L-TAM Module 1 Section 8 Exercises

Unless told or implied otherwise, assume all lives are independent.

For numbers 1 and 2, suppose deaths for males is uniformly distributed over the interval [0, 120] and female mortality follows an exponential distribution with mean 40.

1. Determine ${}_{30}q_{40:45}$ where (40) is female and (45) is male.

2. If (40) is male and (45) is female, determine

   (a) ${}_{10|30}q_{\overline{40:45}}$

   (b) ${}_{10}p_{\overline{40:45}}$

   (c) ${}_{30}q_{\overline{50:55}}$ where (50) is male and (55) is female

   (d) Is ${}_{10|30}q_{\overline{40:45}} = {}_{10}p_{\overline{40:45}} \cdot {}_{30}q_{\overline{50:55}}$? (Comparing (a), (b), and (c).)

   (e) ${}_{t}p_{40:45}$ for $t < 80$

   (f) ${}_{t}p_{40:45}$ for $t > 80$

   (g) $\overset{o}{e}_{40:45}$

   (h) $\overset{o}{e}_{\overline{40:45}}$

   (i) ${}_{20}q_{40:45}$

   (j) ${}_{20}q_{\overset{1}{40}:45}$

   (k) ${}_{20}q_{40:\overset{1}{45}}$

   (l) ${}_{10}q_{\overline{40:45}}$

   (m) ${}_{10}q_{\overset{2}{40}:45}$

   (n) ${}_{10}q_{40:\overset{2}{45}}$

   (o) ${}_{30}q_{\overset{1}{40}:45} - {}_{30}q_{40:\overset{2}{45}}$

3. Suppose both smoker and non-smoker mortality follow a constant force model with the force of mortality for smokers, μ, being twice the force of mortality for non-smokers. You are given that $\mu > 0.05$.

   (a) Given ${}_{20|20}q_{30:50} = 0.09$ where (30) is a non-smoker and (50) is smoker, determine μ.

   (b) Repeat (a) if the non-smoker is 50 years old and the smoker is 30 years old.

4. Given CF(μ) mortality for all lives,

   (a) determine ${}_tp_{xy}$ (in terms of μ)

   (b) Compare your answer in (a) to ${}_tp_x$ with CF(2μ) mortality

   (c) determine $\overset{o}{e}_{xy}$

   (d) determine $\overset{o}{e}_{\overline{xy}}$

5. Given DML(ω) mortality for all lives

   (a) determine ${}_tp_{xx}$

   (b) Compare your answer in (a) to ${}_tp_x$ with GDML(ω,2) mortality

   (c) determine $\overset{o}{e}_{xx}$

   (d) determine $\overset{o}{e}_{\overline{xx}}$

6. Given DML(100) mortality

   (a) determine ${}_tp_{70:70}$

   (b) determine $\overset{o}{e}_{70:70}$

   (c) determine $\overset{o}{e}_{70:70:\overline{20|}}$

7. For a common shock model with $\lambda = .01$, in the absence of the shock the future lifetimes of (x) and (y) follow constant force models with $\mu_x^* = .02$ and $\mu_y^* = .04$. Determine

   (a) the probability that (x) dies within 10 years

   (b) the expected age at which (y) dies

8. For a common shock model, in the absence of the shock the future lifetime of (x) and (y) follow constant force models. The expected time until the death of (x) in the absence of the shock is equal to the expected time until the death of (y) in the presence of the shock. If the hazard rate (i.e. force of mortality) for the joint life status is 0.20, determine the probability that (x) dies first and within the next 5 years.