\[ aeir = .125 \]
\[ \nu = \frac{1}{1.125} \]

\[ I_n = 153.86 = R(1 - \nu) \Rightarrow R = 1384.74 \]

\[ \sum_{k=1}^{n-1} P_k = 6009.12 \]

\[ = R\nu^1 + R\nu^{2-1} + \ldots + R\nu^{n-1} \]

\[ = R\nu \left( \nu + \nu^2 + \ldots + \nu^{n-1} \right) \]

\[ = \frac{1384.74}{1.125} \cdot \frac{\nu}{n-1.125} = 6009.12 \]

\[ \Rightarrow n-1 = 8 \Rightarrow n = 9 \]

\[ \therefore L = 1384.74 \cdot \frac{\nu}{91.125} = 7240.09 \]

\[ \Rightarrow I_1 = .125 \cdot L = 905.01 \]

\[ \Rightarrow P_1 = Y = 1384.74 - I_1 = 479.73 \]
Original: \( \text{APR} = 0.03 \)

\[ \Rightarrow \text{EIR} = (1.03)^2 - 1 = 0.0609 \]

\[ 12000 = 750 \times a_{11.0609} \quad \Rightarrow \quad n \approx 62 \]

(That's close enough, just use 62)

Refinance:

\[ \text{Balance} = 750 \times a_{541.0609} \times (1.03) \]

\[ \quad \text{time 8 value} \]

New timeline: \( \text{meir} = \frac{0.09}{12} = 0.0075 \)

\[ \text{Balance} = 750 \times a_{541.0609} \times (1.03) = R \times a_{301.0075} \times 0.0075 \]

\[ \Rightarrow R = 461.13 \]
26/May 2003

Course 2 Exam

\[ \frac{19800}{36} = 550 = P_k \quad k = 1, 2, \ldots, 36 \]

\[ I_1 = 19800 \times 0.01 = 198 \]

\[ I_2 = 19250 \times 0.01 = 192.5 \]

\[ I_3 = 18700 \times 0.01 = 187 \]

\[ B_1 = 19800 - 550 = 19250 \]

\[ B_2 = 19250 - 550 = 18700 \]

\[ P = \text{Price to Bank} \ Y \]

\[ \text{yield} = 0.07 \text{ semi} \]

\[ \text{merr} = (1.07)^{0.5} - 1 = \dot{i} \]

\[ P = 550 \times a_{0.07} + 5.5 \times (1 \times 0.07) \times a_{0.07} \]

\[ P = 10857.28 \]
Presently:

\[
\begin{array}{cccccccc}
\text{0} & \text{1} & \text{2} & \ldots & \text{8} & \text{9} & \ldots & \text{20} \\
33.75 & 33.75 & \ldots & 33.75 & 33.75 & \ldots & 33.75 & 1000
\end{array}
\]

Proposal:

\[
\begin{align*}
P &= \left[ 33.75 \frac{a_{10|i}}{i} + (1000 + 33.75 S_{8|\frac{1}{2}(1+i)^{12}}) \cdot 2^{12} \right] \cdot 2^8 \\
&= 33.75 \frac{a_{10|i}}{i} \cdot 2^8 + 1000 \cdot 2^8 + 33.75 S_{8|\frac{1}{2}(1+i)^{12}} \\
&= 33.75 \frac{a_{20|i}}{i} + 1000 \cdot 2^8 = 954.63
\end{align*}
\]

= same as the price before the proposal (makes sense, since the missing coupons are replaced, with interest at the same rate as the yield on the bond)
\[
B_5 = 40 \cdot a_{\frac{5}{0.0375}} + 1000 \cdot 0.0375^5 = 1011.21
\]

\[
I_6 = i \cdot B_5 = 0.0375 \cdot (1011.21) = 37.92
\]

\[
P_6 = F_r - I_6 = 40 - 37.92 = 2.08
\]
\[
\sum_{K=1}^{10} I_K = 10R - L
\]

\[
L = 10(1000) - L
\]

\[
\implies L = 5000
\]

\[
5000 = 1000 a_{10.0.0.1}
\]

\[
\implies i = 15.1\%
\]

\[
I_1 = i \cdot L = (15.1\%)(5000) = 755
\]
\[
\begin{align*}
\text{years} & \quad 0 \quad 1 \quad 2 \quad \cdots \quad 20 \\
75 & \quad 75(1.03) \quad \cdots \quad 75(1.03)^{19} \\
\text{aeir} & = 1050 \\
\text{P}_{\text{VEP}} & = 75 v + 75(1.03) v^2 + \cdots (20 \text{ terms}) + 1050 v^{20} \\
& = 1050 v^{20} + \frac{75}{1.0825} \left( 1 + \frac{1.03}{1.0825} + \cdots (20 \text{ terms}) \right) \\
& = 1050 v^{20} + \frac{75}{1.0825} \cdot \frac{v^{20}(1.0825 - 1)}{1.03 - 1} \\
& = 215.10 + 900.02 = 1115.12
\end{align*}
\]
$150000 = 5483.36 a_{\overline{40.02}} \checkmark \text{ (given)}$

