4/23/19

See following pages for more Module 5 Examples.
Module 5: Financial Analysis
Section 3: Pension Mathematics:
Part 5: Projected Unit Credit (PUC) and Traditional Unit Credit (TUC) Funding Methods

Example 1: TUC – Final Year Salary Plan

The annual normal retirement benefit is 2% of final year’s salary per year of service. For a particular participant, you are given:

Date of Birth (DOB): January 1, 1980
Date of Hire (DOH): January 1, 2000
Valuation Date: January 1, 2015
2014 Salary: 100,000
Salary Scale: 5% per year
Normal Retirement Age: 65
Normal Form of Payment: life annuity payable monthly in advance
Pre-Retirement Decrements: None
\(\ddbar^{(12)} = 10\)

Using the TUC funding method, determine:

(a) \(B_{35}\)

(b) \(b_{35}\)

(c) the actuarial liability for this participant as of the 1/1/2015 valuation, using a valuation interest rate of 8% annual effective.

(d) the normal cost for this participant as of the 1/1/2015 valuation, using a valuation interest rate of 8% annual effective.
(a) \( B_{35} = \overline{102 \times S} \cdot (15) \)
\[
\overline{S} \overline{\text{TUC}} \overline{S}_{34} = 100000
\]
\[
\therefore B_{35} = 30000
\]

(b) \( b_{35} = B_{36} - B_{35} \)
\[
B_{36} = \overline{0.02 \times S} \cdot (16) \]
\[
\overline{S} \overline{\text{TUC}} \overline{S}_{36} = 100000(1.05)
\]
\[
\therefore b_{35} = \frac{0.02 \times 100000(1.05)(16) - 0.02 \times 100000 \cdot 1.15}{33600} = 3600
\]

(c) \( AL_{35} = \text{APV}(B_{35}) \)
\[
= 30000 \cdot \overline{a}^{(12)}_{65} \cdot 30E_{35}
\]

(d) \( NC_{35} = \text{APV}(b_{35}) \)
\[
= 3600 \cdot \overline{a}^{(12)}_{65} \cdot 30E_{35}
\]

Notes:
1) \( 30E_{35} = \overline{70} \) since there are no pre-retirement decrements
2) \( NC_{35} = \frac{3600}{30000} \cdot AL_{35} \)
Module 5: Financial Analysis
Section 3: Pension Mathematics:
Part 5: Projected Unit Credit (PUC) and Traditional Unit Credit (TUC) Funding Methods

Example 2: PUC – Final 5-year Average Salary Plan (include Early Retirement)

The annual normal retirement benefit is 2% of final five-year average salary per year of service. For a particular participant, you are given:

Date of Birth (DOB): January 1, 1980
Date of Hire (DOH): January 1, 2000
Valuation Date: January 1, 2015
2014 Salary: 100000
Salary Scale: 5% per year
Normal Retirement Age: 65
Normal Form of Payment: life annuity payable monthly in advance
Pre-Retirement Decrement: Death only
Selected Annuity Values: $a_{63}^{(12)} = 12$, $a_{64}^{(12)} = 11.5$, $a_{65}^{(12)} = 11$
Early Retirement Eligibility: Ages 63 and 64 (with reduced benefit)
Early Retirement Reduction: 5% per year for each year before age 65
Early Retirement Assumptions:
10% of age 63 workers will retire at age 63
20% of age 64 workers will retire at age 64
100% of age 65 workers will retire at age 65

Using the PUC funding method, determine:

(a) the accrued retirement benefit for this participant, $B_{35}$

(b) the accrued retirement benefit earned in the upcoming year for this participant, $b_{35}$

(c) the actuarial liability for this participant’s retirement benefit as of the 1/1/2015 valuation, using ILT preretirement assumptions.

