. A portfolio consists of the following three bonds:

Bond I is a 5-year zero-coupon bond, redeemable at C.

Bond II is a 30-year annual coupon bond. The first coupon is X and each subsequent coupon is 5% more than its previous one. The redemption value is Y. Using an annual effective interest rate of 3%, the price is 1000 and the Macaulay duration is 23.

Bond III is a 10-year zero-coupon bond, redeemable at D. It is priced at 1000 to yield 3% annual effective.

The modified duration of the portfolio is 15 when using a 3% annual effective interest rate. Determine C

(A) 233

(B) 297

(C) 348

(D) 402

(E) 470

$$\text{Mac } D^{\text{Portfolio}} = 15(1.03) = 15.45$$

$$P^{I} = C v^5 \qquad \text{Mac } D^{I} = 5$$

$$P^{II} = 1000 \qquad \text{Mac } D^{II} = 23$$

$$P^{III} = 1000 \qquad \text{Mac } D^{III} = 10$$

$$\Sigma = 2000 + C v^5$$

$$15.45 = \frac{C v^5}{2000 + C v^5}(5) + \frac{1000}{2000 + C v^5}(23) + \frac{1000}{2000 + C v^5}(10)$$

$$\Rightarrow C \doteq 232.96$$

9. The present value of a sequence of cash flows is 1403 when calculated using an annual effective interest rate of 4.5%, but is 1395.25 when calculated using an annual effective interest rate of i. Using an annual effective interest rate of 4.5%, the modified duration of the payments is 7 and the modified convexity of the payments is 64. Determine i.

(A) 4.52%

(B) 4.54%

(C) 4.56%

(D) 4.58%

(E) 4.60%

$P(.045) = 1403 \qquad P(i) = 1395.25$

$\text{Mod } D_{.045} = 7 \qquad \Delta i = i - .045$

$\text{Mod } C_{.045} = 64 \qquad \Delta P = 1395.25 - 1403 = -7.75$

$\Delta P \approx -P(.045) \cdot \text{Mod } D_{.045} \cdot \Delta i + \frac{1}{2} P(.045) \cdot \text{Mod } C_{.045} (\Delta i)^2$

$\Rightarrow -7.75 = -1403(7) \cdot \Delta i + \frac{1}{2}(1403)(64)(\Delta i)^2$

quadratic in Δi $\quad a = 44896,\ b = -9821,\ c = 7.75$

$\Rightarrow \Delta i = .217958$ (extraneous)

or $.000792$

$\therefore\ i = .045 + .000792 = .045792$

10. You are given the following table of interest rates:

Calendar Year of Investment	Investment Year Rates			Portfolio Rates
Y	i_1^Y	i_2^Y	i_3^Y	i^{Y+3}
2008	0.07	0.06	0.05	0.04
2009	0.06	0.06	0.06	0.06
2010	0.10	0.08	0.08	0.04

1000 is invested at the beginning of each of years 2008, 2009, and 2010. Determine the amount of interest paid for year 2013.

(A) Less than or equal to 140

(B) Greater than 140 but less than or equal to 150

(C) Greater than 150 but less than or equal to 160

(D) Greater than 160 but less than or equal to 170

(E) Greater than 170

$I = 1000(1.07)(1.06)^2(1.05)(1.04)(.04)$

$+ 1000(1.06)^4(.04)$

$+ 1000(1.1)(1.08)^2(.04) = 154.34$