$B_{12} = 116692.92 \quad \text{(I changed N to 12 and)}$

$\text{computed FV (i.e. Retro) }$

$116692.92 = 5134.62 a_{\overline{28.015}} \checkmark \text{ (given)}$

$B_{20} \overset{\text{Pro}}{=} 5134.62 a_{\overline{20.015}} = 88154.44$

\[ g_{\text{est}} = \frac{.07}{4} = .0175 \]

\[ X \quad 4265.73 \quad 4265.73 \quad \cdots \quad 4265.73 \]

\[ B_{20} = 88154.44 \]

\[ 88154.44 = X + 4265.73 a_{\overline{20.0175}} \]

\[ \Rightarrow X = 16691.17 \]
\[ P_2 = 977.19 \quad P_4 = 1046.79 \]

\[ P_2 (1 + i)^2 = P_4 \Rightarrow i = 0.35 = \text{seir (yield rate)} \]

\[ \text{Premium} = \sum_{k=1}^{30} P_k = P_1 \cdot S_{301.035} \]

\[ \sum = P_1 + P_1 (1 + i) + \ldots + P_1 (1 + i)^{39} = P_1 [1 + (1 + i) + \ldots + (1 + i)^{39}] \]

\[ P_1 = \frac{P_2}{1 + i} = \frac{977.19}{1.035} \]

\[ \therefore \text{Premium} = \frac{977.19}{1.035} \cdot S_{301.035} = 48739.29 \]
\[ B_{12} = 8000 (1.08)^{12} - 800 S_{\overline{12}|08} = 4963.66 \]

The SF deposits need to accumulate to \( B_{12} \)

\[ 4963.66 = X S_{\overline{12}|04} \]

\[ \Rightarrow X = 330.34 \]
Using Lump Sum Method:

\[ \text{Amount of Interest Paid} = X(1.06)^{10} - X \]
\[ = X \left[ (1.06)^{10} - 1 \right] = A \]

Using Amortization Method:

\[ X = R \cdot A \text{ at } i = 0.06 \]

\[ \text{Amount of Interest Paid} = 10R - X \]
\[ = 10 \left( \frac{X}{a_{10i}} \right) - X = B \]

\[ \therefore \quad A - B = 356.54 \]
\[ \left[ X (1.06)^{10} - X \right] - \left[ \frac{10X}{a_{10i}} - X \right] = 356.54 \]
\[ \therefore \quad X \left[ (1.06)^{10} - \frac{10}{a_{10i,06}} \right] = 356.54 \]
\[ \Rightarrow \quad X = 825 \]
i) \[ 20000 = X \cdot a_{\frac{50}{0.05}} \]

\[ \Rightarrow X = 1815.13 \]

ii) \[ R^I = 20000 \cdot (0.08) = 1600 \]

\[ \Rightarrow R^F = 1815.13 - 1600 = 215.13 \]

\[ 215.13 \cdot s_{\frac{50}{j}} = 20000 \]

\[ \Rightarrow j = 14.18\% \]
S: \[ 5000(1.06)^{10} = 8954.24 \]
Seth pays \[ 8954.24 - 5000 = 3954.24 \] in interest

J: Janice pays \[ 5000(0.06) = 300 \] every 6 months for 5 years, totaling 3000 in interest

L: \[ 5000 = R A_{101.06} \] (Amortization Method)

\[ \Rightarrow R = 679.34 \]

Lori Pays \[ 10R - L = 6793.4 - 5000 = 1793.40 \] in interest

Total amount of interest on all three loans is

\[ 3954.24 + 3000 + 1793.40 = 8747.64 \]
\[ P = 30 \frac{a_{30|0.03}}{a_{30|0.03}} + 1000 \frac{1}{0.03} \]

\[ \therefore P = 1000 \text{ (price Bill paid)} \]

Bill's yield is 7% per annum.

\[ P = 1000 \]

\[ j = \text{semi-annual interest rate} \]

\[ A = 30 \times \frac{1}{0.07} + 1000 = 1000 (1.07)^{10} \]

\[ \Rightarrow j = 4.76\% \]

\[ i = \text{effective annual rate} \Rightarrow i = (1+j)^2 - 1 \approx 9.75\% \]
(i) \( 2000 = R A_{10.08} \Rightarrow R = 299.00 \)

Sum of payments under option (i) is 2990

(ii) \( P_k = 200 \quad k = 1, 2, \ldots, 10 \)

\[ I_1 = 2000 \cdot i \quad B_1 = 2000 - 200 = 1800 \]
\[ I_2 = 1800 \cdot i \quad B_2 = 1800 - 200 = 1600 \]
\[ I_3 = 1600 \cdot i \]

\[ \vdots \]

\[ I_{10} = 200 \cdot i \]

Sum of payments under option (ii) is

\[ 200(10) + (200i + 400i + \ldots + 1800i + 2000i) \]

\[ = 2000 + 200i(1+2+\ldots+10) \]

\[ = 2000 + 200i(55) \]

\[ = 2000 + 11000i \]

\[ \therefore 2000 + 11000i = 2990 \Rightarrow i = .09 \]
$9/\text{Nov. 2001}$

Course 2 Exam

$\text{meir} = \frac{.09}{12} = .0075$

\[B_{40} \frac{P_{40}}{V_{EP}} \left[ 1000(.98)^{40} + 1000(.98)^{41} + \ldots \right. \quad (20 \text{ terms})

= \frac{1000(.98)^{40}}{1.0075} \left( 1 + \frac{.98}{1.0075} + \ldots \right. \quad (20 \text{ terms})

= \frac{1000(.98)^{40}}{1.0075} \cdot \frac{A}{20 \left( \frac{1.0075}{.98} - 1 \right)}$

