(New) M254 Exercises (Solutions)

1) \[
\begin{align*}
5000 & \quad 5000 \\
2 & \quad 35 \quad 4
\end{align*}
\]
\[
\text{APV} = 5000 \cdot A_{35}^{\text{HT}} \approx 643.6
\]

2) \[
\begin{align*}
350 & \\
10 & \quad 40 \quad 4
\end{align*}
\]
\[
\text{APV} = 350 \cdot A_{40}^{\text{HT}} \approx 350 \cdot 10 \cdot E_{40}^{\text{HT}} \approx 187,834.5
\]

3) \[
\begin{align*}
10000 & \\
17 & \quad 35 \quad 4
\end{align*}
\]
\[
\text{APV} = 10000 \cdot A_{35}^{\text{HT}} \approx 10000 \cdot 17 \cdot E_{35}^{\text{HT}}
\]
\[
17 \cdot E_{35}^{\text{HT}} = 2.17 \cdot P_{35} = 2.17 \cdot \frac{\bar{E}_{35}}{\bar{A}_{35}} \approx 3485
\]
\[\therefore \text{APV} \approx 3,485\]

4) \[
\begin{align*}
500 & \quad 500 \\
17 & \quad 18 \quad 19 \quad 52 \quad 35 \quad 4
\end{align*}
\]
\[
\text{APV} = 500 \cdot 17 \cdot A_{35}^{\text{HT}} \approx 500 \cdot 17 \cdot E_{35}^{\text{HT}} \cdot A_{52}^{\text{HT}}
\]
\[\therefore \text{APV} \approx 47,135\]

5) \[
\begin{align*}
750 & \quad 750 \\
32 & \quad 35 \quad 4
\end{align*}
\]
\[
\text{APV} = 750 \cdot A_{32}^{\text{HT}} \quad \text{(Since it's only for 2 years, it's easiest to just VEP)}
\]
\[\therefore \text{APV} = 750 \cdot B_{32} + 750 \cdot 2 \cdot P_{32}^{\text{HT}} \cdot B_{33} = 11 \cdot B_{32} \approx 2,403\]

6) \[
\begin{align*}
2500 & \quad 2500 \\
16 & \quad 17 \quad 18 \quad 19 \quad 35 \quad 4
\end{align*}
\]
\[
\text{APV} = 2500 \cdot A_{35}^{\text{HT}} \approx 2500 \cdot (A_{35} - 17 \cdot E_{35} \cdot A_{52})
\]
\[\therefore \text{APV} \approx 86,123\]

\[\therefore \text{APV} \approx 86,123\]
7) \[ \text{APV} = 25000 \cdot A_{35:11} \] (Since it's only for 1 year, it's easiest to just VEP)

\[ \text{APV} = 25000 \cdot \delta + 25000 \cdot \nu \cdot \frac{\delta + \nu}{1} \]

\[ = 23,584.905 \ldots \]

8) \[ \text{APV} = 8000 \cdot A_{35:11} = 8000 \left( \frac{A_{35:17}}{17} + \frac{A_{35:17}}{17} \right) \]

\[ = 17E_{35} \text{ (see b.3)} \]

\[ \therefore \text{APV} = 3,063.636 \ldots \]

9) \[ \text{APV} = 1000 \cdot A_{30:40} \]

\[ \text{ILT} = 195.84 \]

10) \[ \text{APV} = 1000 \cdot A_{30:40} \]

\[ = 1000 \left( A_{30:40} - 10 \cdot E_{30:40} \cdot A_{40:60} \right) \]

\[ = 5260 \ldots \]

\[ \therefore \text{APV ILT} = 41.353 \ldots \]

11) \[ \text{APV} = 500 \cdot A_{30:40} \]

\[ = 500 \left( A_{30:40} + A_{40:60} - A_{30:40} \right) \]

\[ = 33.98 \]

12) \[ \text{APV} = 500 \cdot A_{30:40} \]

\[ = 500 \left( A_{30:40} + A_{40:60} - A_{30:40} \right) \]

\[ = 0.246 \ldots \]
13) \( \text{APV} = 3000 \cdot A_x \cdot \frac{v}{p} + 3000 \cdot v^2 \cdot q_x + 3000 \cdot v^3 \cdot q_{x+1} + \cdots \)

\( \mu = -\ln(0.9) \Rightarrow P_x = e^{-\mu} = 0.1 \Rightarrow q_x = 0.9 \) (i)