(d) the normal cost for this participant’s retirement benefit as of the 1/1/2015 valuation, using ILT preretirement assumptions.
(a) \( B_{35} = 0.02 \cdot \bar{S} \cdot (15) = 0.3 \cdot \bar{S} \)

\[ \bar{S} \overset{\text{PUC}}{\leq} \begin{cases} \frac{1}{5} (S_{62} + S_{61} + S_{60} + S_{59} + S_{58}) & r = 63 \\ \frac{1}{5} (S_{63} + S_{62} + \ldots + S_{59}) & r = 64 \\ \frac{1}{5} (S_{64} + \ldots + S_{60}) & r = 65 \end{cases} \]

(b) \( b_{35} = B_{36} - B_{35} \)

\[ B_{36} = 0.02 \cdot \bar{S} \cdot (16) \]

\( \bar{S} = \text{the same as above} \)

\[ \therefore b_{35} = 0.02 \cdot \bar{S} \]

(c) \( AL_{35} = APV(B_{35}) \)

\[ AL_{35} = AL_{35}^{r=63} + AL_{35}^{r=64} + AL_{35}^{r=65} \]

\[ AL_{35}^{r=63} = 0.3 \cdot \bar{S}^{r=63} \cdot (0.9) \cdot a_{63}^{(12)} \cdot (1.1) \cdot 28E_{35} \]

\[ AL_{35}^{r=64} = 0.3 \cdot \bar{S}^{r=64} \cdot (0.95) \cdot a_{64}^{(12)} \cdot (1.2) \cdot (1.9) \cdot 29E_{35} \]

\[ AL_{35}^{r=65} = 0.3 \cdot \bar{S}^{r=65} \cdot a_{65}^{(12)} \cdot (1.9)(1.8) \cdot 30E_{35} \]

\[ NC_{35} = APV(b_{35}) = \frac{0.02}{0.3} \cdot AL_{35} = \frac{AL_{35}}{15} \]
Module 5: Financial Analysis
Section 3: Pension Mathematics:
Part 6: Unit Credit (UC) Funding Method

Example 1: UC – Career Average Salary Plan (includes use of a Service Table)

An employer offers a career average pension plan, with annual accrual rate 2.5%.
For Smith, you are given:

Age at Valuation Date: 60
Years of Past Service: 30
Average Annual Past Salary: 60,000
Salary in Year following Valuation: 100,000
Normal Retirement Age: 65
Normal Form of Payment: life annuity payable monthly in advance
Pre-Retirement Decrements: Use the following service table extract
(Retirements occur at BOY, other decrements are UDD throughout the year.)

<table>
<thead>
<tr>
<th></th>
<th>l_x^(r)</th>
<th>d_x^(d)</th>
<th>d_x^(w)</th>
<th>d_x^(r)</th>
</tr>
</thead>
<tbody>
<tr>
<td>60</td>
<td>10000</td>
<td>125</td>
<td>125</td>
<td>0</td>
</tr>
<tr>
<td>61</td>
<td>9750</td>
<td>130</td>
<td>100</td>
<td>0</td>
</tr>
<tr>
<td>62</td>
<td>9520</td>
<td>140</td>
<td>50</td>
<td>0</td>
</tr>
<tr>
<td>63</td>
<td>9330</td>
<td>150</td>
<td>0</td>
<td>2180</td>
</tr>
<tr>
<td>64</td>
<td>7000</td>
<td>120</td>
<td>0</td>
<td>1880</td>
</tr>
<tr>
<td>65</td>
<td>5000</td>
<td></td>
<td></td>
<td>5000</td>
</tr>
</tbody>
</table>

Selected Annuity Values: \( \ddot{a}_{63}^{(12)} = 12, \ddot{a}_{64}^{(12)} = 11.5, \ddot{a}_{65}^{(12)} = 11 \)

Annual benefits are reduced by 3% per year for early retirement.

Using the unit credit funding method, determine:

(a)  \( B_{60} \)
(b)  \( b_{60} \)
(c)  the actuarial liability for Smith's retirement benefit as of the valuation date, using a valuation interest rate of 8% annual effective.
(d)  the normal cost for Smith's retirement benefit as of the valuation date, using a valuation interest rate of 8% annual effective.
(a) \( B_{60} = 0.025 \cdot (60000 \cdot 30) = 45000 \)

(b) \( b_{60} = 0.025 \cdot (10000) = 2500 \)

