L-TAM Module 1 Section 10 Exercises

1. Given a 3-state model with $\mu_x^{01} = .05$, $\mu_x^{02} = .10$, $\mu_x^{12} = .20$, and all other forces of transition equal to zero, determine

   (a) ${}_5p_x^{00}$

   (b) ${}_5p_x^{01}$

   (c) ${}_5p_x^{02}$

2. Given a 2-state model with $\mu_{x+t}^{01} = .02t$ and $\mu_{x+t}^{10} = 0$, determine

   (a) ${}_{10}p_x^{00}$

   (b) ${}_{10}p_x^{01}$

3. Given a 3-state model with $\mu_{x+t}^{01} = .01 + .02t$ and $\mu_{x+t}^{02} = .02 + .04t$, determine

   (a) ${}_{10}p_x^{00}$

   (b) ${}_np_x^{10}$

   (c) ${}_kp_x^{11}$

   (d) ${}_{10}p_x^{02}$

4. Given a 4-state model with $\mu_x^{01} = \mu_x^{03} = \mu_x^{23} = .1$, $\mu_x^{10} = \mu_x^{12} = \mu_x^{13} = .2$, and all other forces of transition equal to zero, determine

   (a) ${}_0p_x^{01}$

   (b) ${}_5p_x^{\overline{11}}$

   (c) ${}_{10}p_x^{22}$

   (d) ${}_t\dot{p}_x^{23}$

   (e) ${}_t\dot{p}_x^{10}$

5. Given independent lives (x) and (y), where (x) is the husband and (y) is the wife, define the following states of the joint-life, last-survivor process:

   State 0: Both Husband and Wife are Alive
   State 1: Husband is Dead and Wife is Alive
   State 2: Husband is Alive and Wife is Dead
   State 3: Both Husband and Wife are Dead

   Suppose $\mu_{xy}^{01} = .01 = \mu_x^{23}$, $\mu_{xy}^{02} = .02 = \mu_y^{13}$, and all other forces of transition equal 0.

   Determine the probability that at the end of 5 years the husband is dead and the wife is alive.

6. Given a three state model with $\mu_x^{01} = .02$, $\mu_x^{10} = .01$, $\mu_x^{02} = .03 = \mu_x^{20}$, $\mu_x^{12} = .04$, and $\mu_x^{21} = 0$, you are given ${}_{0.5}p_x^{00} = .975$, ${}_{0.5}p_x^{01} = .010$, and ${}_{0.5}p_x^{02} = .015$

   (a) determine the value of ${}_{0.5}\dot{p}_x^{00}$ according to Kolmogorov differential equations.

   (b) use Euler's method with step size 0.1 to approximate ${}_{0.6}p_x^{00}$

7. The non-zero transition rates for a 4-state model are:

   $\mu_x^{01} = .04$ $\qquad$ $\mu_x^{02} = .02$ $\qquad$ $\mu_x^{21} = .01$

   $\mu_x^{23} = .03$ $\qquad$ $\mu_x^{13} = .001e^{0.1x} = \mu_x^{31}$

   (a) Determine ${}_{10}p_{30}^{12}$

   (b) Determine ${}_{10}p_{30}^{00}$ More generally, determine ${}_{n}p_{30}^{00}$ for any $n \geq 0$.

   (c) Determine ${}_{10}p_{30}^{02}$ More generally, determine ${}_{n}p_{30}^{02}$ for any $n \geq 0$.

   For parts (d), (e), and (f), you are also given ${}_{10}p_{30}^{01} \approx 0.2587$ and ${}_{10}p_{30}^{03} \approx 0.0710$.

   (d) Determine ${}_{10}\dot{p}_{30}^{01}$ and ${}_{10}\dot{p}_{30}^{03}$

   (e) Use an iteration of Euler's Forward Equation with step size equal to 0.2 to approximate ${}_{10.2}p_{30}^{01}$ and ${}_{10.2}p_{30}^{03}$

   (f) Perform another iteration of Euler's Forward Equation with step size equal to 0.2 to approximate ${}_{10.4}p_{30}^{01}$ and ${}_{10.4}p_{30}^{03}$