L-TAM Module 1 Section 2 Exercises

1. In each part, you are given \( \mu_x(t) = 0.1(1.01)^{x+t} \) and assume that “moment” means in the next 12-hour period. Also assume there are 360 days in a year.

(a) approximate the probability that an 80-year old dies the moment after turning 80
(b) given \( 20p_{60} = .018 \), approximate the probability that a 60-year old survives to age 80 and then dies the moment after turning 80

2. Given \( tp_x = e^{-0.5t} \), determine \( \mu_{x+t} \)

3. Given \( tp_x = (.9)^t \), determine \( \mu_{x+t} \)

4. Given \( tq_x = \frac{t}{100-x} \), determine \( \mu_{50} \)

5. Given \( tp_x = \left( \frac{100-x-t}{100-x} \right)^2 \), determine \( \mu_{50} \)

6. Given \( tp_x = \left( \frac{100-x-t}{100-x} \right)^{1/2} \), determine \( \mu_{50} \)

7. Given \( tp_0 = \begin{cases} 1, & 0 \leq t < 1 \\ 1 - \frac{e^t}{100}, & 1 \leq t < 4.5 \text{ determine } \mu_4 \\ 0, & 4.5 \leq t \end{cases} \)

8. Given \( \int_0^{10} tp_{20} \mu_{20+t} \, dt = .05 \), determine \( 10p_{20} \)

9. Given \( 20p_{30} = .85 \) and \( \int_0^{15} tp_{50} \mu_{50+t} \, dt = .2 \), determine \( 35q_{30} \)

10. Given \( \int_0^{30} tp_{30} \mu_{30+t} \, dt = \frac{2}{7} \) and \( 10q_{30} = \frac{1}{7} \), determine \( 30p_{30} \)

11. Given \( \mu_{20+t} = \frac{1}{60-t}, 0 < t < 60 \), determine \( 10p_{20} \)

12. Given \( \mu_t = \frac{1}{80-t}, 0 < t < 80 \), determine \( 10p_{20} \)

13. Given \( \mu_x = \frac{1}{\sqrt{100-x}}, 0 < x < 100 \), determine \( 17p_{19} \)

14. Given \( \mu_{25+t} = 0.1(1-t) \) for \( 0 \leq t \leq 1 \), determine \( p_{25} \)

15. Given \( \mu_x = e^{2x} \), determine \( 0.4p_0 \)
16. Given \( \int_{71}^{75} \mu_x \, dx = .107 \) and \( \int_{0}^{5} \mu_{x0+t} \, dt = .189 \), determine \( q_{70} \)

17. Given \( \mu_x = .02 \), determine \( 3p_{x+10} \)

18. Given \( \mu_x = \begin{cases} .05 & 50 \leq x < 60 \\ .04 & 60 \leq x < 70 \end{cases} \), determine
   (a) \( 4q_{50} \)
   (b) \( 18q_{50} \)

19. Given smokers (s) have a constant force of mortality of 0.2 and non-smokers (ns) have a constant force of mortality of 0.1, determine
   (a) \( 10q_{x}^s \) (i.e. the probability that an \( x \)-year old smoker dies within 10 years)
   (b) \( 10q_{x}^{ns} \)
   (c) \( 10q_x \) for a population of \( x \)-year olds, 30% of whom are smokers
   (d) the 75th percentile of \( T_x \) for the population of \( x \)-year olds in part (c).

20. Given males (m) have a constant force of mortality of 0.1 and females (f) have a constant force of mortality of 0.08, determine
   (a) \( 60p_{0}^m \) (i.e. the probability that a newborn male lives 60 years)
   (b) \( 60p_{0}^f \)
   (c) \( 60p_{0} \) for a population of newborns, 50% of whom are male
   (d) for the population of newborns in part (c) the proportion of 60-year olds who are male
   (e) \( q_{60} \) for the population of newborns in part (c)

21. If the force of mortality, \( \mu \), is constant for (x), determine
   (a) the expression for \( 5p_x \), as a function of \( \mu \)
   (b) the value of \( 5p_x \) if \( \mu = .02 \) for 30% of \( x \)-year olds, and \( \mu = .03 \) for the other 70%
   (c) the value of \( 5p_x \) if \( \mu \) is drawn from the uniform distribution on the interval [0.01,0.02]

22. Given \( \mu_x^m = \mu_x^f + .02 \) and \( p_x^m = .95 \), determine \( p_x^f \)

23. Given \( \mu_x^m = \mu_x^f + k \cdot 10p_x^f = .74 \), and \( 10p_x^m = .64 \), determine \( k \).

24. Given \( \mu_x^s = 1.1\mu_x^{ns} \) and \( k\mu_x^{ns} = .75 \), determine \( k \mu_x^s \)

25. Given \( \mu_x^{ns} = c \cdot \mu_x^s \), \( k\mu_x^{ns} = .54 \), and \( k\mu_x^s = .6 \), determine \( c \)