1) (See Video Solution)

$${}_{30}q_{40:45} = .716580$$

2) Mortality for (40) follows DML(120)

(45) ——— CF($\mu = .025$)

(a) $${}_{10|30}q_{\overline{40:45}} = {}_{40}q_{\overline{40:45}} - {}_{10}q_{\overline{40:45}}$$

$$= {}_{40}q_{40} \cdot {}_{40}q_{45} - {}_{10}q_{40} \cdot {}_{10}q_{45}$$

$$= \left(\frac{40}{120-40}\right)\left(1-e^{-40(.025)}\right) - \left(\frac{10}{120-40}\right)\left(1-e^{-10(.025)}\right) \doteq .28841$$

(b) $${}_{10}p_{\overline{40:45}} = 1 - {}_{10}q_{\overline{40:45}} = 1 - {}_{10}q_{40} \cdot {}_{10}q_{45}$$

$$= 1 - \left(\frac{10}{120-40}\right)\left(1-e^{-10(.025)}\right) \doteq .97235$$

(c) $${}_{30}q_{\overline{50:55}} = {}_{30}q_{50^m} \cdot {}_{30}q_{55^f} = \left(\frac{30}{120-50}\right)\left(1-e^{-30(.025)}\right)$$

$$\doteq .226129$$

(d) $${}_{10|30}q_{\overline{40:45}} \neq {}_{10}p_{\overline{40:45}} \cdot {}_{30}q_{\overline{50:55}}$$

Generally, ${}_{k|n}q_{\overline{xy}} \neq {}_{k}p_{\overline{xy}} \cdot {}_{n}q_{\overline{x+k:y+k}}$ since ${}_{k}p_{\overline{xy}}$ does not imply that both (x) and (y) survive to ages x+k and y+k. We can not discuss ${}_{n}q_{\overline{x+k:y+k}}$ unless we are sure to have x+k and y+k year olds.

2)

e) $_tP_{40:45} = {_tP_{40}} \cdot {_tP_{45}}$

For $t < 80$, $_tP_{40} = \frac{120-40-t}{120-40} = \frac{80-t}{80}$

$_tP_{45} = e^{-.025t}$

$\therefore$ for $t < 80$, $_tP_{40:45} = \frac{80-t}{80} \cdot e^{-.025t}$

(f) For $t > 80$ $_tP_{40} = 0$ since $\omega = 120$.

$\therefore$ for $t > 80$ $_tP_{40:45} = {_tP_{40}} \cdot {_tP_{45}} = 0$

(g) $\mathring{e}_{40:45} = \int_0^\infty {_tP_{40:45}} = \int_0^{80} \frac{80-t}{80} \cdot e^{-.025t}\, dt$

$u = \frac{80-t}{80}$ $\quad v = -40e^{-.025t}$

$du = -\frac{1}{80}dt$ $\quad dv = e^{-.025t}\, dt$

$$\therefore \mathring{e}_{40:45} = \underline{\left(\frac{80-t}{80}\right)\left(-40e^{-.025t}\right)\Big|_0^{80}} - \int_0^{80}\left(-40e^{-.025t}\right)\cdot\frac{-1}{80}\,dt$$

$$= \underline{40} - \frac{1}{2}\int_0^{80} e^{-.025t}\, dt$$

$$= 40 - \frac{1}{2}\cdot\frac{1}{.025}e^{-.025t}\Big|_{80}^{0} = 40 - 20(1-e^{-2})$$

$$= 20 + 20e^{-2} = 22.706706$$

(h) $\mathring{e}_{\overline{40:45}}$: (See Video Solution)

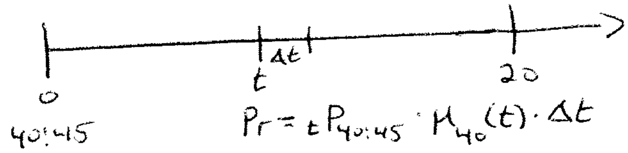
$$\mathring{e}_{\overline{40:45}} = 60 - 20e^{-2} = 57.293294$$

2)

(i) $${}_{20}q_{40:45} = 1 - {}_{20}p_{40:45} = 1 - {}_{20}p_{40} \cdot {}_{20}p_{45}$$

$$= 1 - \left(\frac{120-40-20}{120-40}\right) \cdot e^{-20(.025)} = .545102$$

(j) ${}_{20}q^{\;1}_{40:45}$:



$$\therefore\ {}_{20}q^{\;1}_{40:45} = \int_0^{20} {}_t p_{40:45} \cdot \mu_{40}(t)\, dt = \int_0^{20} \underline{{}_t p_{40}} \cdot {}_t p_{45} \cdot \underline{\mu_{40}(t)}\, dt$$

