\[ P_{k+1} = (kV + \Pi - e)(1+i) - (S+E)q_x + kV_P x+k \]

= profit at \( t=k+1 \) per policy in force at \( t=k \)

**Profit Vector:**

\[ \mathbf{Pr} = \left< P_{r_0}, P_{r_1}, P_{r_2}, P_{r_3}, \ldots \right> \]

\[ P_{r_0} \text{ definition } - (\text{pre-contract expenses}) \]

Now define

\[ \Pi_{k+1} = \text{profit at } t=k+1 \text{ per policy issued } (t=0) \]

\[ = \left( P_{r_{k+1}} \right) \cdot kP_x \]

E.g.

\[ \Pi_0 = P_{r_0} \text{ (negative)} \]

\[ \Pi_1 = P_{r_1} \]

\[ \Pi_2 = P_{r_2} \cdot P_x \]

\[ \Pi_3 = P_{r_3} \cdot 2P_x \]

\[ \vdots \]

**Profit Signature:**

\[ \mathbf{\Pi} = \left< \Pi_0, \Pi_1, \Pi_2, \Pi_3, \ldots \right> \]
**Net Present Value (NPV)**

\[ NPV = \Pi_0 + \Pi_1 \cdot \gamma + \Pi_2 \cdot \gamma^2 + \Pi_3 \cdot \gamma^3 + \ldots \]

**Remark:**
- \( NPV(0) = \Pi_0 \)
- \( NPV(1) = \Pi_0 + \Pi_1 \cdot \gamma \)
- \( NPV(2) = \Pi_0 + \Pi_1 \gamma + \Pi_2 \gamma^2 \)

\( NPV(k) = \text{PV of the } \Pi \text{ components up to and including } \Pi_k \)

**Note:** The interest rate used for these NPV calculations is called the **Risk Discount Rate** (or **Hurdle Rate**). *Warning:* unless told otherwise, this rate is used as an interest rate; \( \gamma = \frac{1}{1+i} \)

**[RDR/HR]**

**Discounted Payback Period (Break-even Year)**

**Idea:**
- \( NPV(0) = \Pi_0 \) (negative)
- \( NPV(1) = \Pi_0 + \Pi_1 \gamma \) (probably negative)

\[ NPV(k-1) < 0 \]
\[ NPV(k) \geq 0 \]

Then \( DPP = k \) = smallest value of \( k \) such that \( NPV(k) \geq 0 \)
Profit Margin: (PM)

\[ PM = \frac{NPV}{APV(\text{Premiums})} > \text{use RDR/HR} \]

Internal Rate of Return: (IRR)

\[ IRR = \text{the value of } i \text{ such that } NPV = 0 \]