Solutions to L-TAM Module 1 Section 7 Exercises

1. Recognize this as a constant force model. For this problem, we would say the future lifetime of (x) follows a CF($\mu = .02$) model, or equivalently, T_x has an exponential distribution with mean $\frac{1}{\mu} = \frac{1}{.02} = 50$.

   (a) $\overset{o}{e}_x = E[T_x] = 50$

   (b) $e_x = E[K_x] = p + p^2 + \cdots = \frac{p}{1-p} = \frac{e^{-.02}}{1-e^{-.02}}$

2. Recognize this as the same problem as Number 1. For this problem, we would say the future lifetime of ($\overline{xy}$) follows a CF($\mu = .02$) model, or equivalently, $T_{\overline{xy}}$ has an exponential distribution with mean $\frac{1}{\mu} = \frac{1}{.02} = 50$.

   (a) $\overset{o}{e}_{\overline{xy}} = E\left[T_{\overline{xy}}\right] = 50$

   (b) $e_{\overline{xy}} = E\left[K_{\overline{xy}}\right] = p + p^2 + \cdots = \frac{p}{1-p} = \frac{e^{-.02}}{1-e^{-.02}}$

3. Note that $T_{\overline{xy}} = Max(T_x, T_y)$. We solve the system of two equations and two unknowns to solve for T_x and T_y. Although not necessary, it will make the equations cleaner to look at if we make the substitutions $a = T_x$ and $b = T_y$. The two equations become $a + b = 40$ and $ab = 346.71$. Using the substitution method, we get $a(40 - a) = 346.71$, or equivalently, $a^2 - 40a + 346.71 = 0$. Solving this quadratic gives $a = 12.7(= T_x)$ or $a = 27.3(= T_x)$. If $T_x = 12.7$ then $T_y = 27.3$. On the other hand, if $T_x = 27.3$ then $T_y = 12.7$. Other than this, we don't have enough information to determine which is the case. However, we seek $T_{\overline{xy}} = Max(T_x, T_y)$, and so regardless of which is the case, $T_{\overline{xy}} = 27.3$.

4. (See Video Solution)

   (a) $\overset{o}{e}_{50:\overline{10|}} = \frac{65}{7}$

   (b) $e_{50:\overline{10|}} = \frac{129}{14}$

5. We have ${}_tp_x = {}_tp = e^{-.025t}$

   (a) $\overset{o}{e}_{x:\overline{5|}} = \int_0^5 e^{-.025t}\, dt = \frac{1}{.025}\left(1 - e^{-.025(5)}\right) = 40(1 - e^{-.125}) \approx 4.7$

   (b) $e_{x:\overline{5|}} = \sum_{k=1}^{5} {}_kp_x = p + p^2 + p^3 + p^4 + p^5 = \frac{p-p^6}{1-p} = \frac{e^{-.025} - e^{-.025(6)}}{1-e^{-.025}} \approx 4.64162$

6. (See Video Solution) $e_{xy:\overline{20|}} \approx 16.35143$

7. We have $p_{80} = .95$ and $p_{81} = .90$.

(a) $e_{80:\overline{2}|} = p_{80} + {}_2p_{80} = p_{80} + p_{80} \cdot p_{81} = .95(1 + .9) = 1.805$

(b) A 2-year recursion for e_{80} is $e_{80} = p_{80} + {}_2p_{80}(1 + e_{82})$. Then we get $6.08 = .95 + (.95)(.90)(1 + e_{82}) \Rightarrow e_{82} = 5$.