<u>Note</u>: mortality for (40) follows DML(120)

$$\Longrightarrow {}_t p_{40} \cdot \mu_{40}(t) = q_{40} = \frac{1}{120-40} = \frac{1}{80}$$

$$\therefore\ {}_{20}q^{\;1}_{40:45} = \int_0^{20} \frac{1}{80} \cdot {}_t p_{45}\, dt = \frac{1}{80} \int_0^{20} e^{-.025t}\, dt$$

$$= \frac{1}{80} \cdot \frac{1}{.025} e^{-.025t} \Big|_{20}^{0} = \frac{1}{2}\left(1 - e^{-.5}\right) = .196735$$

(k) ${}_{20}q^{\;\;1}_{40:45}$: (See Video Solution)

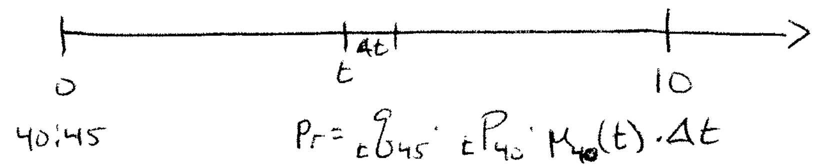
$${}_{20}q^{\;\;1}_{40:45} = \frac{1}{2} - \frac{1}{4} e^{-.5} = .348367$$

2)

(l) $_{10}q_{\overline{40:45}} = {}_{10}q_{40} \cdot {}_{10}q_{45} = \frac{10}{120-40} \cdot \left(1 - e^{-.025(10)}\right)$

$= \frac{1}{8}\left(1 - e^{-.25}\right) = .027650$

(m) $_{10}q^{2}_{40:45}$:



$\therefore \; {}_{10}q^{2}_{40:45} = \int_0^{10} {}_tq_{45} \cdot \underline{{}_tp_{40} \cdot \mu_{40}(t)}\, dt$

Note: mortality for (40) follows DML (120)

$\Longrightarrow \; {}_tp_{40} \cdot \mu_{40}(t) = q_{40} = \frac{1}{120-40} = \frac{1}{80}$

$\therefore \; {}_{10}q^{2}_{40:45} = \int_0^{10} \left(1 - e^{-.025t}\right) \cdot \frac{1}{80}\, dt$

$= \frac{1}{80}\left[10 + 40e^{-.025t}\Big|_0^{10}\right] = \frac{1}{80}\left[40e^{-.25} - 30\right]$

$= \frac{1}{2}e^{-.25} - \frac{3}{8} = .014400$

(n) $_{10}q_{40:\overset{2}{45}}$: (See Video Solution)

$_{10}q_{40:\overset{2}{45}} = \frac{1}{2} - \frac{5}{8}e^{-.25} = .013250$

(o) ~~See Video~~

$_{30}q^{1}_{40:45} - {}_{30}q_{40:\overset{2}{45}} = {}_{30}q_{40} \cdot {}_{30}p_{45} = \frac{30}{120-40} \cdot e^{-30(.025)}$

$= \frac{3}{8}e^{-.75} = .177137$

3) $\mu = \mu^{s} \qquad \mu^{ns} = \frac{1}{2}\mu$

(a) $${}_{20|20}q_{30:50} = {}_{20}p_{30:50} - {}_{40}p_{30:50}$$

$$= {}_{20}p_{30} \cdot {}_{20}p_{50} - {}_{40}p_{30} \cdot {}_{40}p_{50}$$

$$= \underline{e^{-20(\frac{1}{2}\mu)} \cdot e^{-20(\mu)}} - \underline{\underline{e^{-40(\frac{1}{2}\mu)} \cdot e^{-40(\mu)}}}$$

$$\therefore \quad .09 = \underline{e^{-30\mu}} - \underline{\underline{e^{-60\mu}}} \quad \left(\text{quadratic in } e^{-30\mu}\right)$$

$a = -1$
$b = 1$
$c = -.09$

$$\therefore \quad e^{-30\mu} = \frac{-1 \overset{*}{\pm} \sqrt{1-4(-1)(-.09)}}{2(-1)} = \frac{+1+\sqrt{.64}}{+2} = .9$$

$$\Longrightarrow \mu = \frac{\ln(.9)}{-30} = .003512$$

(b) As in (a),

$$ {}_{20|20}q_{30:50} = {}_{20}p_{30} \cdot {}_{20}p_{50} - {}_{40}p_{30} \cdot {}_{40}p_{50}$$

$$= e^{-20(\mu)} \cdot e^{-20(\frac{1}{2}\mu)} - e^{-40(\mu)} \cdot e^{-40(\frac{1}{2}\mu)}$$

$$= e^{-30\mu} - e^{-60\mu}$$

We get the same equation and the same answer as in part (a). Note that this won't always be the case; it just so happened in this problem.