$= 6889.11$
Bond X:

\[ P = 1000 \frac{r}{j} a_{\frac{n}{j}} + C \cdot d^n \]

\[ = 1000 \frac{r}{j} \left( \frac{1 - d^n}{i} \right) + 381.50 \]

\[ = 1000 \left( \frac{r}{i} \right) (1 - d^n) + 381.50 \]

\[ = 1000 (1.03125)(1 - d^n) + 381.50 \]

\[ = 1031.25 - 1031.25 d^n + 381.50 \]

\[ = 1412.75 - 1031.25 d^n \]

Bond Y: (This is a zero-coupon bond, and so the price is equal to the PV of the redemption value.)

\[ P = 647.80 = C \cdot d^n \]

\[ = C \cdot d^n \quad \text{where } d^n = \text{above} \]

\[ : \quad 647.80 = C \cdot d^n \]

From information about Bond X, \( C \cdot d^n = 381.5 \)

\[ \therefore 381.5 = C \cdot d^n \cdot d^n = 647.8 \cdot d^n \Rightarrow d^n = \frac{381.5}{647.8} \]

\[ \therefore \text{for Bond X,} \]

\[ P = 1412.75 - 1031.25 \left( \frac{381.5}{647.8} \right)^2 \]

\[ = 1055.09 \]
1000 = P \ a_{10\,\underline{10}} \Rightarrow P = 162.75

Using the SF, \[ R^1 = 1000 \times 1.1 = 100 \]

\[ \Rightarrow R^{SF} = 62.75 \]

The SF balance at the end of year 10 is

\[ B_{10}^{SF} = R^{SF} \cdot S_{10\,\underline{14}} = 62.75 \times 1213.42 = 1213.42 \]

After repaying the loan of 1000, the balance is 213.42.
Since each of the first 10 payments is interest only, then \( B_{10} = B_9 = \ldots = B_1 = B_0 = 1000 \).

For the next 10 payments, we have:

\[
I_{11} = r \cdot B_{10}, \quad R_{11} = 1.5 \cdot r \cdot B_{10} = 1.5B_{10} \quad B_{11} = B_{10}(1+r) - R_{11} = 1.1B_{10} - 1.5B_{10} = 0.95B_{10}
\]

\[
I_{12} = r \cdot B_{11}, \quad R_{12} = 1.5 \cdot r \cdot B_{11} = 1.5B_{11} \quad B_{12} = B_{11}(1+r) - 1.5B_{11} = 0.95B_{11}
\]

Continue: \( B_{20} = (0.95)^{10}, B_{10} = 1000(0.95)^{10} \).

For the last 10 payments, we have:

\[
B_{30} = 1000(0.95)^{10} = X \cdot A_{10,10}
\]

\[
\Rightarrow X = 97.44
\]
May 2003
Course 2 Exam

\[ I_n = i \cdot B_e \]

\[ = 0.06 \left[ 800 \cdot a_{41.06} + 10000 \cdot 2^{0.06} \right] = 641.58 \]
2/ May 2005 (Course FM Exam)

\[ R^I = 10000 \times (0.09) = 900 \]

\[ R^{SF} \cdot S_{10\%, 0.8} = 10000 \Rightarrow R^{SF} = 690.29 \]

\[ R = R^I + R^{SF} = 1590.29 \]

Total paid by Lori over the 10-year period is

\[ X = 10R = 15902.90 \]
Zero-Coupon Bond: \[ 624.60 = 1000 \times i^{12} \Rightarrow i = 0.04 \times \text{eir} \]

Coupon Bond: \[ r = 0.03 \]

semi-annual periods

\[ j = \text{seir} = (1.04)^{\frac{1}{2}} - 1 \]

\[ X = 30 \times \frac{a_{\overline{30}\,|\,j}}{j} + 1000 \times \frac{1}{j} = 1167.04 \]
8/May/2005
Course FM Exam

\[ B_{10} \overset{\text{Pro}}{=} 300A \frac{15^{1.08}}{15^{1.08}} = 2567.84 \]

After making the extra payment, the new balance is 1567.84.

\[ \therefore 1567.84 = R A_{10^{1.08}} = R = 233.65 \]
Since the investor buys the bond at the highest price to guarantee her desired yield, then she bought at $P = 897$.

The bond is called after 20 years. We have

\[897 = 80 a_{20|c} + 1050 \frac{1}{v}^{20}\]

\[\text{TVM} \implies i = 9.24\%\]
\[ \beta_3 = 500(1.04)^3 - 20S_{31.04} = 500 \]

\[ I_4 = i \cdot \beta_3 = \cdot04(500) = 20 \]

\[ : P_4 = 0 \]

(This loan lasts forever, analogous to a perpetuity)
$118.20 = 4 A_{20.03} + C 20 \Rightarrow C = 106$
Let $X = \text{amount borrowed}$

For the bond: $r = 0.04$ and $i = 0.03$

\[ X = 40 A_{20|0.03} + 1000 \left( \frac{20}{0.03} \right)^{20} = 1148.77 \]

At the end of 10 years, the investor repays

\[ X (1.05)^{10} = 1871.23 \]

The investor has, at the end of 10 years,

\[ 40 S_{20|0.02} + 1000 = 1971.89 \]

\[ \therefore \text{net gain} = 1971.89 - 1871.23 = 100.66 \]
P = 925 = amount Dan paid at t=0

Invests coupons at 0.035 semi-annually.

\[ AV = 45 \times \frac{S_{50,0.035}}{1000} + 1000 = 2272.59 \]

