

Each problem is worth 10 points. Show sufficient work for full credit.

1. A special 3-year bond with annual coupons has an initial coupon of 100. Each subsequent coupon is 10 greater than its preceding coupon. The redemption value of the bond is 1000. Determine the Macaulay convexity of the bond using an annual effective interest rate of 3%.

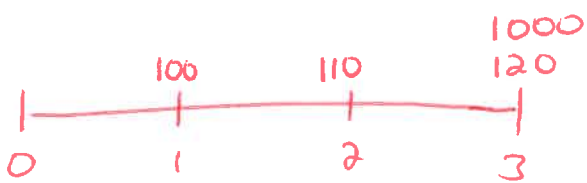
A) 7.5

B) 7.7

☒ C) 7.9

D) 8.1

E) 8.3



$$MacC = w_1 \cdot 1^2 + w_2 \cdot 2^2 + w_3 \cdot 3^2$$

$$P = P(.03) = 100v + 110v^2 + 1120v^3 = 1225.731...$$

$$w_1 = \frac{100v}{P} \quad w_2 = \frac{110v^2}{P} \quad w_3 = \frac{1120v^3}{P}$$

$$\therefore MacC = 7.943...$$

2. The 1-year, 2-year, and 3-year spot rates are 3%, 3.5%, and 4% respectively. Determine the implied yield rate on 3-year 5% annual coupon bonds.

A) 3.48%

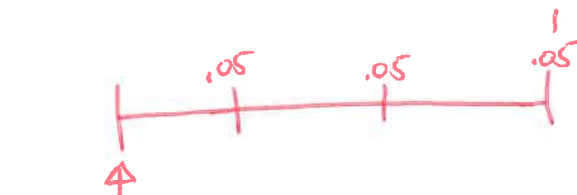
B) 3.60%

C) 3.73%

D) 3.85%

☒ E) 3.97%

Per dollar of face value



$$P = \frac{.05}{1.03} + \frac{.05}{1.035^2} + \frac{1.05}{1.04^3} = .05a_{\overline{3}|i} + v_i^3$$

$$\Rightarrow i = .03967...$$

3. For a given term structure of interest rates,

the 2-year spot rate is $s_2 = 8.00\%$

the 1-year forward rate from time 1 to time 2 is $f_{[1,2]} = 9.01\%$

the 2-year forward rate from time 1 to time 3 is $f_{[1,3]} = 9.64\%$

Determine the 1-year forward rate from time 2 to time 3, $f_{[2,3]}$, that is consistent with these rates.

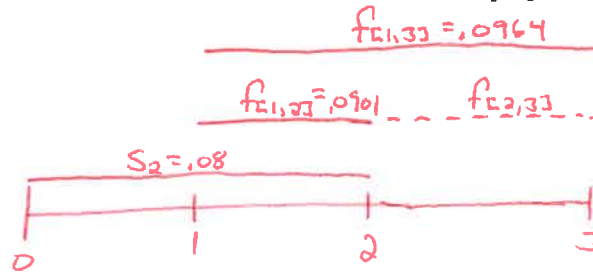
A) 9.5%

B) 9.7%

C) 9.9%

D) 10.1%

☒ E) 10.3%



$$(1.0964)^2 = (1.0901)(1 + f_{[2,3]})$$

$$\Rightarrow f_{[2,3]} = .10273...$$

4. A bond sells for \$977.69 to yield 4% annual effective. The bond's Macaulay duration at 4% annual effective is 14.75. If the annual effective yield is changed to i , the price of the bond is 839.03 when using the modified duration to approximate the new price. Determine i .

A) 3.0%

B) 3.5%

C) 4.5%

☒ D) 5.0%

E) 5.5%

$$\Delta P = -P \cdot \text{Mod } D \cdot \Delta i$$

$$\Delta P = 839.03 - 977.69 = -138.66$$

$$P = 977.69$$

$$\text{Mod } D = \text{Mac } D \cdot v = \frac{14.75}{1.04}$$

$$\therefore -138.66 = -977.69 \cdot \frac{14.75}{1.04} \cdot \Delta i$$

$$\Rightarrow \Delta i = .00999...$$

$$\Rightarrow i = .04 + \Delta i = .04999...$$

5. The balance in an investment account on January 1 is X . On July 1, 100,000 is withdrawn from the account and 150,000 is deposited into the account. The balance in the account immediately before the transactions is 450,000. There are no other transactions within the account during the year, and the balance on December 31 is Y .

The dollar weighted return for the year is 9.5% and the time weighted return for the year is 8.0%. Determine Y .

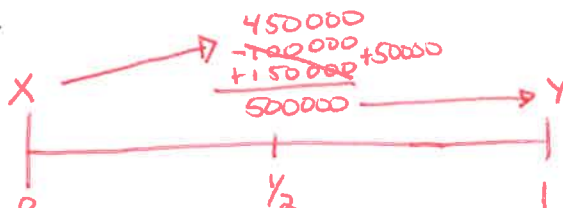
(A) 600,000

(B) 610,000

(C) 620,000

(D) 630,000

(E) 640,000



$$i_{DW} = .095 \Rightarrow X(1.095) + 50000(1 + \frac{1}{2}(.095)) = Y$$

$$i_{TW} = .08 \Rightarrow 1.08 = \frac{450000}{X} \cdot \frac{Y}{500000} \Rightarrow Y = 1.2X \Rightarrow X = \frac{Y}{1.2}$$

$$\therefore \frac{Y}{1.2}(1.095) + 50000(1.0475) = Y$$

$$\Rightarrow Y = 598,571.428 \dots$$

6. A liability of 10,000 at the end of 1 year and another liability of 10,000 at the end of 2 years are to be exactly matched using a 1-year 1000-par value 4% annual coupon bond, redeemable at par, and a 2-year 1000-par value 8% annual coupon bond, redeemable at par. Both bonds can be bought to yield 6% annual effective. Determine the cost to exactly match the liabilities.