\( 3000 \cdot v_x = P_x \cdot q_{x+1} = 0.1 \cdot 0.9 = 0.09 \) (i)

\( 2000 \cdot v_x = 0.9^3 \cdot 0.1 = 0.081 \) (i)

\( \therefore \text{APV} = 3000 \cdot v \cdot 0.1 + 3000 \cdot v^2 \cdot 0.09 \cdot 0.1 + 3000 \cdot v^3 \cdot 0.081 \cdot 0.1 + \cdots \)

\( \text{geometrically} \quad \frac{3000 \cdot v \cdot 0.1}{1 - 0.9} = \frac{3000 \cdot v}{0.1} = 11666.66 \cdots \)

*Remark: Symbolically, \( A_x \cdot \frac{v \cdot q_x}{1 - v \cdot p} \cdot \frac{1 + i}{1 + i} = \frac{q_x}{1 + i - p} = \frac{q_x}{q + i} \)

\( A_x \cdot \frac{v \cdot q_x}{q + i} \)

14) \( \text{APV} = 100 \cdot A_{x:90} = 100 \cdot 12 \cdot E_x = 100 \cdot 12 \cdot P_x \cdot \frac{v}{p} = \text{APV} \frac{i = 0.8}{p = 0.9} = 11.215 \cdots \)

15) \( \text{APV} = 1000 \cdot A_{x:0} \cdot 30 = 1000 \left( A_{40} - 20E_{40} \cdot A_{60} \right) \)

\( A_{40} \cdot \frac{v}{q + i} \cdot \frac{q_{60}}{q + i} = A_{60} \cdot 20E_{40} \cdot \frac{v}{p} = (v \cdot p) \)

\( \therefore \text{APV} = 1000 \cdot \frac{q_{60}}{q + i} \left( 1 - (v \cdot p) \right) \frac{i = 0.8}{p = 0.9} = 541.064 \cdots \)

16) \( -18) \)

\( \text{DML (w=110) mortality} \Rightarrow q_x = 11q_x = 21q_{x} = \cdots = \frac{1}{110 - x} = \frac{1}{110 - x} \)

16) \( \text{APV} = 3000 \cdot A_{60} \cdot \frac{v}{p} + 3000 v \cdot q_{60} + 3000 v^2 \cdot q_{60} + \cdots + 3000 v^{50} \cdot q_{60} \)

\( \therefore \text{APV} = 3000 \cdot \frac{1}{110 - 60} \cdot (v + v^2 + \cdots + v^{50}) = 3000 \cdot \frac{1}{50} \cdot A_{50:110} = 734.009 \cdots \)

*Remark: \( A_x \cdot \frac{\text{DML(w)}}{w - x} \cdot A_{x:w-1} \)
17) \[ \text{APV} = 100 \times A_{s:121} = 100 \times E_{s:50} = 100 \times 1.2 \times P_{50} = 100 \times (1.08)^{-12} \times \frac{110-50}{110-50} = 31.769 \ldots \]

18) \[ \text{APV} = 1000 \times A_{40:201} = 1000 \left( A_{40} - \frac{20}{12} E_{40} \cdot A_{60} \right) \]

\[ A_{40} \xrightarrow{DML(w=10)} \frac{1}{70} \cdot A_{70} \quad A_{60} \xrightarrow{DML(w=10)} \frac{1}{50} \cdot A_{50} \]

\[ \therefore \text{APV} = 1000 \left( \frac{1}{70} A_{70} - (1.08)^{-20} \cdot \frac{5}{7} \cdot \frac{1}{50} A_{50} \right) = 140.259 \ldots \]

*Remark: Using VEP, we get \( A_{1/x:11} \xrightarrow{DML} \frac{1}{w-x} \cdot A_{11} \). Alternatively, we get, \[
\therefore \text{APV} = 1000 \cdot \frac{1}{70} \cdot A_{201} = 140.259 \ldots
\]