His yield over 10-year period is determined by

\[ AV = P \left( 1 + \frac{i^{(2)}}{2} \right)^{20} \]

\[ 2272.59 = 925 \left( 1 + \frac{i^{(2)}}{2} \right)^{20} \]

\[ \Rightarrow i^{(2)} = 9.29\% \]
Nov. 2005)  

\[ I_8 = 789 \quad P_8 = 211 \]

\[ \therefore R = 1000 \text{ (level)} \]

\[ P_{18} = P_8 (1+i)^9 = 211 (1.07)^9 \]

\[ \therefore I_{18} = R - P_{18} = 585 \]
F = 1000 \quad r = .045^3 \quad F_r = 45
\quad i = .05

45 \quad 45 \quad \ldots \quad 45

0 \quad 1 \quad 2 \quad \ldots \quad n

\text{semi-annual periods}

\begin{align*}
P &= 918 = 45a_{\overline{n}|.05} + 1100 2^{\frac{n}{2}} \quad \text{TVM} \quad n = 49.35
\end{align*}

49.35 semiannual periods corresponds to
24.675 years

Best Answer Choice: (B) 25
\( r = .03 \)

\[ P = 850 = \sum_{t=0}^{120} 30 \cdot \frac{a_{100}}{1.001^t} + 1000 \cdot 2^{120} \]

\[ i = 9e^{0.01} \approx .0354 \]

\[ i^{(4)} = 4i = 14.2\% \]
\[ I_t = i \cdot B_{t-1} = i \cdot \frac{A_{n-t}}{i} = 1 - \frac{n-t+1}{i} = 1 - \frac{n-t}{i} \]

\[ P_{t+1} = R - I_{t+1} = 1 - I_{t+1} \]
\[ I_{t+1} = i \cdot B_t = i \cdot A_{n-t} = i \frac{1 - \frac{n-t}{i}}{i} = 1 - \frac{n-t}{i} \]
\[ \therefore P_{t+1} = 1 - (1 - \frac{n-t}{i}) = 2 - n-t \]

\[ I_t + P_{t+1} = 1 - \frac{n-t+1}{i} + 2 - n-t \]

\[ \text{rewrite} \quad 1 + \frac{2-n}{i} - \frac{n-t+1}{i} \quad \frac{n-t+1}{i} = \frac{n-t}{i} \]

\[ \text{factor} \quad 1 + \frac{n-t}{i} (1 - \frac{n}{i}) \quad 1 - \frac{n}{i} = d \]

\[ = 1 + \frac{n-t}{i} d \]
This is an easy question once we understand what the company is doing. Think of r = 0.05 and F = 822703, and so we get

\[ AV = 41135.15 \times S_{41.05} + 822703 = 1000000 \]

exactly what they need!

The company anticipates reinvestment rates to be 5%. Then at time 4, they have

The question asks, what happens if reinvestment rates are 4.5% or 5.5% instead. At 4.5%, they will have

\[ AV = 41135.15 \times S_{41.045} + 822703 = 998,687 \]

but they need 1000000. They will have a loss of

\[ 1000000 - 998,687 = 1313 \]

From here we get answer choice D.

You can show that at 5.5% reinvestment rates, the company will have a gain of 1323.
\[ .08 = ae^{ir} \]

\[ B_3 = 559.12 \]

\[ R = 559.12 \left( 1.08 \right) \]

\[ X = R A_{47.08} = 2000 \]

\[ I_1 = .08 X = 160 \]

\[ P_1 = R - I_1 = 443.85 \]
Set up a table:

<table>
<thead>
<tr>
<th>Cell Time ((n))</th>
<th>X (to yield the guaranteed yield of 0.03)</th>
</tr>
</thead>
<tbody>
<tr>
<td>30</td>
<td>1722.25 = X (0.04 A_{30}^{0.03} + 30^{0.03}) (\Rightarrow) X = 1440</td>
</tr>
<tr>
<td>31</td>
<td>1722.25 = X (0.04 A_{31}^{0.03} + 31^{0.03}) (\Rightarrow) X = 1435</td>
</tr>
<tr>
<td>32</td>
<td>Continue X = 1430 (\downarrow) X's are decreasing (\Rightarrow) Answer will be an extreme value</td>
</tr>
<tr>
<td>39</td>
<td>X = 1402</td>
</tr>
<tr>
<td>40</td>
<td>X = 1399</td>
</tr>
</tbody>
</table>

Think: If \(X = 1399\) but the bond is called at time \(n = 30\), then we would have

\[
1722.25 = 0.04(1399) A_{30}^{0.03} + 1399 \times 30^{0.03} \Rightarrow i = 2.84\%
\]

but then we would not have the guaranteed yield of 3%. So \(X\) cannot equal 1399.

On the other hand, if \(X = 1440\) but the bond is called at time \(n = 40\), then we would have

\[
1722.25 = 0.04(1440) A_{40}^{0.03} + 1440 \times 40^{0.03} \Rightarrow i = 3.13\%
\]

above the guarantee of 3%, and so OK.

