Solutions to L-TAM Module 1 Section 5 Exercises

1. We have $q_{50} = .02$, $q_{51} = .025$, and $q_{52} = .03$.

(a) Working forward through the 2-year select period, we have:

$l_{[50]+1} = l_{[50]} \cdot p_{[50]}$ and $l_{[50]+2} = l_{52} = l_{[50]+1} \cdot p_{[50]+1}$

So if we knew l_{52} then $l_{[50]+1} = \frac{l_{52}}{1-q_{[50]+1}} = \frac{l_{52}}{1-.9q_{51}} = \frac{l_{52}}{.9775}$

Then $l_{[50]} = \frac{l_{[50]+1}}{1-q_{[50]}} = \frac{l_{[50]+1}}{1-.8q_{50}} = \frac{l_{[50]+1}}{.984}$

We can use ultimate rates and $l_{50} = 1000$ to work forward to get l_{52} as follows:

$l_{51} = l_{50} \cdot p_{50} = 1000(.98) = 980$ and $l_{52} = l_{51} \cdot p_{51} = 980(.975) = 955.5$.

Therefore, $l_{[50]+1} = \frac{955.5}{.9775}$ and $l_{[50]} = \frac{955.5}{(.9775)(.984)} \approx 993.38781$

(b) We use the same relationships as in part (a). Namely,

$l_{[50]+1} = l_{[50]} \cdot p_{[50]} = 1000\big(1 - .8(.02)\big) = 1000(.984) = 984$ and

$l_{[50]+2} = l_{52} = l_{[50]+1} \cdot p_{[50]+1} = 984\big(1 - .9(.025)\big) = 984(.9775) = 961.86$

Then $l_{51} = \frac{l_{52}}{p_{51}} = \frac{961.86}{.975}$ and $l_{50} = \frac{l_{51}}{p_{50}} = \frac{961.86}{(.975)(.98)} \approx 1006.65620$

(c) If we start with $l_{[50]} = 1000$, then as we saw in part (b) we have $l_{52} = 961.86$
Then $l_{53} = l_{52} \cdot p_{52} = 961.86(.97) = 933.0042$. The probability we seek is

$${}_{2|}q_{[50]} = \frac{l_{52} - l_{53}}{l_{[50]}} = .0288558$$

(d) If we start with $l_{[50]} = 1000$, then as we saw in previous parts of this problem, we have $l_{[50]+1} = 984$, $l_{52} = 961.86$, and $l_{53} = 933.0042$. The probability we seek is

$${}_{1.4|1.3}q_{[50]} = \frac{l_{[50]+1.4} - l_{52.7}}{l_{[50]}}$$

$l_{[50]+1.4} = l_{([50]+1)+.4} \overset{UDD}{=} .6 \cdot l_{[50]+1} + .4 \cdot l_{52} = .6(984) + .4(961.86) = 975.144$ and

$l_{52.7} \overset{UDD}{=} .3(961.86) + .7(933.0042) = 941.66094$, and so ${}_{1.4|1.3}q_{[50]} \overset{UDD}{\approx} .033483$

(e) We use all the same integer age values as in part (d). We just have to adjust the fractional age values by using the CF assumption.

$l_{[50]+1.4} = l_{([50]+1)+.4} \overset{CF}{=} (l_{[50]+1})^{.6} \cdot (l_{52})^{.4} = (984)^{.6} \cdot (961.86)^{.4}$ and

$l_{52.7} \overset{CF}{=} (961.86)^{.3} \cdot (933.0042)^{.7}$, and so ${}_{1.4|1.3}q_{[50]} = \frac{l_{[50]+1.4} - l_{52.7}}{l_{[50]}} \overset{CF}{\approx} .033515$

(f) Since there's a 2-year select period, recall the 2-year recursion for e_{50}; namely,

$e_{50} = p_{50} + {}_2p_{50}(1 + e_{52})$. Applied in this context to a selected 50-year old, we have

$e_{[50]} = p_{[50]} + {}_2p_{[50]}(1 + e_{52})$ Since $p_{[50]} = .984$ and $p_{[50]+1} = .9775$, then

$e_{[50]} = .984 + .984(.9775)(1 + 9) = 10.6026$. Finally, recall that $\overset{o}{e}_x \overset{UDD}{=} e_x + .5$. This formula is valid in the context of select and ultimate rates too, so $\overset{o}{e}_{[50]} = 11.1026$.

2. Since in all 3 parts we seek a probability associated to a person selected at age 60, let's start with $l_{[60]} = 100{,}000$. Then

   $l_{[60]+1} = l_{[60]} \cdot p_{[60]} = 100{,}000(.91) = 91{,}000,$
   $l_{[60]+2} = l_{[60]+1} \cdot p_{[60]+1} = 910(.89) = 80{,}990,$
   $l_{[60]+3} = l_{63} = l_{[60]+2} \cdot p_{[60]+2} = 80{,}990(.87) = 70{,}461.3$, and
   $l_{64} = l_{63} \cdot p_{63} = 70{,}461.3(.85) = 59{,}892.105$

   (a) ${}_{2|}q_{[60]} = \frac{l_{[60]+2} - l_{63}}{l_{[60]}} = .105287$

   (b) ${}_{3|}q_{[60]} = \frac{l_{63} - l_{64}}{l_{[60]}} \approx .105692$

   (c) ${}_{1|2}q_{[60]+1} = \frac{l_{[60]+2} - l_{64}}{l_{[60]+1}} = .231845$

3. With a 1-year select period, we have the 1-year recursions $e_{[80]} = p_{[80]}(1 + e_{81})$ and $e_{[81]} = p_{[81]}(1 + e_{82})$. From the first recursion we get $e_{81} = \frac{e_{[80]}}{p_{[80]}} - 1 = \frac{800}{91} - 1 = \frac{709}{91}$. Knowing e_{81}, we can get e_{82} from the (ultimate rates) 1 year recursion $e_{81} = p_{81}(1 + e_{82})$; namely, $(1 + e_{82}) = \frac{e_{81}}{p_{81}} = \frac{709/91}{83/91} = \frac{709}{83}$. We could subtract 1 from both sides of the last equation to get e_{82}, but notice that what we really need in the second recursion formula in the first sentence is $1 + e_{82}$. From that recursion formula we get $e_{[81]} = p_{[81]}(1 + e_{82}) = \frac{83}{92}\left(\frac{709}{83}\right) = \frac{709}{92}$.