19) \[ \begin{array}{c}
\text{APV} = \left[ 10000 \times A_{60:21} \right] + \left\{ 10000 \times A_{60:21} \right\} \\
\text{VEP} = \left[ 10000 \times P_{60}^{(a)} + 10000 \times 2 \times 11 \times b_{60}^{(a)} \right] \\
+ \left\{ 10000 \times P_{60}^{(a)} + 10000 \times 2 \times 11 \times b_{60}^{(a)} \right\}
\end{array} \]

\[ b_{60}^{(a)} = .015 \quad 11 \times b_{60}^{(a)} = P_{60}^{(a)} \cdot b_{60}^{(a)} = (985)(.012) \]

\[ 11 \times b_{60}^{(a)} = P_{60}^{(a)} \cdot b_{60}^{(a)} = (985)(.008) \]

\[ \therefore \text{APV} = [339.410 \ldots ] + \left\{ 119.092 \ldots \right\} = 458.503 \ldots \]
20) \[ \text{APV} = 2000 \cdot A_{x:1}^{(\text{Dec 1})} + 3000 \cdot n_1 A_{x}^{(\text{Dec 2})} \]

\[ A_{x:1}^{(\text{Dec 1})} = A_{x:1}^{(\text{Dec 1})} - n E_x = 0.41763 - 0.30158 = 0.11605 \]

\[ n_1 A_{x}^{(\text{Dec 2})} = n E_x \cdot A_{x+1}^{(\text{Dec 2})} = A_{x}^{(\text{Dec 2})} - A_{x:1}^{(\text{Dec 2})} = A_{x}^{(\text{Dec 2})} - (A_{x:1}^{(\text{Dec 2})} - n E_x) = 0.46576 - (0.58378 - 0.30158) = 0.18356 \]

\[ \therefore \text{APV} = 2000 \cdot (0.11605) + 3000 \cdot (0.18356) = 782.78 \]

21) \[ \begin{array}{c|c|c|c}
0 & 750 & 1250 \\
\hline
35 & 1 & 2 \\
\hline
4 & & \\
\end{array} \]

\[ n = 1 - a = 0.95 \]

\[ \text{APV} = \sum_{35} 1000 \cdot b_{35} + 750 \cdot b_{35}^2, 11 \cdot b_{35} + 1250 \cdot b_{35}^3, 21 \cdot b_{35} \]

\[ b_{35} = 0.02 \]

\[ 11 b_{35} = P_{35} \cdot b_{35} = (0.98)(0.025) \]

\[ 21 b_{35} = 2 P_{35} \cdot b_{35} = (0.98)(0.975)(0.03) \]

\[ \therefore \text{APV} = 66.304 \ldots \]
22) \[ \text{APV}^A = \frac{585}{585} \times \text{VEP} = 1000 \cdot 2 \cdot 0.93 + 1000 \cdot 2^2 \cdot 0.93^3 + \ldots + 1000 \cdot 2^{20} \cdot 0.93^{20} \]

\[
\begin{array}{cccc}
\text{B} & 1000(1.02) & 1000(1.02)^2 & \cdots & 1000(1.02)^{20} \\
0 & 1 & 2 & \cdots & 19 & 20 \\
\uparrow \\
0.93 & 1 & 2 & \cdots & 19 & 20 \\
\end{array}
\]

\[ \text{APV}^B = \text{VEP} = 1000 \cdot 2 \cdot 0.93 + 1000 \cdot 2^2 \cdot 0.93^2 + \ldots + 1000 \cdot 2^{20} \cdot 0.93^{20} \]

If \( i = 0.05 \), then \( 1.05^{20} = \frac{1.02}{1.05} = \frac{1.02}{1.05} = 2 \cdot 0.93 \)

\[ \therefore \text{APV}^B = \frac{585}{585} \]

23) \[ \text{APV} = 1000 \cdot 2 \cdot 0.95 + 1000 \cdot (1.05)^2 \cdot 0.95^2 + \ldots + 1000 \cdot (1.05)^2 \cdot 0.95^2 \]

\[ = 1000 \cdot 2 \left( b_{30} + 1.05^2 \cdot b_{30} + \ldots + 2.95 \cdot b_{30} \right) = 1000 \cdot 2 \cdot 30 \cdot 0.95 \]

\[ \therefore \text{APV} = \frac{1000}{1.05^2} (1 - 0.4) = 571.428 \]

24) \[ \text{APV} = 100 \cdot (IA) \]
25) \[ APV = 3A_{25} + 2 \cdot (IA)_{25} \]

or

\[ APV = 5A_{25} + 2 \cdot (IA)_{26} \cdot E_{25} \]

26) \[ APV = 500A_{50:101} + 100(DA)_{50:101} \]

or

\[ APV = 1600A_{50:101} - 100(IA)_{50:101} \]

(There are many other correct answers!)