You may check that if \(X = 1440\), then no matter when the bond is called, the yield is greater than or equal to 3%. Answer \(X = 1440\).
\[
\begin{array}{c|c}
\lambda & P(\lambda) \\
30 & 40a_{30\text{.}03} + 1200\delta_{03}^{30} = 1278.40 \\
\vdots & \vdots \\
39 & 40a_{39\text{.}03} + 1200\delta_{03}^{39} = 1291.23 \\
40 & 40a_{40\text{.}03} + 1100\delta_{03}^{40} = 1261.80
\end{array}
\]

The maximum price to guarantee a yield of at least 3% semiannual effective is 1262.
Coupon amount = 0.03X

\[ i = 0.03 = \text{secr} \]

\[
\begin{array}{c|c}
\hline
n & P(0.03) = 1021.50 \\
\hline
10 & 1021.50 = 0.03 \times a_{10|0.03} + X \times 2^{10}_{0.03} = X \times (0.02 \times a_{10|0.03} + 2^{10}_{0.03}) \Rightarrow X = 1116.76 \\
\vdots & \vdots \\
20 & 1021.50 = 0.03 \times a_{20|0.03} + X \times 2^{20}_{0.03} \quad \Rightarrow \quad X = 1200.03 \\
\hline
\end{array}
\]

Think! If \( X = 1116.76 \) and the bond is held to maturity \( (n=20) \) then what is the yield? 

\[ 1021.50 = 0.03(1116.76) a_{20}^{0.03} + 1116.76 2^{20} \]

\[ \Rightarrow i_{TVM} \approx 2.55\% < 3\% \], which is supposed to be Sue's lowest yield. \( \Rightarrow \; X \neq 1116.76 \)

Answer: \( X = 1200.03 \) (or just 1200)

Note: If the bond is called at time \( n=10 \), then Sue's yield is determined as

\[ 1021.50 = 0.03(1200) a_{10}^{0.03} + 1200 2^{10} \xrightarrow{TVM} i = 3.8\% \],

which is consistent with Sue getting a yield of at least \( i=3.9\% \).
coupon amount = 1100(0.02) = 22

\[
\begin{array}{c|c}
N & P(i) = 1021.50 \\
10 & 1021.50 = 22a_{\overline{10}|i} + 1200v_i^{10} \implies i = 3.63\% \\
\vdots & \vdots \\
19 & 1021.50 = 22a_{\overline{19}|i} + 1200v_i^{19} \implies i = 2.86\% \\
20 & 1021.50 = 22a_{\overline{20}|i} + 1100v_i^{20} \implies i = 2.46\% \\
\end{array}
\]

\[i^{(a)} = 2i = 4.92\% \quad \text{(or 4.9\%)}\]
\[ L = \frac{2000}{1.07} \left( 1 + \frac{1.03}{1.07} + \ldots \ (8 \text{ terms}) \right) \]

\[ + \frac{2000(1.03)^7 (0.97)(1.07)^{-9}}{X} \left( 1 + \frac{0.97}{1.07} + \ldots \ (8 \text{ terms}) \right) \]

\[ \therefore L = \frac{2000}{1.07} \cdot \frac{a^*}{8} \left( \frac{1.07}{1.03} - 1 \right) + 2000(1.03)^7 (0.97)(1.07)^{-9} \cdot \frac{a^*}{8} \left( \frac{1.07}{0.97} - 1 \right) \]

\[ = 20688.63 \]
\[ \sum_{k=1}^{40} p_k = p_1 + p_2 + \ldots + p_{40} = P - C \]

\[ p_{15} = -194.82 \quad p_{20} = -306.69 \]

\[ p_{20} = p_{15} (1+i)^5 \quad \Rightarrow \quad i = 0.095 \]

\[ p_i = p_{15} \cdot 2^{1.4} \]

\[ \sum_{k=1}^{40} p_k = p_1 + p_2 + \ldots + p_{40} = p_i (1 + (1+i) + \ldots + (1+i)^{39}) \nu_e^p = p_i \cdot s_{40|i} \]

\[ \therefore \quad P - C = p_i \cdot s_{40|i} = -194.82 \cdot 2^{1.4} \cdot s_{40|i} \]

This is negative because the bond was bought at a discount.

The amount of discount is

\[ 194.82 \cdot 2^{1.4} \cdot s_{40|i} = 21,135 \]
\[ \sum_{k=1}^{8} P_k = L. \text{ Total amount of interest paid is } 8R - L. \]

\[ P_5 = 699.68 \quad \text{or} \quad i = 0.0475 \]

\[ P_1 = P_5 \cdot u^4 \]

\[ \sum_{k=1}^{8} P_k = P_1 + P_2 + \ldots + P_8 \]

\[ = P_1 (1 + (1+i) + \ldots + (1+i)^7) \quad \text{VAR} \quad P_1 \cdot S_{8i} \]

\[ \therefore L = P_1 \cdot S_{8i} = 699.68 \cdot u^4 \cdot S_{8i} = 5500 \]

Also, \[ L = RA_{8i} \quad \Rightarrow \quad R = \frac{842.39}{1239} \]

\[ \therefore \text{the total amount of interest paid is} \]

\[ 8R - L = 1239 \]
We don’t need to find n.

\[ B_{18} = 16,337.10 \]

We determine the new payment using

\[ 16,337.10 = R \cdot d_{34/12} \quad n = m \cdot r = \frac{0.048}{12} \]

\[ \Rightarrow R \approx 715.27 \]
\[ F = C = 5000 \]
\[ r = \frac{.076}{2} = .038 \]
\[ n = 14 \]
\[ P = 5000 \text{ (since no premium or discount)} \]
\[ \text{(this is also referred to as buying at par, which means } P = C \text{)} \]

Coupons are \( F = 190 \)

From the basic pricing formula,
\[ 5000 = 190 \frac{1}{i/2} + 5000 \left(1 + \frac{.038}{2}\right)^{14} \]
\[ \implies i = .038 \ (r = r) \]

Since \( F = C \) and \( r = i \), we can use the shortcut for the MacD of the bond; namely,
\[ \text{MacD} = \frac{1}{1 + \frac{.038}{2}} = 11.11 \text{ (the time unit here is in semi-annual periods, since the coupons are paid semi-annually.)} \]

