MAP 4170 Name: ______________________

Test 4 Date: July 27, 2016

Show sufficient work and clearly mark your answers. Each problem is worth 10 points.

1. The price of a 5-year 1000 face value bond with 3% annual coupons, redeemable at par, is 1100. The price of a 5-year 100 face value bond with 6% annual coupons, redeemable at par is 130. Determine the 5-year spot rate that is consistent with the pricing of these bonds.

(A) 2.03%

(B) 2.13%

(C) 2.23%

(D) 2.33%

(E) 2.43%

$$1100 = 30v_{s_1} + 30v_{s_2}^2 + 30v_{s_3}^3 + 30v_{s_4}^4 + 1030v_{s_5}^5$$

$$+ \ (130 = 6v_{s_1} + 6v_{s_2}^2 + 6v_{s_3}^3 + 6v_{s_4}^4 + 106v_{s_5}^5)(-5)$$

$$450 = 500v_{s_5}^5 \Rightarrow v_{s_5}^5 = .9$$

$$\Rightarrow s_5 \doteq .02129 \approx 2.13\%$$

2. Sofia wants to buy an item in 3 years. The item currently costs 5000, and she assumes inflation will increase the price of the item by 2% each year. Sofia will make monthly deposits into an account beginning one month from today in order to have exactly enough to purchase the item outright. Determine the amount Sofia needs to deposit each month if she receives 4% compounded monthly on the deposits.

(A) 109

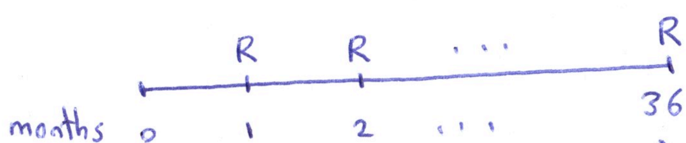
(B) 119

(C) 129

(D) 139

(E) 149

$i = \text{meir} = \frac{4}{12}\%$



$$R s_{\overline{36}|\frac{4}{12}\%} = \underbrace{5000(1.02)^3}_{\text{the price of the item after 3 years}}$$

$$\Rightarrow R \doteq 139$$

3. On January 1, 2014, an investment account is worth 100,000. On July 1, 2014, the value has increased to 108,000 and 3,000 is withdrawn. On January 1, 2016, the account is worth 115,000. Find the dollar weighted return, i_{DW}, for the year 2014, if it is equal to the time weighted return, i_{TW}, for the year 2015.

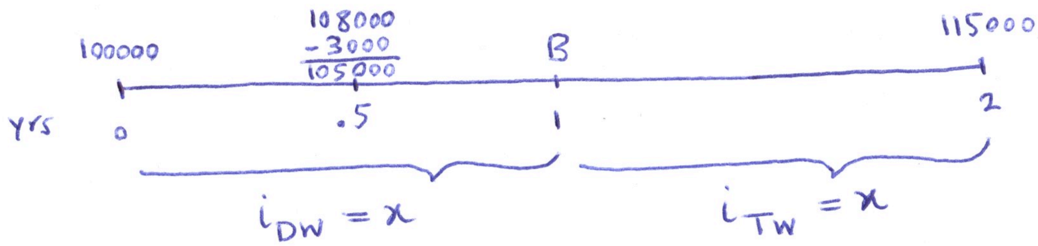
(A) 8.5%

(B) 8.6%

(C) 8.7%

(D) 8.8%

(E) 8.9%



$$100000(1+x) - 3000(1+.5x) = B \Rightarrow 97000 + 98500x = B$$

$$1+x = \frac{115000}{B} \Rightarrow B = \frac{115000}{1+x}$$

$$\Rightarrow 97000 + 98500x = \frac{115000}{1+x} \Rightarrow 98500x^2 + 195500x - 18000 = 0$$

$$x = \frac{-195500 \pm \sqrt{195500^2 - 4(98500)(-18000)}}{2(98500)} \doteq .088 = 8.8\%$$

$\ominus$ Not acceptable

4. Determine the modified duration of a perpetuity immediate with level annual payments of K, when calculated using an annual effective interest rate of 4%.

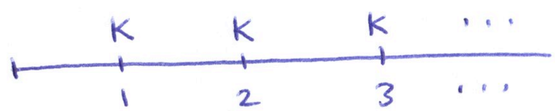
(A) 21

(B) 22

(C) 23

(D) 24

(E) 25



$$MacD = \frac{Kv + 2Kv^2 + 3Kv^3 + \cdots}{Kv + Kv^2 + Kv^3 + \cdots} = \frac{\cancel{K}(v + 2v^2 + 3v^3 + \cdots)}{\cancel{K}(v + v^2 + v^3 + \cdots)}$$

$$= \frac{(Ia)_{\overline{\infty|}}}{a_{\overline{\infty|}}} = \frac{\frac{\ddot{a}_{\overline{\infty|}}}{i}}{\frac{1}{i}} = \ddot{a}_{\overline{\infty|}}$$

$$ModD = v \cdot MacD = v \cdot \ddot{a}_{\overline{\infty|}} = a_{\overline{\infty|}} = \frac{1}{i}$$

$$i = aeir = 4\% = .04 \Rightarrow ModD = \frac{1}{.04} = 25$$

5. A portfolio consists of three bonds, A, B, and C. Using an annual effective interest rate of 8%, you are given:

   (i) Bond A has a price of 885.
   (ii) Bond B is a 100 par value 10-year zero-coupon bond, redeemable at par.
   (iii) Bond C is an n-year bond with a duration of 9 and price of 1000.

