See the following pages for pension examples.
Example 1: Flat Dollar Benefit with Early Retirement Factor

Amy and Bob both work for companies that have defined benefit pension plans. Both started working for their respective companies at age 30. Both Amy's and Bob's pension plans have the following provisions:

(i) the benefit formula is $1000 per year per year of service, payable annually starting at normal retirement age
(ii) the normal form of payment is a life annuity due
(iii) the normal retirement age is 65
(iv) early retirement is allowed starting at age 60

For Amy's plan, the early retirement benefit is determined by reducing the normal retirement benefit by 5% per year for each year prior to her normal retirement date, whereas for Bob's plan, the early retirement benefit is determined by actuarial equivalence to the normal retirement benefit at the age of benefit disbursement, using ILT actuarial assumptions and normal form of payment.

Amy and Bob both retire, and receive their benefit, at age 60. Amy's annual benefit is \( X \) and Bob's is \( Y \).

Determine \( X - Y \).

Amy: \[
N RB = 30 \times 10,000 = 300,000 \text{ payable at age 65}
\]
\[
ER F = 1 - 5(0.05) = 0.75
\]
\[
ER B = X = 30,000 \times 0.75
\]

Bob:

\[
Y \cdot \ddot{a}_{60} = 30,000 \times 5 \times \ddot{a}_{60} = 30,000 \times 6 \cdot E_{60} \cdot \ddot{a}_{65}
\]
Module 5: Financial Analysis  
Section 3: Pension Mathematics:  
Part 2: Defined Benefit Plans

Example 2: Percent of Salary with Optional Form of Payment

Amy and Bob both work for companies that have defined benefit pension plans. Both started working for their respective companies at age 22, each making $50,000 per year. Both Amy's and Bob's pension plans have the following benefit formula:

1% of "average salary" per year of service, payable annually starting at age 65

Amy's plan defines "average salary" as a career average.  
Bob's plan defines "average salary" as the final three year average

Amy's plan has a normal form of payment as a life annuity due.  
Bob's plan has a normal form of payment as a 10-year certain and life annuity due.

Both plans assume a 3% per year salary increase assumption and both plans use ILT actuarial assumptions.

Amy and Bob both retire at age 65. The actuarial present value at age 65 of Amy's benefit is $X$ and the actuarial present value at age 65 of Bob's is $Y$.

Determine $X - Y$.

Amy:  
\[ B_{65} = (0.01) \cdot \bar{s} \cdot (42) \]

\[ \bar{s} = \frac{1}{43} \left( S_{62} + S_{63} + \ldots + S_{64} \right) \]

\[ = \frac{1}{43} \cdot 50000 \cdot \left( \frac{1 + 1.03 + 1.03^2 + \ldots + 1.03^{42}}{S_{43}, 0.03} \right) \]

\[ X = B_{65} \cdot \ddot{a}_{65} \]

Bob:  
\[ B_{65} = (0.01) \cdot \bar{s} \cdot (45) \]

\[ \bar{s} = \frac{1}{3} \left( S_{64} + S_{65} + S_{66} \right) = \]

\[ Y = B_{65} \cdot \ddot{a}_{65, 10} = B_{65} \cdot (\ddot{a}_{101} + 10E_{65} \cdot \ddot{a}_{75}) \]
Module 5: Financial Analysis  
Section 3: Pension Mathematics:  
Part 3: Replacement Ratio Example

Amy works for a company that has both a defined contribution plan and a defined benefit plan. Amy started working for the company at age 22, making $50,000 per year. Amy’s company’s plans’ contribution and benefit formulas are:

DC:  The company matches Amy’s contribution dollar for dollar, up to a maximum of a 3% company contribution.

DB:  1% of career average salary per year of service, payable annually starting at age 65

You are given:

(i)  Amy works at this company until age 65  
(ii) Amy receives a 3% increase in salary each year  
(iii) Amy contributes 3% of her salary at the end of each year into the DC account, and the company’s match is deposited at the same time as Amy’s deposit  
(iv) Deposits into Amy’s DC account earn interest at 5% annual effective

Using IILT actuarial assumptions to convert Amy’s DC account balance to a benefit, determine Amy’s replacement ratio at age 65.

\[
DB: \quad B_{65} = (0.01) \cdot \left( \frac{1}{r} \cdot 50000 \cdot s_{65 | 0.03} \right) \cdot (43)
\]

\[
DC:
\]

\[
A_V = 0.06 \cdot 50000 \cdot (1.05)^{42} + (1.03)(1.05)^{41} + (1.03^2)(1.05)^{40} + \ldots
\]

\[
= 0.06 \cdot 50000 \cdot (1.05)^{42} \cdot \left( 1 + \frac{1.02}{1.03} + - - - \right)
\]

\[
\text{Defining } \overline{i} = \frac{1.05}{1.03} - 1
\]

\[
= 0.06 \cdot 50000 \cdot (1.05)^{42} \cdot \overline{\overline{a}}_{42 \rvert i} = X \cdot \overline{a}_{65}
\]

Then  
\[
RR = \frac{B_{65} + X}{S_{64}}
\]