4)

(a) $_tP_{xy} = {_tP_x} \cdot {_tP_y} = e^{-\mu t} \cdot e^{-\mu t} = e^{-2\mu t} \Rightarrow T_{xy} \sim Exp(\text{mean} = \frac{1}{2\mu}$

(b) Using CF(2μ) mortality, $_tP_x = e^{-2\mu t}$ also.

(c) $\mathring{e}_{xy} = \frac{1}{2\mu}$ since $T_{xy} \sim Exp(\text{mean} = \frac{1}{2\mu})$

(d) $\mathring{e}_{\overline{xy}} = \mathring{e}_x + \mathring{e}_y - \mathring{e}_{xy} = \frac{1}{\mu} + \frac{1}{\mu} - \frac{1}{2\mu} = \frac{2+2-1}{2\mu} = \frac{3}{2\mu}$

5) (See Video Solution)

(a) $_tP_{xx} = \left(\frac{\omega - x - t}{\omega - x}\right)^2$

(b) $_tP_x \overset{GDML(\omega,2)}{=\!=} \left(\frac{\omega - x - t}{\omega - x}\right)^2$

(c) $\mathring{e}_{xx} = \frac{\omega - x}{3}$

(d) $\mathring{e}_{\overline{xx}} = \frac{2(\omega - x)}{3}$

6) (a) $_tP_{70:70} = ({_tP_{70}})^2 = \left(\frac{100-70-t}{100-70}\right)^2 = \left(\frac{30-t}{30}\right)^2$ Note $0 \le t \le 30$

(b) $\mathring{e}_{70:70} = \int_0^{30} \left(\frac{30-t}{30}\right)^2 dt = \frac{30}{3}\left(\frac{30-t}{30}\right)^3 \Big|_{30}^{0} = 10$

Note: (See #5(c)) $\mathring{e}_{70:70} \overset{DML(100)}{=\!=} \frac{100-70}{3} = \frac{30}{3} = 10$

(c) $\mathring{e}_{70:70:\overline{20|}} = \int_0^{20} \left(\frac{30-t}{30}\right)^2 dt = \frac{30}{3}\left(\frac{30-t}{30}\right)^3 \Big|_{20}^{0}$

$= 10(1)^3 - 10\left(\frac{1}{3}\right)^3 = 10\left(1 - \frac{1}{27}\right) = \frac{260}{27}$

7) $\mu_x^* = .02 \quad \mu_y^* = .04 \quad \lambda = .01 \Longrightarrow \mu_x = .03 \quad \mu_y = .05 \quad \mu_{xy} = .07$

(a) ${}_{10}q_x = 1 - {}_{10}p_x \stackrel{CF}{=} 1 - e^{-10(.03)} = 1 - e^{-.3}$

(b) expected age when (y) dies $= y + E[T_y] = y + \mathring{e}_y$

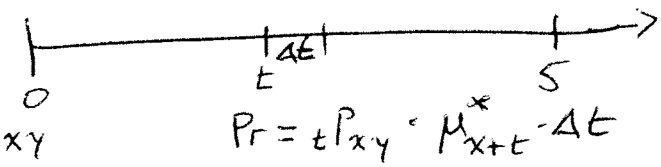
$$\mathring{e}_y \stackrel{CF}{=} \frac{1}{\mu_y} = \frac{1}{.05} = 20$$

$\therefore$ expected age when (y) dies $= y + 20$

8) Given $\underbrace{E[T_x^*]}_{=\frac{1}{\mu_x^*}} = \underbrace{E[T_y]}_{=\frac{1}{\mu_y} = \frac{1}{\mu_y^* + \lambda}} \Longrightarrow \mu_x^* = \underline{\mu_y^* + \lambda}$

$$.2 = \mu_{xy} = \mu_x^* + \underline{\mu_y^* + \lambda} = 2\mu_x^* \Longrightarrow \mu_x^* = .1$$

We seek ${}_5q^1_{xy}$:



$Pr = {}_tp_{xy} \cdot \mu^*_{x+t} \cdot \Delta t$

↳ We use μ^* and not μ since we seek the probability that (x) dies before (y)

$$\therefore {}_5q^1_{xy} = \int_0^5 {}_tp_{xy} \cdot \mu^*_{x+t}\, dt = \int_0^5 e^{-.2t}(.1)\, dt$$

$$= \frac{.1}{.2} e^{-.2t}\Big|_5^0 = \frac{1}{2}(1 - e^{-1})$$