(c) \( AL_{60} = APV(B_{60}) = AL_{60}^{r=63} + AL_{60}^{r=64} + AL_{60}^{r=65} \)

\[
AL_{60}^{r=63} = 45000 \cdot (0.94) \cdot \ddot{a}_{63}^{(12)} \cdot \frac{2}{10000} = 2180
\]

\[
AL_{60}^{r=64} = 45000 \cdot (0.97) \cdot \ddot{a}_{64}^{(12)} \cdot \frac{2}{10000} = 1880
\]

\[
AL_{60}^{r=65} = 45000 \cdot \ddot{a}_{65}^{(12)} \cdot \frac{5}{10000} = 5000
\]

(d) \( NL_{60} = APV(b_{60}) = \frac{2500}{45000} \cdot AL_{60} \)
Module 5: Financial Analysis  
Section 3: Pension Mathematics  
Part 6: Unit Credit (UC) Funding Method

Example 2: UC- Flat Dollar Plan (with MOY retirement benefits)

The normal retirement benefit is 10 per month for each completed month of service. For Smith, you are given:

Age at Valuation Date: 63  
Years of Past Service: 30

Normal Retirement Age: 65  
Normal Form of Payment: life annuity payable monthly in advance

Pre-Retirement Decrement: Use the following service table extract
(Retirements at age 63 and 64 occur at MOY; Retirements at age 65 occur at BOY)

<table>
<thead>
<tr>
<th>$x$</th>
<th>$l_x^{(r)}$</th>
<th>$d_x^{(d)}$</th>
<th>$d_x^{(w)}$</th>
<th>$d_x^{(r)}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>60</td>
<td>10000</td>
<td>125</td>
<td>125</td>
<td>0</td>
</tr>
<tr>
<td>61</td>
<td>9750</td>
<td>130</td>
<td>100</td>
<td>0</td>
</tr>
<tr>
<td>62</td>
<td>9520</td>
<td>140</td>
<td>50</td>
<td>0</td>
</tr>
<tr>
<td>63</td>
<td>9330</td>
<td>150</td>
<td>0</td>
<td>2180</td>
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<tr>
<td>64</td>
<td>7000</td>
<td>120</td>
<td>0</td>
<td>1880</td>
</tr>
<tr>
<td>65</td>
<td>5000</td>
<td></td>
<td></td>
<td>5000</td>
</tr>
</tbody>
</table>

Selected Annuity Values: \( \bar{a}_{63.5}^{(12)} = 11.75, \bar{a}_{64.5}^{(12)} = 11.25, \bar{a}_{65}^{(12)} = 11 \)

There is no reduction for early retirement.

Using the unit credit funding method, determine:

(a) \( B_{63} \)
(b) \( b_{63} \)
(c) the actuarial liability for Smith’s retirement benefit as of the valuation date, using a valuation interest rate of 7% annual effective.
(d) the normal cost for Smith’s retirement benefit as of the valuation date, using a valuation interest rate of 7% annual effective.
(a) \( B_{63} = 10(12) \div (30) = 3600 / m_0 = 43200 / yr \)

(b) \( b_{63} = \begin{cases} \frac{60}{12} = 720 / yr & \text{if } r = 63.5 \\ \frac{120}{12} = 1440 / yr & \text{if } r = 64.5 \text{ or } 65 \end{cases} \)

(c) \( AL_{63} = AL_{63}^{r=63.5} + AL_{63}^{r=64.5} + AL_{63}^{r=65} \)

\[
AL_{63}^{r=63.5} = 43200 \cdot \ddot{a}_{63.5} \cdot v \cdot 0.5 \cdot \frac{2180}{9330}
\]

\[
AL_{63}^{r=64.5} = 43200 \cdot \ddot{a}_{64.5} \cdot v \cdot 1.5 \cdot \frac{1880}{9330}
\]

\[
AL_{63}^{r=65} = 43200 \cdot \ddot{a}_{65} \cdot v^2 \cdot \frac{5000}{9330}
\]

(d) \( NC_{63}^{r=63.5} = 720 \cdot \ddot{a}_{63.5} \cdot v \cdot 0.5 \cdot \frac{2180}{9330} \)

\[
NC_{63}^{r=64.5} = 1440 \cdot \ddot{a}_{64.5} \cdot v \cdot 1.5 \cdot \frac{1880}{9330}
\]

\[
NC_{63}^{r=65} = 1440 \cdot \ddot{a}_{65} \cdot v^2 \cdot \frac{5000}{9330}
\]