4/9/19

Back to M4S4 Part 1: Net Premium Reserve with matching special statuses

Standard example \( r = x \)

\[
K^V = APV(FID) - APV(FP) \\
= A_{x+k} - P_x \cdot \bar{a}_{x+k} \\
= (1 - d\bar{a}_{x+k}) - \frac{1 - d\bar{a}_x}{\bar{a}_x} \cdot \bar{a}_{x+k} \\
= \frac{\bar{a}_x - d\bar{a}_{x+k} \cdot \bar{a}_x}{\bar{a}_x} - \bar{a}_{x+k} + d\bar{a}_x \cdot \bar{a}_{x+k} \\

\Rightarrow K^V = 1 - \frac{\bar{a}_{x+k}}{\bar{a}_x} \quad (1 \text{ minus } "a \overline{a}" )
\]

M4S5 (Continued)

"DAC" = Deferred Acquisition Costs (at time \( k \))

\[
= \left| K^V \right| \\

"FPT" = Full Preliminary Term (a "modified" net premium reserve)

Modified as follows:

\[ P_{i0} = (\text{premium at } t=0) = P_{x+1} \text{ thereafter use net level premium for a policy adjust to be issued at age } x+1. \]
Real Result:

\[ \text{Def: } V^{FPT}_0 = 0 = V^{FPT}_1 \]

Also \[ V^{FPT}_k = V^{FPT}_{k-1} \]

Example (See next pages)
Example 1: (Deferred Acquisition Costs and Full Preliminary Reserves Example)

For a fully discrete 20-year endowment insurance of 100,000 issued to (30), you are given:

(i) \( \dd{90}{20} = 11.017 \)
(ii) \( \dd{40}{101} = 7.277 \)
(iii) \( i = 0.05 \)
(iv) \( p_{30} = .98 \)
(v) The gross annual premium is \( P^g = 4450 \)
(vi) After issue, the only expenses are 50 at the beginning of each renewal year

Determine

(a) the gross premium reserve at time \( t = 10 \)
(b) the net premium reserve at time \( t = 10 \)
(c) the deferred acquisition cost at time \( t = 10 \) (i.e. the expense reserve at time \( t = 10 \))
(d) the full preliminary reserve at time \( t = 10 \)

\[
10V^g = APV_{\dd{40}} (FB_{\dd{40}} 4E) - APV_{\dd{40}} (FP^g)
\]
\[
= (100000 \dd{40}{101} + 50 \dd{40}{101}) - 4450 \dd{40}{101}
\]
\[
= 100000 (1 - \frac{0.05}{0.05} (7.277)) - 4400 (7.277)
\]

\[
10V^g = 33,329
\]

(b) \( 10V^n = 100000 (1 - \frac{\dd{40}{101}}{\dd{30}{101}}) = 33,948 \)

**Note:** \( 10V^n = 100000 \dd{40}{101} - \Pi \cdot \dd{40}{101} \)

\[
\Pi = \frac{100000 \dd{30}{101}}{\dd{20}{101}}
\]
Module 4: Reserves
Section 5: Incorporating Expenses

Example 1: (Deferred Acquisition Costs and Full Preliminary Reserves Example)

For a fully discrete 20-year endowment insurance of 100,000 issued to (30), you are given:

(i) \( \ddot{a}_{30:20} = 11.017 \)
(ii) \( \ddot{a}_{40:10} = 7.277 \)
(iii) \( i = 0.05 \)
(iv) \( p_{30} = .98 \)
(v) The gross annual premium is \( P^g = 4450 \)
(vi) After issue, the only expenses are 50 at the beginning of each renewal year

Determine

(a) the gross premium reserve at time \( t = 10 \)
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(c) the deferred acquisition cost at time \( t = 10 \) (i.e. the expense reserve at time \( t = 10 \))
(d) the full preliminary reserve at time \( t = 10 \)

\[
C) \quad 10V^g = 10V^n + 10V^e
\]

\[
\Rightarrow 10V^e = 33329 - 33948 = -619 \quad \text{DAC} = 619
\]

Note: \( P^g = 4450 \) (given) \( P^n = \frac{100000 \cdot \ddot{a}_{30:20}}{\ddot{a}_{20:30}} = 4315 \)

\[\therefore P^e = 135\]

\[\therefore 10V^e = APV_{40} \left( FE \right) - APV_{40} \left( FP^e \right)\]

\[= 50 \cdot \ddot{a}_{40:10} - 135 \cdot \ddot{a}_{40:10} = -85 \cdot \ddot{a}_{40:10} = -85 \cdot \ddot{a}_{40:10}\]

\[
C) \quad 10V^FPT = 10V^n = 100000 \left( 1 - \frac{\ddot{a}_{40:10}}{\ddot{a}_{31:19}} \right)
\]

\[\ddot{a}_{30;20} = 1 + P_{30} \cdot \ddot{a}_{21:19}\]
MH56: Reserve Recursion

\[ kV = S \cdot 2 \cdot \delta_{x+k} - \Pi + k+1V \cdot 2 \cdot P_{x+k} \]

Another example:

\[ l0V = 10000 \cdot y_2 \cdot \delta_{y_0} + 10000 \cdot y_2 \cdot y_{1.5} \cdot \delta_{40} - \Pi - \Pi 2 \cdot y_2 \cdot P_{40} + 11V \cdot 2 \cdot P_{40} \]