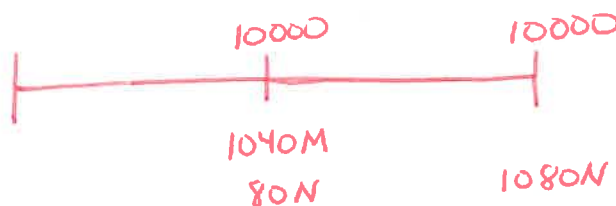
A) 18,115

(B) 18,335

C) 18,555

D) 18,775

E) 18,995



$$1080N = 10000 \Rightarrow N = 9.259 \dots = \# \text{ of 2-year bonds needed}$$

$$80N + 1040M = 10000 \Rightarrow M = 8.903 \dots = \# \text{ of 1-year bonds needed}$$

$$P_1 = \text{price of 1-year bond} = \frac{1040}{1.06} = 981.132 \dots$$

$$P_2 = \text{2-year} = 809.51.06 + 1000v_{.06}^2 = 1036.667 \dots$$

$$\therefore \text{Cost} = M \cdot P_1 + N \cdot P_2 = 18333.926 \dots$$

Remark: Since $i = .06$ for both bonds, then
 $\text{Cost} = 10000v_{.06} + 10000v_{.06}^2$

7. A company has liabilities of 402.11 due at the end of each of the next three years. The company will invest 1000 today to fund these payouts. The only investments available are one-year and three-year zero-coupon bonds, and the yield curve is flat at a 10% annual effective rate. The company wishes to match the duration of its assets to the duration of its liabilities.

Determine how much the company should invest in the one-year bond.

A) 366

B) 484

C) 500

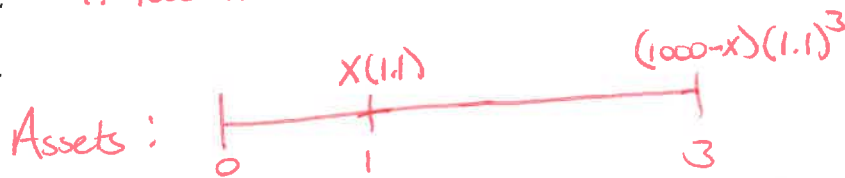
☒ D) 532

E) 634

$$MacD_{.10}^L = \frac{402.11v + 2 \cdot 402.11v^2 + 3 \cdot 402.11v^3}{402.11a_{\overline{3}|.10}} = 1.9365...$$

Let X = amount invested in the 1-year bond

$\therefore 1000 - X$ = 3-year bond



$$MacD^A = \frac{X(1.1)v + (1000-X)(1.1)^3 \cdot v^3}{X(1.1)v + (1000-X)(1.1)^3 \cdot v^3} = \frac{3000 - 2X}{1000}$$

$$\therefore \frac{3000 - 2X}{1000} = 1.9365... \Rightarrow X = 531.722...$$

8. You are given:

- (i) The 1-year spot rate is 6%.
- (ii) The 3-year spot rate is 8%.
- (iii) The price of a 3-year 1000 face-value 6% annual coupon bond, redeemable at par, is 950.97.

Determine the 1-year forward rate from time 1 to time 2 that is consistent with the above information.

A) 6.0%

B) 6.5%

☒ C) 7.0%

D) 7.5%

E) 8.0%

$$P = 950.97 = \frac{60}{1.06} + \frac{60}{(1+f_{1,2})(1.06)} + \frac{1060}{1.08^3}$$

$$\Rightarrow f_{1,2} = .06993...$$

9. A 1000 face value n -year bond, redeemable at par, with 6% annual coupons has a Macaulay duration of 13.55 when calculated using a 6% annual effective interest rate. Determine the Macaulay duration of the bond using a 5% annual effective interest rate.

A) 13.76

B) 14.02

☒ C) 14.27

D) 14.45

E) 14.76

$$MacD_{.06} \frac{F=c}{r=c} \ddot{a}_{\overline{n}|i} \Rightarrow 13.55 = \ddot{a}_{\overline{n}|.06}$$

$$\Rightarrow n = 25$$

$$\therefore MacD_{.05} = \frac{60(Ia)_{\overline{25}|.05} + 25 \cdot 1000 v_{.05}^{25}}{60 a_{\overline{25}|.05} + 1000 v_{.05}^{25}} = 14.270...$$

10. A 3-year 1000 face value bond with 5% annual coupons, redeemable at par, costs \$1,028.85. A 3-year 1000 face value bond with 3% annual coupons, redeemable at par, costs \$972.91. The 1-year spot rate is 3%. Determine the 2-year spot rate.

(A) 3.1%

☒ (B) 3.3%

(C) 3.5%

(D) 3.7%

(E) 3.9%

$$\begin{aligned} (1028.85 &= 50 v_1 + 50 v_2^2 + 1050 v_3^3) (3) \\ - (972.91 &= 30 v_1 + 30 v_2^2 + 1030 v_3^3) (5) \\ \hline -1778 &= -2000 v_3^3 \Rightarrow v_3^3 = .889 \end{aligned}$$

$$v_1 = \frac{1}{1+s_1} = \frac{1}{1.03}$$

$$\therefore 1028.85 = \frac{50}{1.03} + 50 v_2^2 + 1050 (.889)$$

$$\Rightarrow v_2^2 = .9371... \Rightarrow \frac{1}{(1+s_2)^2} = .9371...$$

$$\Rightarrow s_2 = .0330...$$