   Determine the modified duration of bond A, if the Macaulay duration of the portfolio is 7 years.

   (A) 4.14

   (B) 4.24

   (C) 4.34

   (D) 4.44

   (E) 4.54

$$P^A = 885 \qquad P^B = 100\, v_{.08}^{10} \doteq 46.319 \qquad P^C = 1000$$

$$P^{\text{Portfolio}} = P^A + P^B + P^C = 1931.319 \qquad MacD^B = 10 \qquad MacD^C = 9$$

$$7 = MacD^{\text{Portfolio}} = \frac{P^A}{P^{\text{Portfolio}}} \cdot \underbrace{MacD^A}_{\text{the only unknown in this equation}} + \frac{P^B}{P^{\text{Portfolio}}} \cdot MacD^B + \frac{P^C}{P^{\text{Portfolio}}} \cdot MacD^C$$

$$\Rightarrow MacD^A \doteq 4.583$$

$$\Rightarrow ModD^A = v_{.08} \cdot MacD^A \doteq 4.243$$

6. A bond will pay monthly coupons of 50 at the end of each month for the next 10 years and will pay the face value of 10000 at the end of the 10-year period.

   Calculate the bond's Modified duration (in years) using a 6%, compounded monthly, interest rate.

   (A) 7.51

   (B) 7.71

   (C) 7.91

   (D) 8.11

   (E) 8.31

$$\boxed{F = C} \quad (*)$$

$$\left.\begin{array}{l} i = \text{meir} = \frac{6}{12}\% = .5\% = .005 \\ r = \frac{Fr}{F} = \frac{50}{10000} = .005 \end{array}\right\} \Rightarrow \boxed{i = r} \quad (**)$$

From $(*)$ & $(**)$, we know that :

$$MacD = \ddot{a}_{\overline{120}|.5\%}$$

$$ModD = v_{.005}\, \ddot{a}_{\overline{120}|.5\%} = a_{\overline{120}|.5\%} = 90.0734 \text{ (months)}$$

$$= \frac{90.0734}{12} \text{ (years)} \doteq 7.51 \text{ (years)}$$

7. A bond is priced at 1000 to yield 4% annual effective, and the Macaulay duration using this interest rate is 10. Approximate the price of the bond if the interest rate is changed to 4.5%.

$\Delta P = -P \cdot ModD \cdot \Delta i$

(A) 944

(B) 946 $\quad 4\% \longrightarrow 4.5\% \; : \; \Delta i = .5\% = .005$

(C) 950 $\quad ModD_{4\%} = MacD_{4\%} \cdot v_{.04} = \frac{10}{1.04} = 9.615$

(D) 952

(E) 956 $\quad P(4.5\%) - P(4\%) = -P(4\%) \cdot ModD_{4\%} \cdot \Delta i$

$P(4.5\%) - 1000 = -(1000)(9.615)(.005) \Rightarrow P(4.5\%) \doteq 952$

8. You are given the following table of interest rates:

Calendar Year of Investment	Investment Year Rate			Portfolio Rates
Y	i_1^Y	i_2^Y	i_3^Y	i^{Y+3}
2010	0.07	0.07	0.06	0.05
2011	0.08	0.08	0.06	0.04
2012	0.08	0.06	0.05	0.03

1000 is invested at the beginning of years 2010 and 2012. Determine the total amount of interest paid for year 2015.

(A) 64.31 $\quad 1000(1.07)^2(1.06)(1.05)(1.04)(.03)$

(B) 67.93 $\quad + 1000(1.08)(1.06)(1.05)(.03) \doteq 75.82$

(C) 70.05

(D) 73.46

(E) 75.82

9. A liability of 10000 at the end of 5 years is to be fully immunized, at an annual effective interest rate of i, using an asset of 2000 at the end of 3 years and another asset of X at the end of 6 years. Determine X.

(A) 8156

(B) 8556

(C) 8607

(D) 9003

(E) 9175

A: 2000 at 3, X at 6

L: 10000 at 5

$$\left(10000v^5 = 2000v^3 + Xv^6\right)(-6)$$

$$+ \quad 50000v^5 = 6000v^3 + 6Xv^6$$

$$-10000v^5 = -6000v^3 \Rightarrow v^2 = .6 \Rightarrow v \doteq .7746$$

Plug this v into the first (or second) equation, and solve for X to find: $X \doteq 8607$

10. A liability of 3000 at the end of 1 year and another liability of 8000 at the end of 2 years are to be exactly matched using the following two bonds:

Bond A: a 1-year zero-coupon bond, redeemable at par, which can be bought to yield 3% annual effective.

Bond B: a 2-year 6% annual coupon bond, redeemable at par, which can be bought to yield 5% annual effective.

Determine the cost to exactly match the liabilities.

(A) 9584

(B) 9820

(C) 9953

(D) 10160

(E) 10875

A: $\begin{cases} .06F_2 \\ F_1 \end{cases}$ at 1, $\begin{cases} F_2 \\ .06F_2 \end{cases}$ at 2

L: 3000 at 1, 8000 at 2

$$\begin{cases} F_2 + .06F_2 = 8000 \Rightarrow \boxed{F_2 = 7547.169} \\ F_1 + .06F_2 = 3000 \Rightarrow \boxed{F_1 = 2547.169} \end{cases}$$

$$P = \underbrace{\left(F_1 v_{.03}\right)}_{\text{Price of bond A}} + \underbrace{\left(.06F_2\, a_{\overline{2}|5\%} + F_2 v_{.05}^2\right)}_{\text{Price of bond B}} = 10160.481 \doteq 10160$$