As a remark, note that we could have written the 2-year recursion formula as $e_{80} = (p_{80} + {}_2p_{80}) + {}_2p_{80} \cdot e_{82} = e_{80:\overline{2}|} + {}_2p_{80} \cdot e_{82}$. Written this way, we could have used part (a) as follows: $6.08 = 1.805 + (.95)(.90)e_{82} \Rightarrow e_{82} = 5$

8. (a) $\overset{o}{e}_{30} = E[T_{30}] = \frac{12.7+8.6+26.3+47.9+34.5}{5} = 26$

(b) The corresponding K_{30} values are: 12, 8, 26, 47, 34

$e_{30} = E[K_{30}] = \frac{12+8+26+47+34}{5} = 25.4$

(c) Since $T_{30:\overline{10}|} = Min(T_{30}, 10)$, the $T_{30:\overline{10}|}$ values are: 10, 8.6, 10, 10, 10

$\overset{o}{e}_{30:\overline{10}|} = E\left[T_{30:\overline{10}|}\right] = \frac{10+8.6+10+10+10}{5} = 9.72$

(d) Since $K_{30:\overline{10}|} = Min(K_{30}, 10)$, the $K_{30:\overline{10}|}$ values are: 10, 8, 10, 10, 10

$e_{30:\overline{10}|} = E\left[K_{30:\overline{10}|}\right] = \frac{10+8+10+10+10}{5} = 9.6$

(e) Since $T_{30:\overline{30}|} = Min(T_{30}, 30)$, the $T_{30:\overline{30}|}$ values are: 12.7, 8.6, 26.3, 30, 30

$\overset{o}{e}_{30:\overline{30}|} = E\left[T_{30:\overline{30}|}\right] = \frac{12.7+8.6+26.3+30+30}{5} = 21.52$

(f) Since $K_{30:\overline{30}|} = Min(K_{30}, 30)$, the $K_{30:\overline{30}|}$ values are: 12, 8, 26, 30, 30

$e_{30:\overline{30}|} = E\left[K_{30:\overline{30}|}\right] = \frac{12+8+26+30+30}{5} = 21.2$

(g) Since 4 of the 5 T_{30} values are greater than 10, ${}_{10}p_{30} = \Pr(T_{30} > 10) = \frac{4}{5} = .8$

9. (a) the four T_{40} values are: 2.7, 16.3, 37.9, 24.5

(b) $\overset{o}{e}_{40} = E[T_{40}] = \frac{2.7+16.3+37.9+24.5}{4} = 20.35$

(c) The corresponding K_{40} values are: 2, 16, 37, 24

$e_{40} = E[K_{40}] = \frac{2+16+37+24}{4} = 19.75$

(d) Since $T_{40:\overline{20|}} = Min(T_{40}, 20)$, the $T_{40:\overline{20|}}$ values are: 2.7, 16.3, 20, 20

$\overset{o}{e}_{40:\overline{20|}} = E\left[T_{40:\overline{20|}}\right] = \frac{2.7+16.3+20+20}{4} = 14.75$

(e) Since $K_{40:\overline{20|}} = Min(K_{40}, 20)$, the $K_{40:\overline{20|}}$ values are: 2, 16, 20, 20

$e_{40:\overline{20|}} = E\left[K_{40:\overline{20|}}\right] = \frac{2+16+20+20}{4} = 14.5$

10. Plug in the corresponding values and verify:

(a) $\overset{o}{e}_{30} \overset{?}{=} \overset{o}{e}_{30:\overline{10|}} + {}_{10}p_{30} \cdot \overset{o}{e}_{40} \Rightarrow 26 \overset{?}{=} 9.72 + .8(20.35)$ YES

(b) $e_{30} \overset{?}{=} e_{30:\overline{10|}} + {}_{10}p_{30} \cdot e_{40} \Rightarrow 25.4 \overset{?}{=} 9.6 + .8(19.75)$ YES

(c) $\overset{o}{e}_{30:\overline{30|}} \overset{?}{=} \overset{o}{e}_{30:\overline{10|}} + {}_{10}p_{30} \cdot \overset{o}{e}_{40:\overline{20|}} \Rightarrow 21.52 \overset{?}{=} 9.72 + .8(14.75)$ YES

(d) $e_{30:\overline{30|}} \overset{?}{=} e_{30:\overline{10|}} + {}_{10}p_{30} \cdot e_{40:\overline{20|}} \Rightarrow 21.2 \overset{?}{=} 9.6 + .8(14.5)$ YES