\* In years, \( \text{MacD} = \frac{11.11}{2} = 5.56 \)

Remark: If you don’t recognize that you can use the shortcut, then
\[ \text{MacD} = \frac{190(1 + i)^{14} + 5000(14) 2^{14}}{190(1 + i = \frac{.038}{2})^{14}} = 11.11 \text{ as above} \]
Bond A: \( r = \frac{i}{2} + 0.02 \Rightarrow \text{coupons} = 5000i + 200 \)

Bond B: \( r = \frac{i}{2} - 0.02 \Rightarrow \text{coupons} = 5000i - 200 \)

\[
\begin{align*}
    P_A &= (5000i + 200) \cdot \frac{1}{2} + 10000 \cdot 2^{30} \\
    P_B &= (5000i - 200) \cdot \frac{1}{2} + 10000 \cdot 2^{30} \\
    P_A - P_B &= 400 \cdot \frac{1}{2} = 6341.12 \\
    \Rightarrow \quad i &= 0.084
\end{align*}
\]
\[
\frac{.09}{1.12} = .0875
\]

\[400000 = R \frac{a}{1.8014975} \quad \Rightarrow \quad R = 4057.07 \text{ (original payments)}
\]

\[B_{36} = R \frac{a}{1.8014975 - 1.149} \quad \text{(could have used retrospective)}
\]

\[\Rightarrow B_{36} = 356,498.85
\]

\[R_{\text{new}} = 4057.07 - 409.88 = 3647.19
\]

\[\therefore 356,498.85 = 3647.19 a \frac{1.149}{1.12}
\]

\[\Rightarrow \frac{j}{.12} = .575\%
\]

\[\Rightarrow j = 6.9\%
\]
For the first bond,

\[ P = 25a_{\overline{60}|.025} + 1200v_{.025}^{60} = 1045.46 \]

\[ \text{for the second bond} \]

\[ 1045.46 = 25a_{\overline{60}|.1/2} + 800v_{.1/2}^{60} \]

\[ \text{TVM} \quad \frac{i}{2} = 2.2\% \quad \Rightarrow \quad j = 4.4\% \]
\[ R^{SF} = 2(1000 \cdot \bar{i}) = 2000 \cdot \bar{i} \]

\[ \therefore \quad 2000 \cdot \bar{i} \cdot S_{5,0.8\bar{i}} = 1000 \]

\[ \Rightarrow \quad 2000 \cdot \bar{i} \cdot \frac{(1+0.8\bar{i})^5 - 1}{0.8\bar{i}} = 1000 \]

\[ \Rightarrow \quad (1+0.8\bar{i})^5 = 1.4 \quad \Rightarrow \quad \bar{i} = 8.7\% \]
If we subtract the second equation from the first, we would have as one of the terms, 

\[ B_{26} \cdot 2^{26} - B_{25} \cdot 2^{25} \]

which doesn't factor. So, let's first multiply the first equation by \( (\text{ii}) \) and then subtract. We get

\[ L = 2500 \cdot A_{25} \cdot 2^{25} + B_{26} \cdot 2^{26} \]

\[ P_{26} = X = B_{26} - B_{25} \cdot 2^{25} \]
\[
\text{aeir} = 0.65
\]
\[
L = 100(Ia)_{51.05} + X A_{751.05} \cdot 2_{0.5}^5 = 10000
\]

\[
\Rightarrow X = 1075.08
\]
\[ 65000 = R a_{\frac{1801 \cdot 0.08}{12}} \implies R = 621.17 \text{ (original payments)} \]

\[ B_{1a} = R a_{\frac{180-121 \cdot 0.08}{12}} = 62661.40 \]

\[ \therefore 62661.40 = R^{\text{new}} a_{\frac{1681 \cdot 0.06}{12}} \implies R^{\text{new}} = 552.19 \]
90
Exarn FM
Sample Questions)

\[ F = 1000 \]
\[ r = \frac{.09}{3} = .045 \]
\[ n = 40 \]
\[ C = 1200 \]
\[ i = \frac{.06}{3} = .05 \]

\[ P = 45 A_{40 | .05} + 1200 2_{.05}^{40} = 942.61 \]
Exam FM
Sample Questions

\[ F = 1000 = c \]
\[ r = \frac{10}{3} = 0.5 \]
\[ i = \frac{12}{3} = 0.6 \]

\[
\begin{array}{c|c}
 n & P(0.06) \\
2 & 50a_{\frac{2}{0.06}} + 1000\, v_{0.06} = 981.67 \\
\vdots & \vdots \\
20 & 50a_{\frac{20}{0.06}} + 1000\, v_{0.06} = 885.30 \\
\end{array}
\]

Any price above 885.30 is paying too much to yield \( i = 0.06 \) seeir
\[ PV = 1050.50 = 22.5a_{14.03} + x \cdot (Ia)_{14.03} + 300 \cdot 2^{14} \]

\[ \Rightarrow x = 7.54 \]
\[ L = 1000 a_{30} \cdot 2^9 + 500 (I_a) \cdot 2^{10} \]

\[ = 64207 \cdot 64,257.02 \]
D = amount of "drop payment" 
\[ \text{extra payment that pays off the loan} \]
\[
\begin{array}{c}
0 & 1 & 2 & 3 & 4 & \cdots & D \\
\hline
4 & 1200000 & 1200000 & 1200000 & \ldots & 1200000
\end{array}
\]
\[ a e d r = 0.055 \Rightarrow \dot{c} = a e i r = \frac{0.055}{1.055} \]
\[ L = 15000000 \]

Changing the valuation date to \( t = 2 \) allows us to find the number of payments of 1200000 as follows:

\[
15000000 \left(1 - 0.055\right)^2 = 1200000 \frac{a_{n|c}}{q_{n|c}}
\]

\[ \Rightarrow n = 29 + \]

\[ \therefore 15000000 \left(1 - 0.055\right)^2 = 1200000 \frac{a_{29|c}}{30|c} + D \cdot 30 \]

\[ \Rightarrow D = 960723.59 \]

Remark: The SOA solution uses rounded numbers, e.g., the solution uses \( i = 5.82\% \) whereas this solution does not use this rounded value. This is why the value of \( D \) is a little off from the SOA solution.
\[ I_{n-1} = 0.525 \cdot I_{n-3} = 0.1427 \cdot I_1 \]

Since \( I_k = i \cdot B_{k-1} \), we can write the above equations in terms of balances as follows:

\[ B_{n-2} = 0.525 \cdot B_{n-4} = 0.1427 \cdot B_0 \]

Note: 1) \( B_{n-4} = RA_{n-1} + B_{n-2} \cdot \gamma^2 \)

2) \( B_{n-2} = \frac{P_0}{RA_{n-1}} \)

\[ \therefore B_{n-4} = B_{n-2} + B_{n-2} \cdot \gamma^2 = B_{n-2} \left( 1 + 2 \gamma^2 \right) \]

We're given \( \frac{B_{n-4}}{B_{n-2}} = \frac{1}{0.525} \cdot \frac{B_{n-2}}{B_{n-2}} \)

\[ \therefore 1 + \gamma^2 = \frac{1}{0.525} \Rightarrow \gamma = 5 \cdot 131.5\% \]

We also have \( B_{n-2} = 0.1427 \cdot B_0 \)

\[ \Rightarrow RA_{n-1} = 0.1427 \cdot (RA_{n-1}) \Rightarrow A_{n-1} = \frac{a_{n-1}}{0.1427} \]

\[ \Rightarrow \frac{1 - \gamma^2}{i} = \frac{A_{n-1}}{0.1427} \Rightarrow \gamma = 1 - i \cdot \frac{a_{n-1}}{0.1427} \]

Since \( i = 5.1315\% \), then \( n = 22 \)
i) \[ L = R_{SF} \cdot S_{III.047} \]

ii) \[ L - AV_{7}^{SF} = 6241 \]

\[ AV_{7}^{SF} = R_{SF} \cdot S_{71.047} \]

\[ 6241 = R_{SF} \cdot S_{III} - R_{SF} \cdot S_{71} = R_{SF} (S_{III} - S_{71}) \]

\[ R_{SF} = \frac{6241}{S_{III.047} - S_{71.047}} = 1054.57 \]
\[ m = \frac{.06}{12} = .005 = m \]

\[ 200,000 = R A\frac{r}{360\text{m}} \implies R = 1199.10 \text{ (amount of monthly payments)} \]

As of 12/31/2007 we have:

1) If there were no extra payments made, then
\[ B_{60}^{(\text{no extra payments})} = R A\frac{r}{300\text{m}} = 186108.72 \]

2) The accumulated value of the extra payments is
\[ AV^{(\text{extra payments})} = 10000 S_{51}^{i} \quad i = \text{aeir} = (1.005)^{12} - 1 \]
\[ = 56,560.07 \]

\[ \therefore \text{as of 12/31/2007, with the extra payments,} \]
\[ B_{12/31/2007} = 186108.72 - 56560.07 = 129548.65 \]

We get the number of remaining payments as follows:
\[ 129548.65 = 1199.10 A_{110.05} \implies n = 155+ \]

The drop payment is made 156 months (exactly 13 years) after 12/31/2007, which is 12/31/2020.
\[ ae_{cr} = 0.08 \]

\[ \Rightarrow ae_{cr} = \frac{0.08}{1 - 0.08} = \frac{8}{92} = i \]

\[ 500,000 = R \ a_{\overline{5}|i} \implies R = 1275.32.72 \]

We calculate \( X \) by solving

\[ 500,000 = 128000 \ a_{\overline{5}|i} + X \ \bar{a}^5_i \]

\[ \Rightarrow X = 125,220.38 \]
\[ \begin{align*}
\text{i)} & \quad \boldsymbol{I}_1 = 3600 = i \cdot B_0 = i \cdot (Ra_{10}) = R(1-\nu^{10}) \\
& \quad \text{since } a_{10} = \frac{1-\nu^{10}}{c} \\
& \quad \therefore 3600 = R(1-\nu^{10}) \\
\text{ii)} & \quad P_6 = 4871 \\
& \quad P_6 = R - I_6 = R - i \cdot B_5 = R - \left[ i \cdot \left( \frac{Ra_{10}}{c} \right) \right] = R(1-\nu^{5}) \\
& \quad \therefore P_6 = R - [R(1-\nu^{5})] = R - [R - R\nu^{5}] \\
& \quad \Rightarrow P_6 = R\nu^{5} \Rightarrow 4871 = R\nu^{5} \\
& \quad \therefore \begin{cases} 
3600 = R(1-\nu^{10}) \\
4871 = R\nu^{5} \quad \Rightarrow \quad R = \frac{4871}{\nu^{5}} 
\end{cases} \\
& \quad \therefore 3600 = \frac{4871}{\nu^{5}} (1-\nu^{10}) \Rightarrow 3600\nu^{5} = 4871 - 4871\nu^{10} \\
& \quad \Rightarrow 4871\nu^{10} + 3600\nu^{5} - 4871 = 0 \quad \Rightarrow \quad \nu^{5} = 1.69656 \\
& \quad \Rightarrow i = 0.075 \quad \text{and} \quad R = \frac{4871}{\nu^{5}} = 6992.94 \\
& \quad X = Ra_{10}i = 48000
\end{align*} \]
10000 = F(0.035) \frac{a_{501c}}{0.035} + F \nu_i^{50}

\Rightarrow \frac{C}{F} = \frac{10000}{0.035 \frac{a_{501c}}{0.035} + \nu_i^{50}} = 9917.99
At time 0, Jeff invests $8000$ of his own money.

For the next 10 years, at the end of each month, he receives a coupon of $10000 \times (0.0075) = 75$, but makes an interest payment of $2000 \times \left(\frac{0.08}{12}\right) = 13.33$. So he receives $75 - 13.33 = 61.67$ each month for 10 years.

At the end of 10 years, the bond matures for $10000$ but the loan amount of $2000$ is due. So he receives $8000$.

$: \text{the timeline is}$

\[
\begin{array}{cccccc}
\text{months} & 0 & 1 & 2 & \cdots & 59 & 60 \\
\uparrow & 61.67 & 61.67 & \cdots & 61.67 & 61.67 & 8000 \\
\end{array}
\]

\[
P \left( 8000 \right) = 8000 \times a_{\overline{60}|m} + 8000 \times 2_{\overline{60}|m}
\]

$: \text{Jeff's monthly effective yield is } m = 0.00770875$

\[
\Rightarrow \text{his annual effective yield rate is } i = (1 + m)^{12} - 1 = 9.65\% 
\]
\( p^I = 52.8 a_m + 1000 v^2 \) \\{ \begin{align*} p^I &= p^I \\ p^I &= 48.4 a_m + 1000 v^2 \end{align*} \} \Rightarrow 0 = 4.4 a_m - 1000 v^2 \Rightarrow v^2 = 0.4 a_m

\( p^\Pi = 1320 r a_m + 1320 v^2 \) \\{ \begin{align*} p^\Pi &= p^I \\ p^I &= 52.8 a_m + 1000 v^2 \end{align*} \} \Rightarrow 0 = a_m (1320r - 52.8) + 320 v^2

\( v^2 = 0.044 a_m \Rightarrow 320 v^2 = 14.08 a_m \)

\( 0 = a_m (1320r - 52.8 + 14.08) \Rightarrow 1320r - 52.8 + 14.08 = 0 \Rightarrow r = \frac{14.08 - 52.8}{1320} = .029 \bar{3} \)
semi-annual periods

\[ a e i r = 0.04 \Rightarrow i = \log(1 + 0.04) = (1.04)^{1/2} - 1 \]

\[ PV = 582.53 = \sum_{r=1}^{12} 1.02c \cdot v + 1.02^2c \cdot v^2 + \ldots + 1.02^{12}c \cdot v^{12} + 250v^{12} \]

\[ = \frac{1.02c}{1+i} \left( 1 + \frac{1.02}{1+i} + \ldots + (12 \text{ terms}) \right) + 250v^{12} \]

\[ = \frac{1.02c}{1+i} \cdot \sum_{r=1}^{12} \left( \frac{1.02}{1+i} \right)^{r-1} + 250v^{12} \]

\[ c = (1.04)^{1/2} - 1 \]

\[ \Rightarrow c = 3.204 \]
\[ B_3 = F_r a_n + B_4 \cdot \nu \]

\[ \Rightarrow 1254.87 = F_r a_n + 1277.38 \nu_{oo} \Rightarrow F_r = 52.7822 \]

\[ P = F_r a_n + B_4 \cdot \nu^4 \Rightarrow P = 1194.70 \]

\[ P = F_r a_n + C \cdot \nu^n \Rightarrow \nu^n = 20 \]
\[ \begin{align*} \text{Exam FM} \quad F &= C = 2500 \quad Fr = 87.5 \\
&= \frac{.0?}{i} = .035 \\
&= \frac{.08}{i} = .04 \\

B_4 &= B_3 (1+i) - Fr = B_3 + .04B_3 - 87.5 \\
\Rightarrow B_4 - B_3 &= .04B_3 - 87.5 \quad \text{(by)} \quad 8.44 \\
\Rightarrow B_3 &= 2398.50 \\

P &= Fr \cdot a_{31} + B_3 \cdot \nu^3 = 87.5a_{31,04} + 2398.50\nu_{04}^3 \\
\Rightarrow P &= 2375.08 \\

\text{Then} \quad P &= Fr \cdot a_{\text{#of coupons}} + C \cdot \nu^{\text{# of coupons}} \\

\text{We normally use } n \text{ to denote the number of coupons, but since we seek } n, \text{ and it denotes the number of years, we have } # \text{ of coupons = } 2n \\

\therefore 2375.08 = 87.5a_{31,04} + 2500\nu_{04}^{2n} \quad \Rightarrow 2n \overset{TVM}{=} 1.3 \\
\Rightarrow n = 6.5 \\

\text{Remark: We didn't really need to find } P \text{, we could have used } B_3 \text{ as follows:} \\
B_3 &= Fr \cdot a_{31-3} + C \cdot \nu^{2n-3} \quad \Rightarrow 2n-3 \overset{TVM}{=} 10 \\
\Rightarrow n \overset{=}{} 6.5 \end{align*} \]